DE PERFECTIONE TELESCOPIORVM TERTII GENERIS, DVAS IMAGINES REALES CONTINENTIVM.

Auctore

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§. 1.

Quia pro hoc genere tres lentes non sufficiunt, statim consideremus quatuor: quarum distantiae focales sint p, q, r, s et ex earum distantiis determinatricibus vt ante formemus has quantitates:

$$P = -\tfrac{\alpha}{b};\ Q = -\tfrac{\beta}{c} \quad \text{et} \quad R = -\tfrac{\gamma}{d}$$

quarum productum PQR aequetur multiplicationi m positiue sumtae, quia repraesentatio debet esse erecta, ita vt sit $PQR = m$; et quia duae imagines requiruntur reales, litterarum P, Q et R, duas negatiuas esse oportet. Porro autem statuamus vt ante

$$B = \tfrac{\beta}{b};\ C = \tfrac{\gamma}{c} \quad \text{et} \quad D = \tfrac{\delta}{d} = \infty, \text{ ob } \delta = \infty$$

hincque fiat

$$\mathfrak{B} = \tfrac{B}{1+B};\ \mathfrak{C} = \tfrac{C}{1+C} \text{ et } \mathfrak{D} = \tfrac{D}{1+D} = 1$$

ex quibus colliguntur vicissim distantiae determinatrices

$b = -$

$$b = -\tfrac{\alpha}{P};\quad c = \tfrac{B\alpha}{PQ};\quad d = -\tfrac{BC\alpha}{PQR} = -\tfrac{BC\alpha}{m}$$

$$\beta = -\tfrac{B\alpha}{P};\quad \gamma = \tfrac{BC\alpha}{PQ};\quad \delta = \infty$$

vnde colliguntur interualla lentium

$$\mathrm{I}-\mathrm{II} = \alpha + b = \alpha(1 - \tfrac{1}{P})$$

$$\mathrm{II}-\mathrm{III} = \beta + c = -\tfrac{B\alpha}{P}(1 - \tfrac{1}{Q})$$

$$\mathrm{III}-\mathrm{IV} = \gamma + d = \tfrac{BC\alpha}{PQ}(1 - \tfrac{1}{R})$$

quae omnia debent esse positiua,

denique distantiae focales hinc ita definiuntur

$$p = \alpha;\ q = \mathfrak{B}b;\ r = \mathfrak{C}c \quad \text{et} \quad s = d.$$

§. 2. Cum nunc primae lentis semidiameter aperturae sit $= X$, ponatur semidiameter aperturae secundae lentis $= \pi q$, tertiae lentis $= \pi' r$, et quartae lentis $= \pi'' s$: vbi litterae π, π' et π'' denotant fractiones, quartam partem vnitatis non superantes, siue positiuas siue negatiuas; hincque semidiameter campi apparentis Φ statim ita determinatur, vt sit $\Phi = \frac{-\pi+\pi'-\pi''}{m-1}$; ab his autem porro vicissim superiores litterae ita pendent vt sit

$$\tfrac{\mathfrak{B}\pi - \Phi}{\Phi} = -P \quad \text{seu} \quad \mathfrak{B} = \tfrac{\Phi(1-P)}{\pi}$$

$$\tfrac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = PQ \quad \text{hinc} \quad \mathfrak{C} = \tfrac{\Phi(PQ-1)+\pi}{\pi'}$$

$$\tfrac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = -PQR = -m \quad \text{vnde fit}$$

$$\mathfrak{D} = \tfrac{\Phi(1-m) - \pi + \pi'}{\pi''}$$

quia igitur $\mathfrak{D} = +1$ hinc sequitur

$$\Phi = \tfrac{\pi - \pi' + \pi''}{1-m} \quad \text{siue} \quad \Phi = \tfrac{-\pi + \pi' - \pi''}{m-1}$$

vti

vti oportet: vt autem repraesentatio ab omni margine colorato liberetur, satisfieri oportet huic aequationi

