DE PERFECTIONE TELESCOPIORVM SECVNDI GENERIS SEV ASTRONOMICORVM, VNICAM IMAGINEM REALEM CONTINENTIVM.

Auctore

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§. 1.

Cum vnica lens ocularis non sufficiat, vt margo coloratus tolli possit, statim duas lentes oculares introducamus, ita, vt cum lente obiectiua vicaria tres habeamus lentes, ex eadem vitri specie, puta coronaria formatas, quarum distantiae focales sint p, q et r, ideoque $p = \Phi$ et semidiameter aperturae primae lentis $= X$, at semidiameter aperturae secundae lentis $= \pi q$, tertiae vero $= \pi' r$; vbi litterae π et π' denotant denotant fractiones, siue positiuas, siue negatiuas, quadrantem vnitatis non superantes; vnde pro campo apparente et multiplicatione $= m$ statim habebimus, ~~semidiametrum~~ campi $\Phi = \frac{\pi - \pi'}{m + 1}$. Hinc autem si ξ denotet maximam fractionem, quam litterae π et π' habere possunt, ponamus breuitatis gratia $\frac{\pi - \pi'}{m + 1} = M\xi$.

§. 2. Sint iam pro nostris tribus lentibus, distantiae determinatrices a, α; b, $\mathfrak{b}$; et c, γ eritque

1°. $a = \infty$; $\alpha = p = \Phi$ et $\gamma = \infty$

hincque fiet

$\frac{1}{q} = \frac{1}{b} + \frac{1}{\mathfrak{b}}$ et $\frac{1}{r} = \frac{1}{c} + \frac{1}{\gamma}$ siue $r = c$,

hinc autem porro sequentes litterae definiantur

$P = -\frac{\alpha}{b}$ et $Q = -\frac{\mathfrak{b}}{c}$

vnde pro data multiplicatione $= m$, ob situm inuersum erit $PQ = -m$, ita, vt litterarum P et Q altera debeat esse positiua, altera negatiua; praeterea vero ponatur

$B = \frac{\mathfrak{b}}{b}$ et $C = \frac{\gamma}{c} = \infty$ vnde fit

$\mathfrak{B} = \frac{B}{1 + B}$ et $\mathfrak{C} = \frac{C}{1 + C} = 1$,

hinc autem vicissim erit

$b = -\frac{\alpha}{P}$, $\mathfrak{b} = -\frac{B\alpha}{P}$; $c = +\frac{B\alpha}{PQ} = -\frac{B\alpha}{m}$

hincque porro

$q = \mathfrak{B} b = -\frac{\mathfrak{B}\alpha}{P}$ et $r = \mathfrak{C} c = -\frac{B\alpha}{m}$

ex quibus valoribus deducuntur interualla lentium

I°. ; $\text{I} - \text{II} = \alpha + b = \alpha(1 - \frac{1}{P})$ et

II°; $\text{II} - \text{III} = \mathfrak{b} + c = -B\alpha(\frac{1}{P} + \frac{1}{m})$

quae ambo necessario debent esse positiua.

§. 3. Iam consideremus campum adparentem; quem vt reddamus maximum statim sumamus

$\pi = \xi$ et $\pi' = -\xi$

vt fiat

$\Phi = \frac{2\xi}{m + 1}$ ideoque $M = \frac{2}{m + 1}$.

 qui

qui valor cum in partibus radii fit expreſſus, ob radium $1 = 3437$ min. ſumto $\xi = \frac{1}{2}$ erit $\Phi = \frac{1718}{m+1}$ minutis primis; nunc vero, pro margine colorato deſtruendo ſatisfieri opportet huic aequationi

$$0 = \frac{\pi}{P} - \frac{\pi'}{PQ} \text{ ſiue } \frac{1}{P} + \frac{1}{PQ} = 0$$

vnde fit

$$Q = -1 \quad \text{et} \quad P = m$$

hinc autem deductae ſunt iſtae aequationes

$$\frac{\mathfrak{B}\pi - \Phi}{\Phi} = -P \text{ et } \frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = PQ \text{ quae ob } \pi = \xi;\ \pi' = -\xi$$

et $\Phi = M\xi$ abeunt in has

$$\frac{\mathfrak{B} - M}{M} = -P \text{ et } -\frac{\mathfrak{C} - 1 + M}{M} = PQ = -m$$

quae poſterior, ob $\mathfrak{C} = 1$ praebet $M = \frac{2}{1+m}$ prorſus vt ante; ex illa vero reperitur

$$\mathfrak{B} = (1-P)M = -\frac{2(m-1)}{m+1}$$

vnde fit

$$B = \frac{\mathfrak{B}}{1-\mathfrak{B}} = -\frac{2(m-1)}{3m-1}$$

denique diſtantia oculi poſt vltimam lentem eſt

$$\frac{r}{Mm} = \frac{m+1}{2m}r.$$

§. 4. Nunc igitur omnia elementa, quibus conſtructio teleſcopii continetur, penitus ſunt determinata, quae ita ſe habebunt

$$\alpha = \Phi;\ b = -\frac{\alpha}{m};\ \mathfrak{c} = +\frac{2(m-1)\alpha}{m(3m-1)};\ c = \frac{2(m-1)\alpha}{m(3m-1)}$$

vnde ſtatim prodeunt interualla lentium

$$\mathrm{I} - \mathrm{II} = \alpha\left(1 - \frac{1}{m}\right);\ \mathrm{II} - \mathrm{III} = \frac{4(m-1)\alpha}{m(3m-1)}$$

diſtan-

diſtantiae denique focales erunt

$$p = \alpha;\ q = \frac{2(m-1)\alpha}{m(m+1)} \text{ et } r = \frac{2(m-1)\alpha}{m(3m-1)}.$$