$$\frac{\pi}{P} - \frac{\pi'}{PQ} + \frac{\pi''}{PQR} = 0.$$

§. 3. Statim nunc videamus, an campo apparenti maximum valorem conciliare liceat, id quod praestaretur, si denotante ξ maximum valorem, quem litterae π, π' et π'' recipere possunt, poneretur

$$\pi = -\xi,\ \pi' = +\xi \text{ et } \pi'' = -\xi$$

tum enim prodiret

$$\Phi = +\frac{2\xi}{m-1} = M\xi \text{ sumto } M = \frac{2}{m-1}$$

ex quo valore determinatur distantia oculi post vltimam lentem $= \frac{s}{Mm}$, quam ergo pariter positiuam esse necesse est, quia alioquin campus assignatus ab oculo conspici non posset; His igitur positis destructio marginis colorati hanc exigit aequationem

$$\frac{1}{P} + \frac{1}{PQ} + \frac{1}{PQR} = 0 \text{ siue } 1 + \frac{1}{Q} + \frac{1}{QR} = 0.$$

Casus quo $P > 0$ §. 4. Quia litterarum P, Q et R binae debent esse negatiuae, sumamus primo litteram P esse positiuam, vnde tam Q quam R negatiuas esse oportet; quocirca ponamus $Q = -k$ atque reperietur $R = \frac{1}{k-1}$, sicque debet esse $k < 1$ vt fiat

$$R = \frac{-1}{1-k} \text{ ideoque } PQR = \frac{Pk}{1-k} = m$$

consequenter $P = \frac{m(1-k)}{k}$; quia igitur

$$P = \frac{(1-k)}{k} m \text{ et } PQ = -(1-k)m \text{ ob}$$

$$\pi = -\xi,\ \pi' = +\xi,\ \pi'' = -\xi \text{ et } \Phi = M\xi;$$

repe-

reperiemus

$$\mathfrak{B} = M\left(\frac{m(1-k)}{k} - 1\right) = \frac{2m(1-k)-2k}{k(m-1)}$$

$$B = \frac{2m(1-k)-2k}{m(4k-3)+2k} \text{ deinde}$$

$$\mathfrak{C} = -M((1-k)m+1) - 1 = -\frac{m(4-3k)-3}{m-1}$$

$$C = -\frac{m(4-3k)-2}{m(5-2k)+1}.$$

§. 5. Nunc igitur ad interualla lentium respiciamus, ac primum quidem erit $= \alpha\left(1 - \frac{k}{m(1-k)}\right)$ quod semper est positiuum ob $k < 1$, secundum autem erit $= -\frac{B\alpha}{P}\left(1 + \frac{1}{k}\right)$, quod positiuum esse nequit nisi B sit numerus negatiuus. Cum igitur numerator ipsius B sit positiuus, denominator negatiuus esse oportet, vnde sequitur $4k < 3$ et $k < \frac{3}{4}$, tertium autem interuallum est $= -\frac{BC\alpha}{(1-k)m}(2-k)$ vnde patet BC esse debere negatiuum; quia igitur B iam est negatiuum, oportet esse C positiuum; ob numeratorem vero ipsius C negatiuum, videamus an denominator etiam negatiuus reddi possit: quod cum fieri nequeat, patet hunc casum locum habere non posse, quo erat $P > 0$.

§. 6. Tribuamus igitur litterae P valorem negatiuum, et nunc vel Q vel R debet esse negatiuum. Sit primo Q negatiuum et ponamus $Q = -k$ erit $1 - \frac{1}{k} - \frac{1}{kR} = 0$, vnde fit $R = \frac{1}{k-1}$, sicque R debet esse positiuum; erit igitur Casus quo $P < 0$ et $Q > 0$.