§. 5. Formula autem pro confuſione tollenda quae ex apertura lentium oritur, ſi λ, λ' et λ'' denotent numeros arbitrarios, ſingulis lentibus reſpondentes, ita ſe habet

$$\lambda - \frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) + \frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)$$

vbi eſt vti vidimus

$$P = m \text{ et } PQ = -m,\ \mathfrak{B} = -\frac{2(m-1)}{m+1};\ B = -\frac{2(m-1)}{3m-1};$$
$$\mathfrak{C} = 1 \text{ et } C = \infty;$$

Quibus valoribus ſubſtitutis prodit iſta formula

$$\lambda + \frac{1}{m}\left(\frac{\lambda'(m+1)^3}{8(m-1)^3} - \frac{\nu(m+1)(3m-1)}{4(m-1)^2}\right) + \frac{\lambda''(3m-1)^3}{8m(m-1)^3}.$$

Hinc ergo confuſio ex ſecunda et tertia lente nata quam littera Ω ſumus complexi erit

$$\Omega = \frac{1}{m}\left(\frac{\lambda'(m+1)^3}{8(m-1)^3} - \frac{\nu(m+1)(3m-1)}{4(m-1)^2}\right) + \frac{\lambda''(3m-1)^3}{8m(m-1)^3}.$$

Quia autem his lentibus maximam aperturam tribuimus, cuius ſint capaces, numeri λ' et λ'' ex his formulis definiri debent

$$\sqrt{\lambda' - 1} = \frac{(\sigma - \rho)}{\tau}\left(\mathfrak{B} - \frac{1}{2}\right) \text{ et } \sqrt{\lambda'' - 1} = \left(\frac{\sigma - \rho}{\tau}\right)\frac{1}{2}$$

vbi pro vitro coronario eſt

$$l\frac{\sigma - \rho}{\tau} = 0,1901924 \text{ et } l\nu = 9,3412366$$

inuento autem hoc numero Ω, ſupra oſtendimus, quemadmodum lens compoſita ſiue duplicata ſiue triplicata, loco lentis primae ſubſtituenda, determinari debeat; quo facto teleſcopium omnibus numeris erit

abſo-

absolutum. Quia autem hic in genere nihil definire licet, casus aliquot pro datis multiplicationibus euolvamus.

Exemplum primum.

§. 6. Sit multiplicatio $m = 50$, erunt primo litterae

$$P = 50;\ Q = -1;\ \mathfrak{B} = -\tfrac{98}{51};\ \text{et } B = -\tfrac{98}{149}$$

hincque

$$l\mathfrak{B} = (-)0,2836559;\ lB = 9,8180389(-)$$

hinc igitur erunt distantiae determinatrices

$$a = \Phi;\ b = -\tfrac{a}{50} = -0,020.a;\ \mathfrak{C} = 0,01316.a;$$
$$c = 0,01316\,a$$

vnde interualla lentium colliguntur

$$I - II = 0,980.a \text{ et } II - III = 0,02632.a$$

distantiae denique focales

$$p = a;\ q = 0,03843.a \text{ et } r = 0,01316.a.$$

Quia igitur binae lentes posteriores vtrinque sunt aeque conuexae, erit radius vtriusque faciei secundae lentis

$$= 1,06\,q = 0,04071\,a$$

et tertiae lentis

$$= 1,06\,r = 0,01394.a,$$

at locus oculi post lentem tertiam

$$= 0,00671.a,$$

denique semidiameter campi adparentis $= 33'.\ 41''$.

§. 7. Nunc igitur quaerantur numeri λ' et λ'', et quia est

$$\mathfrak{B} = -1,92157 \text{ erit } \mathfrak{B} - \tfrac{1}{2} = -2,42157$$

vnde calculus ita se habebit:

$l(\frac{\sigma-\rho}{\tau}) = 0,1901924$	$l(\frac{\sigma-\rho}{\tau}) = 0,1901924$
$l(\mathfrak{B}-\frac{1}{2}) = 0,3830971$	$l\,2 = 0,3010300$
$l\sqrt{\lambda'-1} = 0,5732895$	$l\sqrt{\lambda''-1} = 9,8891624$
$l(\lambda'-1) = 1,1465790$	$l(\lambda''-1) = 9,7783248$
hinc $\lambda' = 15,01455$	hincque $\lambda'' = 1,60024$

quia igitur est

$$l\tfrac{1}{p} = 8,3010300;\ l\mathfrak{B}^3 = 0,8509677(-)$$

et $lB\mathfrak{B} = 0,1016957$ tum vero $l\frac{1}{B^3 m} = 8,8469806$ hinc calculus pro littera Ω ita instituatur

$l\frac{1}{m} = 8,3010300$	$l\frac{1}{p} = 8,3010300$	$l\frac{1}{B^3 m} = 8,8469806$
$l\lambda' = 1,1765112$	$l\nu = 9,3412366$	$l\lambda'' = 0,2041851$
$9,4775412$	$7,6422666$	$9,0511657$
$l\mathfrak{B}^3 = 0,8509677$	$lB\mathfrak{B} = 0,1016957$	Pars III $= 0,11250$
$8,6265735$	$7,5405709$	
Pars I $= 0,04232$	Pars II $= -0,00347$	

vnde colligitur numerus $\Omega = 0,15135$.

§. 8. Adhibeamus statim lentem obiectiuam triplicatam postremam, vtpote perfectissimam, cuius distantia focalis sit $= \text{II}$, ac supra vidimus fore

$$\Phi = 1,02311\,\text{II} = a$$

et $X = x$; tum vero pro prima lente erit numerus arbitrarius

$$\lambda = 2,50862 - 0,08257\,\Omega = 2,49612,$$

vnde patet, ob confusionem ω, numerum λ tam parum immutari, vt effectus in constructione lentis prorsus euadat insensibilis, vnde tuto assumere poterimus

$$\lambda = 2,50862$$

ita vt huius lentes constructio futura sit

Radius faciei anterioris $= 0,85048\,\Pi$

Radius faciei posterioris $= 0,32689\,\Pi$.

Cum igitur hic sit $m = 50$, si capiamus

$$\Pi = 12\tfrac{1}{2} = 12,50 \text{ dig. erit } \alpha = 12,789 \text{ dig.}$$

hincque deducitur sequens.

CONSTRVCTIO

Tubi astronomici pro multiplicatione $m = 50$.

Tab. IV. Fig. 3.

1°. Lens igitur obiectiua est triplicata, distantiam focalem habens $= 12\tfrac{1}{2}$ dig. et aperturae semidiametrum $= 1$ dig.

1°. Eius prima lens coronaria distantiam focalem habet $= 5,569$ dig. et ita construatur vt sit

Radius faciei $\begin{cases} \text{anterioris} = 10,631 \text{ dig.} \\ \text{posterioris} = 4,086 \text{ dig.} \end{cases}$

2°. Ab huius medio, ad medium secundae statuatur interuallum $= 0,283$ dig.

3°. Secunda lens crystallina vtrinque aeque concaua, distantiam focalem habet $= -3,398$ dig. et radium vtriusque faciei $= -3,942$.

4°.

4°. Ab huius medio, vsque ad medium lentis tertiae statuatur interuallum $= 0,283$.

5°. Tertiae lentis coronariae distantia focalis est $= 5,505$ dig. et

Radius vtriusque faciei $= 5,836$ dig.

II. Ab hac lente vsque ad lentem quartam seu primam ocularem interuallum est $= 12,244$.

1°. Istius lentis coronariae vtrinque aequaliter conuexae distantia focalis $= 0,491$ dig.

Radius vtriusque faciei $= 0,520$ dig. et semidiameter aperturae $= 0,123$ dig.

2°. Hinc vsque ad lentem vltimam distantia $= 0,337$ digitor.

III. Haec autem lens coronaria vtrinque, aeque conuexa distantiam habet focalem $= 0,168$ dig.

Radium vtriusque faciei $= 0,178$ dig.

et semidiametrum aperturae $= 0,042$ dig.

Ab hac lente vsque ad oculum distantia $= 0,086$ dig.

Longitudo totius telescopii $= 13,516$.

Semidiameter campi adparentis $= 33\tfrac{1}{3}'$, qui apparebit instar spatii circularis in coelo, cuius radius $= 26°.3'$ ideoque diameter $= 52°.6'$.

§. 9. Circa tubum autem sequentia sunt notanda: primo quod lens ocularis prodierit nimis parva, id quod in praxi non satis commodum videtur; deinde lens penultima nimis videtur propinqua foco lentis obiectiuae, scilicet vnius tantum quadrantis digiti

giti; vnde verendum eſt, ne maculae vel ſtriae huius lentis cum repreſentatione obiecti miſceantur. Praeterea vero, vt totus campus apparens vbique aeque lucidus videatur, non ſufficit vt ſemidiameter huius lentis ſit $= \pi q$, ſed requiritur vt is ſit $= \pi q + \frac{x}{m}$. His igitur incommodis, vt remedium, afferatur de campo adparente aliquid eſt remittendum quod, ſit ſi loco π non valorem maximum ξ accipiamus, ſed tantum eius partem quandam, veluti $\pi = \frac{1}{2}\xi$, manente $\pi' = -\xi$ ſicque erit

$$\Phi = \frac{\frac{1}{2}\xi}{m+1} \text{ et } M = \frac{\frac{3}{2}}{m+1} = \frac{3}{2(m+1)}.$$

§. 10. Poſito igitur $\pi = \frac{1}{2}\xi$ et $\pi' = -\xi$, aequatio pro margine colorato tollendo erit

$$0 = \frac{1}{2P} + \frac{1}{PQ}, \text{ vnde fit } Q = -2 \text{ ergo } P = \frac{m}{2}$$

deinde reperitur

$$\frac{\pi}{\Phi} = \frac{m+1}{2} \text{ ideoque } \frac{\mathfrak{B}(m+1)}{2} - 1 = -\frac{m}{2} \text{ ideoque}$$

$$\mathfrak{B} = \frac{3 - \frac{3}{2}m}{m+1} = \frac{6 - 3m}{2(m+1)} \text{ hincque } B = \frac{6 - 3m}{5m - 4}$$

ſequentes igitur habebimus determinationes

$$b = -\frac{2\alpha}{m}; \quad \mathfrak{C} = \frac{(6m - 12)}{m(5m - 4)}\alpha; \quad c = \frac{(3m - 6)\alpha}{m(5m - 4)}$$

diſtantiae focales

$$p = \alpha; \quad q = \frac{(2m - 6)\alpha}{m(m+1)} \text{ et } r = \frac{(3m - 6)\alpha}{m(5m - 4)}$$

hinc autem, etſi ſecundae lenti conſulitur, tamen lens ocularis adhuc ſit minor quam ante, vnde hac emendatione ſuperſedebimus.

§. 11.