$$PQR = -\frac{Pk}{k-1} = +m$$

ideoque

$$P = -\frac{m(k-1)}{k} \text{ et } PQ = +m(k-1)$$

vnde

vnde colligimus

$$\mathfrak{B} = -M\left(1 + \frac{m(k-1)}{k}\right) = -\frac{3m(k-1)-3k}{k(m-1)}$$

$$B = -\frac{3m(k-1)-3k}{m(4k-3)+2k} \quad \text{deinde}$$

$$\mathfrak{C} = M(m(k-1)-1) - 1 = \frac{m(5k-4)-2}{m-1} \quad \text{et}$$

$$C = \frac{m(5k-4)-2}{m(5-3k)+1}.$$

§. 7. Examinemus nunc pariter fingula interualla, ac primum quidem $\alpha(1-\frac{1}{P})$ certe eft pofitiuum, fecundum $-\frac{B\alpha}{P}(1+\frac{1}{k})$ vbi ob P negatiuum B debet effe pofitiuum, cuius numerator cum fit negatiuus etiam denominator debet effe negatiuus, quod autem fieri nequit: vnde etiam hic cafus $P<0$ et $Q<0$ locum habere nequit, quare tertium cafum euoluamus quo $P<0$ et $R<0$.

Cafus quo $P<0$ et $R<0$. §. 8. Hic igitur nil aliud opus eft nifi vt loco k fcribamus $-k$ eritque

$$P = -\frac{m(k+1)}{k}; \quad Q = +k \quad \text{et} \quad R = \frac{-1}{k+1} \quad \text{hincque}$$

$$PQ = -m(k+1)$$

ex his porro colligitur

$$\mathfrak{B} = \frac{-3m(k+1)-3k}{k(m-1)}; \quad B = \frac{-3m(k+1)-3k}{m(4k+3)+2k}$$

$$\mathfrak{C} = \frac{-m(5k+4)-2}{m-1}; \quad C = \frac{-m(5k+4k)-2}{m(5+3k)+1}$$

iam quia primum interuallum nulla laborat difficultate; fecundum interuallum eft $-\frac{B\alpha}{P}(1-\frac{1}{k})$ vbi duo cafus occurrunt prout fuerit $k>1$ vel $k<1$. Sit

I°. $k>1$ et quia P eft negatiuum neceffe eft vt fit $B>0$ quod autem fieri nequit.

II°. Sit igitur $k<1$ et fieri debet B negatiuum quod fponte euenit; tertium autem interuallum eft $\frac{BC\alpha}{-m(k+1)}(2+k)$ vnde patet C effe debere pofitiuum, cuius numerator cum fit manifefto negatiuus denominator vero pofitiuus, etiam hic cafus locum habere nequit: quocirca nunc quidem certum eft, in hoc genere per quaternas lentes campum apparentem $\Phi = \frac{3\xi}{m-1}$ obtineri non poffe.

§. 9. Hinc igitur intelligimus omnes tres lentes fimul adhiberi non poffe ad campum apparentem augendum, quocirca affumamus effe

$$\pi = +\tfrac{3}{2}\xi \text{ et } \pi' = +\xi \text{ et } \pi'' = -\xi$$

eritque pro campo apparente

$$\Phi = \frac{3\xi}{2(m-1)} \quad \text{hinc} \quad M = \frac{3}{2(m-1)}$$

tum vero erit

$$\mathfrak{B} = \frac{3(1-P)}{m-1}; \quad \mathfrak{C} = \frac{3PQ+m-4}{2(m-1)}$$

aequatio autem pro margine colorato erit

$$\frac{3}{2P} - \frac{3}{PQ} - \frac{1}{PQR} = 0 \text{ fiue } 1 - \frac{2}{Q} - \frac{2}{QR} = 0$$

vbi notandum, trium litterarum P, Q et R duas effe debere negatiuas; ponamus primo P et R effe negatiuas et fit $Q = +k$ eritque $1 - \frac{2}{k} - \frac{2}{kR} = 0$ vnde reperitur $R = \frac{-2}{2-k}$

hinc patet effe $k<2$ vnde erit

Cafus $P<0$ et $R<0$.

$$PQR = -\frac{2kP}{2-k} = m \quad \text{ideoque} \quad P = -\frac{m(2-k)}{2k} \quad \text{et}$$

$$PQ = -\frac{m(2-k)}{2}.$$

ex his iam valoribus habebimus

$$\mathfrak{B} = \frac{3(2k + m(2-k))}{2k(m-1)}; \quad \mathfrak{C} = \frac{m(3k-4)-8}{4(m-1)}$$

vnde sequitur

$$B = \frac{3(2k + m(2-k))}{m(5k-6)-8k} \text{ et } C = \frac{m(3k-4)-8}{m(8-3k)+8}.$$

§. 10. Quod ad interualla attinet, primum per se est positiuum

$$a\left(1 - \frac{1}{P}\right) = \alpha\left(1 + \frac{2k}{m(2-k)}\right)$$

secundum autem interuallum $= -\frac{B\alpha}{P}\left(1 - \frac{1}{k}\right)$ vbi duo casus considerandi veniunt prout fuerit vel $k > 1$ vel $k < 1$.

I°. Primo sit $k > 1$, ideoque $1 - \frac{1}{k} > 0$, quia P est negatiuum, B debet esse positiuum, quod vt eueniat, esse debet $k > \frac{6}{5}$ attamen $k < 2$.

II. Sin autem sit $k < 1$; B debet esse negatiuum, cuius numerator cum sit positiuus, deberet esse $k < \frac{6}{5}$ quod per se euenit.

§. 11. Tertium interuallum erat $\frac{BC\alpha}{PQ}\left(\frac{4-k}{2}\right)$; vnde patet ob PQ negatiuum etiam BC negatiuum esse debere; quare binos casus praecedentes percurramus.

I. Priore casu $k > \frac{6}{5}$ quo B positiuum, debet esse C negatiuum, cuius denominator cum sit per se positiuus, numerator debet esse negatiuus quod fit si sit $k < \frac{4}{3}$, vnde hos duos limites habemus $k < \frac{4}{3}$ et $k > \frac{6}{5}$. Praeterea erat $s = -\frac{BC\alpha}{m}$ ideoque ob $B > 0$ et $\mathfrak{C} < 0$ vti requiritur.

§. 12.

§. 12. Examinemus quoque alterum casum vbi ob $k < 1$ et B negatiuum, debet esse C positiuum cuius denominator cum per se sit positiuus debet esse $k > \frac{4}{3}$; quod quia fieri nequit hic casus secundus locum habere nequit.

Euolutio casus prioris.

quo $k > \frac{6}{5}$ et $k < \frac{4}{3}$.

§. 13. Sumamus igitur $k = \frac{5}{4}$ et sequentes nanciscemur determinationes:

$$P = -\tfrac{3}{16}m; \quad Q = \tfrac{5}{4}; \quad R = -\tfrac{8}{5}$$

$$\mathfrak{B} = \frac{9m+30}{10(m-1)}; \quad B = \frac{9m+30}{m-40}; \quad \mathfrak{C} = \frac{-m-32}{16(m-1)}; \quad C = \frac{-m-32}{17m+16}$$

vnde elementa deriuantur

$$b = \frac{10\alpha}{3m}; \quad \beta = \frac{10(3m+10)}{m(m-40)}\alpha; \quad c = -\frac{8(3m+10)}{m(m-40)}\alpha;$$

$$\gamma = +\frac{8(3m+10)(m+32)}{m(m-40)(17m+16)}\alpha; \quad d = \frac{3(3m+10)(m+32)}{m(m-40)(17m+16)}\alpha$$

hinc interualla lentium

$$\text{I} - \text{II} = \alpha + b = \frac{(3m-10)}{3m}\alpha$$

$$\text{II} - \text{III} = \beta + c = \frac{2(3m+10)}{m(m-40)}\alpha$$

$$\text{III} - \text{IV} = \gamma + d = \frac{11(3m+10)(m+32)}{m(m-40)(17m+16)}\alpha$$

distantiae focales

$$p = \alpha; \quad q = \frac{3m+10}{m(m-1)}\alpha; \quad r = +\frac{(m+32)(3m+10)}{2m(m-1)(m-40)}\alpha;$$

$$s = \frac{3(3m+10)(m+32)}{m(m-40)(17m+16)}\alpha$$

distantia autem oculi $= \frac{25(m-1)}{3m}$ et semidiameter campi apparentis

$$\Phi = \frac{3\xi}{2(m-1)} = \frac{1290}{m-1} \text{ min.}$$

§. 14.