§. 11. Quia confuſio a lentibus ocularibus oriunda, vix quicquam in lente obiectiua immutat, vel ſaltem in praxi obſeruari nequit: conſtructionem horum teleſcopiorum pro omni multiplicatione in genere tradere poterimus. Vtamur igitur iterum lente triplicata poſtrema, cui tribuimus ſtatim diſtantiam focalem $\Pi = \frac{m}{4}$, et ſemidiametrum aperturae $= \frac{m}{50}$ dig. vnde colligimus

$$\Phi = \alpha = 0,25578\, m \text{ dig.}$$

qui numerus cum in ſequentibus lentibus vbique occurrit, ponamus breuitatis gratia

$$\vartheta = 0,25578 \text{ vt ſit } \alpha = \vartheta m \text{ et } l\vartheta = 9,4078666.$$

Quia igitur eſt $\frac{\alpha}{m} = \vartheta$, et ſecunda lens tanto interuallo ante primam imaginem conſtituitur, erit diſtantia a lente obiectiua vsque ad primam ocularem $= \frac{m}{4} - \vartheta$ digitis; tum vero haec lens ocularis habebit diſtantiam focalem $= \frac{2\vartheta(m-1)}{m+1}$, quae expreſſio, ob m numerum praegrandem, reducitur ad hanc $2\vartheta - \frac{4\vartheta}{m}$ quae in 1,06 ducta, dabit radium vtriusque faciei; deinde interuallum ab hac lente ad ipſam ocularem erit $\frac{4\vartheta(m-1)}{5m-1} = \frac{4}{5}\vartheta - \frac{16\vartheta}{25m}$ dig. tum vero diſtantia focalis vltimae lentis $= \frac{1}{3}\vartheta - \frac{4\vartheta}{9m}$, quae denuo in 1,06 ducta, praebet radium faciei huius lentis, poſt quam oculus collocari debet ad diſtantiam $= \frac{m+1}{3m}r = \frac{1}{3}\vartheta + \frac{2\vartheta}{9m}$, campi autem adparentis ſemidiameter erit $= \frac{1718}{m+1}$ min.

CONSTRVCTIO GENERALIS
horum telescopiorum pro multiplicatione quacunque $= m$.

I. Lens obiectiua constat ex tribus lentibus; habens distantiam focalem $= \frac{m}{4}$ dig. et aperturam semidiametri $= \frac{m}{50}$ dig. ternae autem eius lentes ita determinantur.

1°. Primae lentis coronariae distantia focalis sit $= 0,11137\,m$ dig.

Tum vero radius faciei $\begin{cases} \text{anterioris} = 0,21262.\,m \\ \text{posterioris} = 0,08172.\,m \end{cases}$

2°. A medio huius lentis ad sequentem, statuatur interuallum $= 0,00566.\,m$.

3°. Secundae lentis crystallinae concauae distantia focalis est $= -0,06796.\,m$ et

Radius vtriusque faciei $= -0,07883.\,m$ dig.

4°. A medio huius lentis ad tertiam, interuallum esto $= 0,00566.\,m$ dig.

5°. Tertiae lentis coronariae conuexae distantia focalis $= 0,11010.\,m$ dig. et

Radius vtriusque faciei $= 0,11671.\,m$ dig.

II. Ab hac lente obiectiua vsque ad lentem quartam, statuatur interuallum $= \frac{1}{4} m - \vartheta$ dig.

III. Quartae lentis coronariae conuexae distantia focalis est $= 2\vartheta - \frac{4\vartheta}{m}$ et

Radius vtriusque faciei $= 1,06\,(2\vartheta - \frac{4\vartheta}{m})$.

IV. Hinc vsque ad lentem vltimam est interuallum $= \frac{4}{3}\vartheta - \frac{4\vartheta}{9m}$.

V. Vltimae lentis coronariae conuexae distantia focalis est $= \frac{2}{3}\vartheta - \frac{4\vartheta}{9m}$ et radius vtriusque faciei $= 1,06\,(\frac{2}{3}\vartheta - \frac{4\vartheta}{9m})$.

VI. Ab hac lente ad oculum distantia $= \frac{1}{3}\vartheta + \frac{\vartheta}{9m}$.

VII. Semidiameter campi apparentis $= \frac{1719}{m+1}$ min.

VIII. Longitudo tubi $= \frac{1}{4} m + \frac{2}{3}\vartheta - \frac{3\vartheta}{9m}$ dig.

§. 12. Quoniam in constructione lentis obiectiuae triplicatae mensuras praescriptas exactissime exsequi vix licet, imprimis autem interualla harum lentium accuratissime in praxi definiri nequeunt, maxime consultum erit: has ternas lentes ita capsulae idoneae includere, vt, ope cochlearum tantillum promoueri, vel a se inuicem remoueri queant, donec repraesentatio distinctissima obtineatur. Caeterum per se intelligitur, etiam vltimam lentem mobilem relinqui debere, vt ad indolem cuiusque oculi adcommodari possit.

DE VLTERIORI
horum telescopiorum perfectione; vnica insuper lente oculari adiiciendo.

§. 13. Praeter primam igitur lentem vicariam, cuius distantia focalis $\Phi = p \pm \alpha$, tres habebimus lentes, quarum distantiae focales sint q, r et s, quibus respondeant distantiae determinatrices

b, $\mathfrak{C}$, c, γ et d, δ; vbi $\delta = \infty$

ita vt fit

$$\frac{1}{q}=\frac{1}{b}+\frac{1}{\beta};\ \frac{1}{r}=\frac{1}{c}+\frac{1}{\gamma} \text{ et } \frac{1}{s}=\frac{1}{\delta} \text{ fiue } s=d$$

tum vero ponatur

$$P=-\frac{\alpha}{b};\ Q=-\frac{\beta}{c} \text{ et } R=-\frac{\gamma}{d}$$

atque ob multiplicationem datam $=m$, debebit esse $PQR=-m$, ficque litterarum P, Q et R vna debet esse negatiua; praeterea ponatur

$$\beta=\mathrm{B}b;\ \gamma=\mathrm{C}c$$

hincque porro

$$\mathfrak{B}=\frac{\mathrm{B}}{1+\mathrm{B}} \text{ et } \mathfrak{C}=\frac{\mathrm{C}}{1+\mathrm{C}}$$

ita vt fit

$$q=\mathfrak{B}b;\ r=\mathfrak{C}c \text{ et } s=d$$

ex his autem litteris viciffim erit

$$b=-\frac{\alpha}{P};\ c=\frac{\mathrm{B}\alpha}{PQ};\ d=-\frac{\mathrm{BC}\alpha}{PQR}$$

$$\beta=-\frac{\mathrm{B}\alpha}{P};\ \gamma=\frac{\mathrm{BC}\alpha}{PQ};\ \delta=\infty$$

vnde interualla lentium erunt

$$\mathrm{I}-\mathrm{II}=\alpha\left(1-\frac{1}{P}\right);\ \mathrm{II}-\mathrm{III}=-\frac{\mathrm{B}\alpha}{P}\left(1-\frac{1}{Q}\right);$$
$$\mathrm{III}-\mathrm{IV}=\frac{\mathrm{BC}\alpha}{PQ}\left(1-\frac{1}{R}\right).$$

§. 14. Quod fi iam femidiametri aperturarum ternarum lentium ftatuantur:

πq; $\pi' r$ et $\pi'' s$ erit

femidiameter campi adparentis $\Phi=\frac{\pi-\pi'+\pi''}{m+1}$

tum vero erit

$\frac{\mathfrak{B}\pi-\Phi}{\Phi}$

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=P \quad \text{fiue} \quad \mathfrak{B}=\frac{\Phi(1+P)}{\pi}$$

$$\frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=PQ \qquad \mathfrak{C}=\frac{\Phi(PQ-1)+\pi}{\pi'}$$

$$\frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=-PQR;\ \mathfrak{D}=\frac{\Phi(1-PQR)-\pi+\pi'}{\pi''}$$

ex quibus valoribus viciffim colligimus

$\mathrm{B}=\frac{\mathfrak{B}}{1-\mathfrak{B}}$; $\mathrm{C}=\frac{\mathfrak{C}}{1-\mathfrak{C}}$; $\mathrm{D}=\frac{\mathfrak{D}}{1-\mathfrak{D}}=\infty$, ob $\mathfrak{D}=1$ ficque omnia elementa per litteras P, Q et R erunt expreffa; vbi ante omnia requiritur vt interualla lentium prodeant pofitiua.

§. 15. Iam fi ξ fuerit maximus valor, quem litteris π, π' et π'' tribuere licet, pro quo affumi poteft $\xi=\frac{1}{4}$; vt maximum campum adparentem obtineamus, ftatuamus

$$\pi=\xi;\ \pi'=-\xi \text{ et } \pi''=+\xi$$

vt prodeat

$$\Phi=\frac{3\xi}{m+1} \text{ vnde fit } M=\frac{3}{m+1} \text{ et } \Phi=M\xi$$

hinc igitur fumendo $\xi=\frac{1}{4}$, in minutis primis fiet

$$\Phi=\frac{3\cdot 3437}{4(m+1)}\text{ min.}=\frac{2578}{m+1}\text{ min.}$$

Ex his igitur valoribus nancifcimur

$$\mathfrak{B}=(1-P)M;\ \mathfrak{C}=M(1-PQ)-1;\ \mathfrak{D}=M(m+1)-2=1$$

vti primae conditiones requirunt.

§. 16. Pro margine autem colorato tollendo huic aequationi fatisfieri oportet

$$\frac{\pi}{P}-\frac{\pi'}{PQ}+\frac{\pi''}{PQR}=0$$

quae ergo abit in hanc

$$\frac{1}{P}+\frac{1}{PQ}+\frac{1}{PQR}=0 \text{ fiue in hanc } 1+\frac{1}{Q}+\frac{1}{QR}=0$$

cum

cum nunc litterarum P, Q et R vna debeat esse negatiua, sit

$$Q = -k, \text{ eritque } 1 - \tfrac{1}{k} - \tfrac{1}{kR} = 0$$

vnde reperitur

$$R = \tfrac{1}{k-1}; \text{ sicque fiet } PQR = -\tfrac{Pk}{(k-1)} = -m$$

ideoque $P = \frac{m(k-1)}{k}$, vbi notandum, vt R fiat positiuum esse debere $k > 1$; qua conditione etiam littera P fiet positiua; hinc igitur nanciscimur

$$\mathfrak{B} = M\left(1 - \tfrac{m(k-1)}{k}\right) = \tfrac{2k - 2m(k-1)}{k(m+1)}$$

$$\mathfrak{C} = M(1 + m(k-1)) - 1 = \tfrac{2 + m(3k-4)}{m+1}.$$

§. 17. Hic primo patet, litteram $\mathfrak{B}$ esse negatiuam, vnde etiam B erit numerus negatiuus; deinde etiam numerus $\mathfrak{C}$ erit positiuus, dummodo non fuerit $3k < 4$, hincque erit

$$\mathfrak{C} = \tfrac{2 + m(3k-4)}{m(5-3k) - 2}$$

qui numerus est positiuus si fuerit $3k < 5$, siue $k < \frac{5}{3}$ attamen $k > \frac{4}{3}$; sin autem esset $k < \frac{4}{3}$, numerator foret negatiuus, et denominator maneret positiuus, ideoque hoc casu C fieret negatiuum; hinc interualla lentium ita se habebunt

$$I - II = \alpha\left(1 - \tfrac{k}{m(k-1)}\right)$$

quod certe est positiuum ob $k > 1$

$$II - III = -\tfrac{B k \alpha}{m(k-1)}\left(1 + \tfrac{1}{k}\right)$$

quod vt fiat positiuum, littera B debet esse negatiua, quod fit si fuerit $k > \frac{4}{3}$

$$III - IV = -\tfrac{B C \alpha}{m(k-1)}(2 - k)$$

quia

quia nunc $-B$ est quantitas positiua, relinquitur vt sit $C(2 - k)$ positiuum; vnde patet si fuerit $k < 2$ tum C debere esse positiuum; sin autem esset $k > 2$ tum C esse debere negatiuum, id quod sponte accidit.