§. 14. Haec igitur species locum habere nequit nisi multiplicatio m multo sit maior quam 40, tum autem vt supra loco lentis obiectiuae substituatur lens nostra triplicata perfecta iam aliquoties descripta, sumendo $\Pi = \frac{m}{4}$ dig. et $\alpha = 0,25578.m$. Interuallum autem inter hanc lentem obiectiuam et lentem q debet esse $= \frac{1}{4}m + b$; reliquae determinationes hinc facile deducuntur, quandoquidem ob confusionem sequentium lentium constructio lentis obiectiuae vix quicquam immutatur.

Euolutio casus

quo $P > 0$ ideoque Q et R negatiuae.

§. 15. Quia igitur Q negatiuum loco k scribamus $-k$ eritque

$$P = + \frac{m(2+k)}{2k^2}; \quad R = \frac{-2}{2+k}$$

existente $Q = -k$

vnde erit vt ante $\Phi = \frac{3\xi}{2(m-1)}$, porro vero erit

$$\mathfrak{B} = \frac{-3m(2+k)+6k}{2k(m-1)}; \quad B = \frac{-3m(2+k)+6k}{m(5k+6)-6k}$$

$$\mathfrak{C} = \frac{-m(3k+4)-8}{4(m-1)}; \quad C = \frac{-m(3k+4)-8}{m(6+5k)+4}.$$

§. 16. Nunc igitur secundum interuallum erit $-\frac{B\alpha}{P}(1+\frac{1}{k})$, ob P positiuum hoc interuallum postulat vt B sit negatiuum, quod sponte euenit, tertium autem interuallum $\frac{BC\alpha}{PQ}(\frac{4+k}{2})$, vnde C debet esse positiuum, quod plane fieri nequit.

EVO-

EVOLVTIO TELESCOPIORVM pro casu in tomo secundo Dioptricae pag. 393. exposito.

§. 17. Haec telescopiorum species imprimis memorabilis sequenti modo est determinata: proposita multiplicatione quacunque m quaeratur numerus $k = -m + \sqrt{2m(m-1)}$ et distantiae focales lentium ita determinabuntur

$$p = \alpha; \quad q = \frac{\alpha}{k}; \quad r = \frac{\vartheta\alpha}{k} \text{ et } s = \frac{\vartheta\alpha}{m}$$

vbi numerum ϑ pro arbitrio assumere licet; tum vero interualla lentium ita se habebunt

$$\mathrm{I} - \mathrm{II} = p + q = \alpha(1 + \tfrac{1}{k})$$

$$\mathrm{II} - \mathrm{III} = \eta\alpha = (\tfrac{k+1}{k^2})\alpha + \frac{\vartheta\alpha(\sqrt{2m(m-1)})}{2kk}$$

$$\mathrm{III} - \mathrm{IV} = r + s = \vartheta\alpha(\tfrac{1}{k} + \tfrac{1}{m})$$

oculi autem distantia est $= \frac{s(m-1)}{\sqrt{2m(m-1)}}$.

§. 18. Nunc igitur ad perfectionem huius speciei telescopiorum nihil aliud requiritur nisi vt loco lentis obiectiuae P nostra lens triplicata substituatur sumendo $\Pi = \frac{m}{4}$ digit. eamque ante lentem secundam statuendo ad distantiam $\frac{1}{4}m + q$, tum vero erit $\alpha = 0,25578\,m$. Praeterea, vt lentes oculares maximam aperturam admittant eas vtrinque aequaliter conuexas parari conuenit, ex quo radius vtriusque faciei pro lente q erit $1,06\,q$, pro lente $r = 1,06.r$ et pro lente $s = 1,06\ s$. Denique campi apparentis semidiameter in minutis primis erit

$$\Phi = \frac{1215}{\sqrt{m(m-1)}} = \frac{1718}{\sqrt{2m(m-1)}} \text{ siue } \Phi = \frac{1718}{m+k}.$$

 §. 19.