Casus I. §. 18. Hic ergo duo casus occurrunt, prout k fuerit vel < 2 vel > 2; sit igitur primo $k < 2$ atque vt C fiat positiuum, esse debebit

$$k > \tfrac{4}{3} \text{ et } k < \tfrac{5}{3}.$$

Casus II. At si $k > 2$, tum C fiat negatiuum. Operae igitur pretium erit hos casus exemplo illustrasse.

Casus prior

quo $k > \frac{4}{3}$ et $< \frac{5}{3}$.

§. 19. Sumamus igitur $k = \frac{3}{2}$ erit $P = \frac{1}{3}m$; $Q = -\frac{3}{2}$ et $R = 2$; porro

$$\mathfrak{B} = \tfrac{3-m}{m+1} = -\tfrac{(m-3)}{m+1} \text{ et } B = \tfrac{3-m}{2m-2}, \quad \mathfrak{C} = \tfrac{4+m}{3(m+1)}$$

$$\text{et } C = \tfrac{4+m}{m-2}$$

hinc autem interualla lentium prodeunt:

$$I - II = \alpha\left(1 - \tfrac{3}{m}\right)$$

$$II - III = \tfrac{3(m-3)}{2m(m-1)}\alpha$$

$$III - IV = \tfrac{(m-3)(m+4)}{2m(m-1)(m-2)}\alpha$$

quae ergo omnia sunt positiua; deinde vero habebimus

$$b = -\tfrac{3\alpha}{m}; \quad \mathfrak{b} = +\tfrac{3(m-3)\alpha}{2m(m-1)}; \quad c = \tfrac{(m-3)\alpha}{m(m-1)};$$

$$\gamma = \tfrac{(m-3)(m+4)\alpha}{m(m-1)(m-2)}; \quad d = -\tfrac{(m-3)(m+4)\alpha}{2m(m-1)(m-2)}$$

hinc-

hincque colliguntur diſtantiae focales

$$q = \frac{2(m-3)}{m(m+1)}\alpha;\; r = \frac{(m-3)(m+4)\alpha}{2m(m-1)(m+1)};\; s = -\frac{(m-3)(m+4)\alpha}{2m(m-1)(m-2)}$$

ſicque vltima lens fieret concaua, vnde diſtantia oculi etiam prodiret negatiua, ita, vt campum adparentem nequidem tueri liceret.

Caſus poſterior

quo $k > 2$.

§. 20. Sit igitur $k = \frac{5}{2}$ erit $P = \frac{3m}{5}$; $Q = -\frac{5}{2}$ et $R = \frac{2}{5}$ porro fit

$$\mathfrak{B} = \frac{15-9m}{5m+5} \text{ et } B = \frac{15-9m}{14m-10};\; \mathfrak{C} = \frac{4+7m}{3m+3};\; C = -\frac{(4-7m)}{5m+2}$$

deinde vero

$$b = -\frac{5\alpha}{3m};\; \mathfrak{b} = \frac{5(3m-5)}{m(14m-10)}\alpha;\; c = \frac{(3m-5)}{m(7m-5)}\alpha;$$

$$\gamma = -\frac{(3m-5)(7m+4)}{m(7m-5)(5m+2)}\alpha;\; d = +\frac{3(3m-5)(7m+4)}{2m(7m-5)(5m+2)}\alpha$$

hinc interualla lentium

$$\mathrm{I}-\mathrm{II} = \alpha\left(1-\frac{5}{3m}\right);\; \mathrm{II}-\mathrm{III} = \frac{7(3m-5)}{m(14m-10)}\alpha;$$

$$\mathrm{III}-\mathrm{IV} = \frac{(3m-5)(7m+4)}{2m(7m-5)(5m+4)}\alpha$$

denique diſtantiae focales

$$q = \frac{3m-5)}{m(m+1)}\alpha;\; r = \frac{(7m+4)(3m-5)}{3m(m+1)(7m-5)}\alpha;\; s = \frac{3(3m-5)(7m+4)}{2m(7m-5)(5m+2)}\alpha.$$

§. 21. Quoniam igitur hic caſus ad praxin videtur accommodatus, ſumamus quoque $k = 3$ eritque

$$P = \frac{2m}{3};\; Q = -3 \text{ et } R = \frac{1}{3} \text{ hinc}$$

$$\mathfrak{B} = \frac{3-2m}{m+1};\; B = \frac{3-2m}{3m-3};\; \mathfrak{C} = \frac{2+5m}{m+1};\; C = -\frac{(2+5m)}{4m+1}$$

porro

porro vero

$$b = -\frac{3\alpha}{2m};\; \mathfrak{b} = \frac{3(2m-3)}{2m(3m-3)}\alpha;\; c = \frac{(2m-3)}{3m(2m-2)}\alpha;$$

$$\gamma = -\frac{(2+5m)(2m-3)}{2m(4m+1)(3m-3)}\alpha;\; d = \frac{(2+5m)(2m-3)}{m(4m+1)(2m-2)}\alpha$$

et interualla lentium

$$\mathrm{I}-\mathrm{II} = \alpha\left(1-\frac{3}{2m}\right);\; \mathrm{II}-\mathrm{III} = \frac{3(2m-3)}{m(3m-2)}\alpha;$$

$$\mathrm{III}-\mathrm{IV} = \frac{(2+5m)(2m-3)}{2m(4m+1)(2m-2)}\alpha$$

at diſtantiae focales erunt

$$q = \frac{3(2m-3)}{2m(m+1)}\alpha;\; r = \frac{(2+5m)(2m-3)}{2m(m+1)(3m-2)}\alpha;\; s = \frac{(5m+2)(2m-3)}{m(4m+1)(3m-2)}\alpha.$$

§. 22. Huic autem caſui poſtremo, praecedens quo $k = \frac{5}{2}$ merito anteferuntur; quia pro vltima lente maiorem praebet diſtantiam focalem, vnde operae pretium erit eum ad praxin accommodare; id quod facile vt ſupra in genere praeſtabitur; dum loco lentis obiectiuae lens illa triplicata perfectiſſima praefigitur. Sumta ſcilicet diſtantia focali $\mathrm{II} = \frac{m}{4}$ dig. et $x = \frac{m}{30}$ dig. tum vero poſito vt ante $\vartheta = 0, 25578$ capi debet $\alpha = \vartheta m$ dig. exiſtente $l\vartheta = 9, 4078666$; quia autem ex ternis lentibus poſterioribus maior confuſio naſci poteſt, prima lens quadam correctione egebit, vnde pro eius conſtructione ſtatuamus

Radium faciei $\begin{cases} \text{anterioris} = 0, 21262\, m + f \\ \text{poſterioris} = 0, 08172\, m + g \end{cases}$

vbi quantitates f et g ex vnico exemplo definire licebit.

§. 23. Sumamus igitur multiplicationem $m = 50$, indeque computentur ſequentes valores numerici

$P = 30$	$lP = (+)1,4771213$	
$PQ = -75$	$lPQ = (-)1,8750613$	
$PQR = -50$	$lPQR = (-)1,6989700$	
$\mathfrak{B} = -\frac{29}{37}$	$l\mathfrak{B} = (-)0,2319491$	$l\mathfrak{B}^3 = (-)0,6958473$
$B = -\frac{29}{42}$	$lB = (-)9,7996402$	$lB^3 = (-)9,3989206$
	$lB\mathfrak{B} = (+)0,0315893$	
$\mathfrak{C} = \frac{59}{37}$	$l\mathfrak{C} = (+)0,5404031$	$l\mathfrak{C}^3 = (+)1,6212093$
$C = -\frac{59}{42}$	$lC = (-)0,1476027$	$lC^3 = (-)0,4428081$
	$l\mathfrak{C}C = (-)0,6880058$	

§. 24. Iam pro tribus lentibus postremis, quia vtrinque debent esse aeque conuexae, quaerantur numeri respondentes λ', λ'' et λ''' ex formulis

$$\sqrt{\lambda'-1} = \frac{\sigma-\varrho}{\tau}(\mathfrak{B}-\tfrac{1}{2});\ \sqrt{\lambda''-1} = \frac{\sigma-\varrho}{\tau}(\mathfrak{C}-\tfrac{1}{2})$$

$$\text{et } \sqrt{\lambda'''-1} = \frac{\sigma-\varrho}{\tau}\,\frac{1}{2},$$

quia igitur

$$\mathfrak{B}-\tfrac{1}{2} = -\tfrac{75}{74} \text{ et } \mathfrak{C}-\tfrac{1}{2} = \tfrac{101}{74}$$

inde calculus ita se habebit.

$l(\frac{\sigma-\varrho}{\tau}) = 0,1901924$	$l(\frac{\sigma-\varrho}{\tau}) = 0,1901924$	At pro vltima lente erit vt ante
$l\,75 = 1,8750613$	$l\,101 = 2,0043214$	$\lambda''' = 1,60924$
$2,0652537$	$2,1945138$	
$l\,34 = 1,5314789$	$l\,34 = 1,5314789$	
$l\sqrt{\lambda'-1} = 0,5337748$	$l\sqrt{\lambda''-1} = 0,6630349$	
$l(\lambda'-1) = 1,0675496$	$l(\lambda''-1) = 1,3260698$	
hinc $\lambda' = 12,68290$	hinc $\lambda'' = 22,18700$	

§. 25. Nunc autem pro calculo confusionis ponamus breuitatis gratia

$$\frac{1}{B^3PQ} = M \text{ et } \frac{-1}{B^3C^3PQR} = N$$

vt fiat

$$\Omega = -\tfrac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right) + M\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu}{C\mathfrak{C}}\right) + N\lambda'''$$

vbi ergo erit

$l\frac{1}{P} = 8,5228787$; $lM = 8,7260181$; $lN = 8,4593013$

vnde calculus ita se habebit

$l\frac{-1}{P} = (-)8,5228787$	$lM = 8,7260181$	$lN = 8,4593013$
$l\lambda' = 1,1032186$	$l\lambda'' = 1,3450986$	$l\lambda''' = 0,2041851$
$(-)9,6260973$	$0,0711167$	$8,6634864$
$l\mathfrak{B}^3 = (-)0,6958473$	$l\mathfrak{C}^3 = 1,6212093$	ideoque $0,04608$
$(+)\ 8,9302500$	$8,4499074$	
ergo $+0,08516$	ergo $+0,02818$	

$l\frac{-1}{P} = (-)8,5228787$	$lM = 8,7260181$
$l\nu = (+)9,9412366$	$l\nu = 9,9412366$
$(-)7,8641153$	$8,0672547$
$lB\mathfrak{B} = (+)0,0315893$	$lC\mathfrak{C} = (-)0,6880058$
$(-)7,8325260$	$(-)7,3792489$
ergo $(-)0,00680$	ideoque $= (-)0,00239$

hinc igitur erit $\Omega = 0,15023$.

§. 26. Hoc igitur valore inuento erit pro prima ternarum lentium obiectiuarum

$$\lambda = 2,50862 - 0,08257\,\Omega^2 \text{ siue } \lambda = 2,49622$$

hinc

$$\lambda - 1 = 1,49622 \text{ et } \sqrt{\lambda - 1} = 1,22320,$$

quocirca pro radio faciei anterioris istius lentis, erit denominator $= 0,57122$; numerator autem erat

$$= 0,48154\, \Pi = 6,0190$$

vnde radius huius faciei fit $= 10,5371$, quem supra supposuimus

$$= 0,21262 . m + f = 10,631 + f$$

vnde concludimus $f = -0,094$ dig.