§. 19. Quo autem facilius haec formulae ad calculum reuocari possint, sequentem tabulam subiungamus:

m	$\sqrt{2m(m-1)}$	k	η
10	13,416	3,416	$0,3784+1,1497.\frac{1}{2}\vartheta$
20	27,568	7,568	$0,1495+0,4813.\frac{1}{2}\vartheta$
30	41,713	11,713	$0,0927+0,3040.\frac{1}{2}\vartheta$
40	55,857	15,857	$0,0670+0,2221.\frac{1}{2}\vartheta$
50	70,000	20,000	$0,0525+0,1759.\frac{1}{2}\vartheta$
75	105,357	30,357	$0,0340+0,1143.\frac{1}{2}\vartheta$
100	140,712	40,712	$0,0252+0,0849.\frac{1}{2}\vartheta$
150	211,424	61,424	$0,0165+0,0560.\frac{1}{2}\vartheta$
200	282,135	82,135	$0,0123+0,0418.\frac{1}{2}\vartheta$
300	423,556	123,556	$0,0081+0,0277.\frac{1}{2}\vartheta$
400	564,978	164,978	$0,0061+0,0207.\frac{1}{2}\vartheta$

APPLICATIO

ad eam telescopiorum speciem, quae in Dioptricae tomo 2do pag. 448. est exposita.

§. 20. Haec species prae caeteris omni attentione digna videtur, quia campus apparens pro lubitu augeri potest, ita vt eius semidiameter fiat $\Phi = \frac{i\xi}{m+\sqrt{m}}$ quia autem ibi lens obiectiua assumta est duplicata hic pro instituto nostro lente simplici vicaria vtamur, vnde litterae P, B et C penitus ex formulis ibi datis sunt excludendae, sequentes vero vno gradu

du remouendae; hinc igitur ex loco citato sequentes habebimus valores

$$P = \frac{2i\sqrt{m}}{1+i};\ Pk = \sqrt{m};\ Pkk' = im;\ Pkk'S = \frac{im}{2};$$
$$Pkk'ST = \frac{im}{3};\ Pkk'STU = \frac{im}{4}\ \text{etc.}$$

ita vt horum productorum vltimum sit $= \frac{im}{i} = m$ sicque omnium lentium numerus erit $= i+3$.

§. 21. Praeterea habebimus sequentes numeros

$\mathfrak{B} = \frac{-2i\sqrt{m}+i+1}{(1+i)(1+\sqrt{m})}$	$B = \frac{-2i\sqrt{m}+i+1}{(1+3i)\sqrt{m}}$
$\mathfrak{C} = \frac{\vartheta}{1+\vartheta}$	$C = \vartheta$
$\mathfrak{D} = \frac{i(1+i\sqrt{m})}{1+\sqrt{m}}$	$D = \frac{-(i+ii\sqrt{m})}{(i-1)(1+(i+1)\sqrt{m})}$
$\mathfrak{E} = \frac{i(2+i\sqrt{m})}{2(1+\sqrt{m})} - 1$	$E = \frac{-(2(i-1)+(ii-1.2)\sqrt{m})}{(i-2)(2+(i+2)\sqrt{m})}$
$\mathfrak{F} = \frac{i(3+i\sqrt{m})}{3(1+\sqrt{m})} - 2$	$F = \frac{-(3(i-2)+(ii-2.3)\sqrt{m})}{(i-3)(3+(i+3)\sqrt{m})}$
$\mathfrak{G} = \frac{i(4+i\sqrt{m})}{4(1+\sqrt{m})} - 3$	$G = \frac{-(4(i-3)+(ii-3.4)\sqrt{m})}{(i-4)(4+(i+4)\sqrt{m})}$
etc.	etc.

Pro priori columna littera penultima erit $\frac{2(i-1)+(3i-2)\sqrt{m}}{(i-1)(1+\sqrt{m})}$ vltima vero $= 1$.