§. 27. Simili modo pro facie posteriore erit denominator $1,46812$; numerator vero manet vt ante $= 6,0190$, vnde ipse radius colligitur $= 4,0998$ dig. quem supposuimus ante

$$= 0,08172 . m + g = 4,086 + g.$$

vnde concluditur fore $g = 0,0138$. Nunc igitur demum certi sumus, hanc correctionem tam esse paruam, vt omnem industriam artificis effugiat.

§. 28. Prosequamur igitur constructionem huius telescopii, quod omnibus numeris absolutum videri potest, et cum sit $a = \vartheta m$, existente $\vartheta = 0,25578$: secunda lens ante imaginem lentis obiectiuae statui debet interuallo $= -b = \frac{5}{7}\vartheta$ dig. ideoque post lentem obiectiuam triplicatam secunda lens collocari debet ad distantiam $= \frac{1}{4} m - \frac{5}{7}\vartheta$; deinde distantia huius secundae lentis a tertia erit

$$= \frac{7\vartheta(5m-5)}{14m-10} = \frac{5}{2}\vartheta - \frac{10\vartheta}{7m}$$

at

at distantia tertiae lentis ad quartam

$$= \frac{(5m-5)(7m+4)\vartheta}{2(7m-5)(5m+4)} = \frac{5}{10}\vartheta - \frac{63}{175m}\vartheta$$

tum vero erit secundae lentis distantia focalis

$$= \frac{(3m-3)\vartheta}{m+1} = 3\vartheta - \frac{6\vartheta}{m} \text{ dig.}$$

tertiae autem lentis distantia focalis

$$= \frac{(7m+4)(3m-5)}{2(m+1)(7m-5)}\vartheta = \frac{3}{2}\vartheta - \frac{29}{14m}\vartheta$$

vltimae denique lentis distantia focalis est

$$= \frac{3\vartheta(3m-5)(7m+4)}{2(7m-5)(5m+2)} = \frac{9}{10}\vartheta - \frac{186}{175m}\vartheta$$

denique distantia oculi

$$= \frac{m+1}{8m} s = \frac{5}{10}\vartheta - \frac{19}{350m}\vartheta$$

et semidiameter campi adparentis $= \frac{2578\, m}{m+1}$, qui apparebit instar spatii circularis in coelo, cuius semidiameter est $36°. 33'$ ideoque diameter $= 73°. 6'$.

CONSTRVCTIO GENERALIS

Tuborum Astronomicorum, perfectissimorum, sex lentibus instructorum, pro multiplicatione quacunque m.

I. Lens obiectiua constat ex tribus lentibus, habens distantiam focalem $= \frac{m}{4}$ dig. et aperturam semidiametri $= \frac{m}{50}$ dig. ternae autem lentes ita determinantur. Tab. IV. Fig. 4

1°. Primae lentis coronariae distantia focalis sit $= 0,11137 . m$ dig.

Tum vero radius faciei $\begin{cases} \text{anterioris} = 0,21262 . m - 0,094 \text{ dig.} \\ \text{posterioris} = 0,08172 . m + 0,014 \text{ dig.} \end{cases}$

2°.

2°. A medio huius lentis vsque ad medium sequentis, statuatur interuallum $= 0{,}00566 . m$.

3°. Secundae lentis crystallinae distantia focalis $= - 0{,}06796 . m$ et

Radius vtriusque faciei $= - 0{,}07883 . m$ dig.

4°. A medio huius, ad sequentem interuallum $= 0{,}00566\, m$.

5°. Tertiae lentis coronariae distantia focalis $= 0{,}11010\, m$ et

Radius vtriusque faciei esto $= + 0{,}11671\, m$ digit.

II. Ab hac lente obiectiua vsque ad lentem quartam, statuatur interuallum $\frac{1}{4} m - 0{,}426$ digit.

III. Quartae lentis coronariae conuexae distantia focalis est $= 0{,}767 - \frac{2{,}045}{m}$ dig. et radius vtriusque faciei $= 0{,}813 - \frac{2{,}167}{m}$ dig.

IV. Hinc vsque ad quintam, interuallum esto $= 0{,}383 - \frac{0{,}365}{m}$ dig.

V. Huius lentis coronariae conuexae distantia focalis est $= 0{,}383 - \frac{0{,}529}{m}$ dig. et radius vtriusque faciei $= 0{,}406 - \frac{0{,}561}{m}$ dig.

VI. Hinc, ad lentem vltimam interuallum $= 0{,}076 - \frac{0{,}090}{m}$ dig.

VII. Vltimae lentis coronariae conuexae distantia focalis $= 0{,}230 - \frac{0{,}270}{m}$ dig. et radius vtriusque faciei $= 0{,}244 - \frac{0{,}286}{m}$ dig.

VIII.

VIII. Hinc, vsque ad oculum interuallum statuatur $= 0{,}076 - \frac{0{,}005}{m}$.

IX. Semidiameter campi adparentis $= \frac{3570}{m+1}$ min. qui instar spatii circularis in coelo cernetur, cuius diameter $= 73°. 6.$ min.

Caeterum tribus lentibus postremis tanta tribuatur apertura, quantam per figuram admittunt, cuius semidiameter circiter est pars quarta distantiae focalis cuiusque lentis. Denique, quia hic binae lentes postremae reuera vicem gerunt lentis ocularis: consultum erit, eas ambas eidem capsulae mobili includere quae ad indolem cuiusque oculi adcommodari possit.

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