§. 22. Ex his litteris distantiae determinatrices singularum lentium sequentibus formulis exprimentur

$b = -\frac{\alpha}{P}$	$\mathfrak{b} = -\frac{B\alpha}{P}$
$c = -\frac{B\alpha}{Pk}$	$\gamma = -\frac{BC\alpha}{Pk}$
$d = -\frac{BC\alpha}{Pkk'}$	$\delta = -\frac{BCD\alpha}{Pkk'}$
$e = +\frac{BCD\alpha}{Pkk'S}$	$\varepsilon = +\frac{BCDE\alpha}{Pkk'S}$
$f = -\frac{BCDE\alpha}{Pkk'ST}$	$\zeta = -\frac{BCDEF\alpha}{Pkk'ST}$
$g = +\frac{BCDEF\alpha}{Pkk'STU}$	$\eta = +\frac{BCDEFG\alpha}{Pkk'STU}$
etc.	etc.

23. Hinc porro diſtantiae focales ſingularum lentium ita definientur

$$p = \alpha;\ q = \mathfrak{B}b;\ r = \mathfrak{C}c;\ s = \mathfrak{D}d;\ t = \mathfrak{E}e \text{ etc.}$$

quarum vltima ſi vocetur $= z$ poſt eam oculus teneri debet ad diſtantiam

$$= \frac{m + \sqrt{m}}{i\,m}\,z = \frac{1 + \sqrt{m}}{i\sqrt{m}}\,z$$

vnde tantus campus conſpicietur cuius ſemidiameter in minutis primis erit $\frac{859\,i}{m + \sqrt{m}}$

caeterum littera ϑ quae hic eſt introducta penitus arbitrio noſtro relinquitur, eamque ita aſſumi conuenit vt diſtantiae focales vltimarum lentium ſatis modicae magnitudinis euadant, ſcilicet ne minores fiant quam vnus digitus vel ad ſummum $\frac{1}{2}$ digit.

§. 24. Denique ſi $\lambda, \lambda', \lambda'', \lambda''', \lambda''''$ etc. denotent ordine numeros arbitrarios ad formationem ſingularum lentium pertinentes, confuſio illa Ω ex qua lentem obiectiuam ſiue duplicatam ſiue triplicatam determinari oportet ſequenti modo expreſſa prodibit

$\Omega =$

$$\Omega = -\frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\,\mathfrak{B}}\right) - \frac{1}{B^3\,Pk}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\,\mathfrak{C}}\right)$$
$$- \frac{1}{B^3\,C^3\,Pkk'}\left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{\nu}{D\,\mathfrak{D}}\right) + \frac{1}{B^3\,C^3\,D^3\,Pkk'S}\left(\frac{\lambda''''}{\mathfrak{E}^3} + \frac{\nu}{E\,\mathfrak{E}}\right) \text{ etc.}$$

vbi pro ſecunda lente ſumi poteſt $\lambda' = 2$ et $\lambda'' = 1$ reliquae autem ita debent eſſe comparatae, vt ſingulae lentes prodeant vtrinque aequaliter conuexae. Caeterum hi termini valorem Ω praebentes plerumque tam erunt exigui vt inde in conſtructionem lentis obiectiuae multiplicatae vix vlla mutatio ſenſibilis ingrediatur, ob quam cauſam etiam parum referet, vtrum lentes ſecunda et tertia etiam parentur aequaliter conuexae nec ne.

§. 24. Quod denique ad aperturas lentium attinet, ſi primae obiectiuae ſemidiameter aperturae fuerit $= X$, ſecundae ſemidiameter debet eſſe

$$= \frac{i}{\sqrt{m}}\,\frac{q}{4} \pm \frac{X(1+i)}{2\,i\sqrt{m}};\ \text{tertiae} = 0.\frac{r}{4} \pm \frac{X}{\sqrt{m}};$$
$$\text{quartae} = \frac{s}{4} \pm \frac{X}{i\,m};\ \text{quintae} = \frac{t}{4} \pm \frac{2X}{i\,m};$$
$$\text{ſextae} = \frac{u}{4} \pm \frac{3X}{i\,m} \text{ etc.}$$

vbi notandum eſt: partes poſteriores nihil ad campum augendum conferre, ſed ideo tantum eſſe adiectas, vt aequabilis claritas per totum campum obtineatur, quam conditionem non adeo neceſſe eſt adimpleri. Hinc igitur pro variis valoribus litterae i ſequentes caſus euoluamus.

CASVS I.

quo $i = 1$, numerus lentium $= 4$ et semidiameter campi $\Phi = \frac{859}{m + \sqrt{m}}$ min.

§. 25. Pro hoc igitur casu habebimus sequentes valores principales, ex quibus omnes determinationes facile deducuntur:

$P = \sqrt{m}$	$\mathfrak{B} = \frac{-\sqrt{m}+1}{1+\sqrt{m}}$	$B = \frac{-\sqrt{m}+1}{2\sqrt{m}}$
$Pk = \sqrt{m}$	$\mathfrak{C} = \frac{\vartheta}{1+\vartheta}$	$C = \vartheta$
$Pkk' = m$	$\mathfrak{D} = 1$	$E = \frac{-\sqrt{m}-1}{6(1+2\sqrt{m})} = \infty.$

CASVS II.

quo $i = 2$, numerus lentium $= 5$ et semidiameter campi $\Phi = \frac{1718}{m + \sqrt{m}}$ min.

§. 26. Pro hoc igitur casu nostrae litterae principales sequentes accipiunt valores

$P = \frac{4\sqrt{m}}{3}$	$\mathfrak{B} = \frac{-4\sqrt{m}+3}{3(1+\sqrt{m})}$	$B = \frac{-4\sqrt{m}+3}{7\sqrt{m}}$
$Pk = \sqrt{m}$	$\mathfrak{C} = \frac{\vartheta}{1+\vartheta}$	$C = \vartheta$
$Pkk' = 2m$	$\mathfrak{D} = \frac{2+4\sqrt{m}}{1+\sqrt{m}}$	$D = \frac{-2-4\sqrt{m}}{1+3\sqrt{m}}$
$Pkk'S = m$	$\mathfrak{E} = 1$	$E = \infty.$

CASVS III.

Quo $i = 3$, numerus lentium 6 et semidiameter campi $= \frac{2577}{m + \sqrt{m}}$ min.

§. 27. Pro hoc igitur casu nostrae litterae principales sequentes accipiunt valores

P =

$P = \frac{3\sqrt{m}}{2}$	$\mathfrak{B} = \frac{-3\sqrt{m}+2}{2(1+\sqrt{m})}$	$B = \frac{-3\sqrt{m}+2}{5\sqrt{m}}$
$Pk = \sqrt{m}$	$\mathfrak{C} = \frac{\vartheta}{1+\vartheta}$	$C = \vartheta$
$Pkk' = 3m$	$\mathfrak{D} = \frac{9\sqrt{m}+3}{1+\sqrt{m}}$	$D = \frac{-9\sqrt{m}-3}{2(1+4\sqrt{m})}$
$Pkk'S = \frac{5m}{2}$	$\mathfrak{E} = \frac{7\sqrt{m}+4}{2(1+\sqrt{m})}$	$E = \frac{-7\sqrt{m}-4}{2+5\sqrt{m}}$
$Pkk'ST = m$	$\mathfrak{F} = 1$	$F = \infty.$

CASVS IV.

Quo $i = 4$, numerus lentium 7 et campi semidiameter $= \frac{3436}{m + \sqrt{m}}$ min.

§. 28. Pro hoc igitur casu sequentes consequimur valores principales

$P = \frac{8\sqrt{m}}{5}$	$\mathfrak{B} = \frac{-8\sqrt{m}+5}{5(1+\sqrt{m})}$	$B = \frac{-8\sqrt{m}+5}{13\sqrt{m}}$
$Pk = \sqrt{m}$	$\mathfrak{C} = \frac{\vartheta}{1+\vartheta}$	$C = \vartheta$
$Pkk' = 4m$	$\mathfrak{D} = \frac{16\sqrt{m}+4}{1+\sqrt{m}}$	$D = \frac{-16\sqrt{m}-4}{2(1+5\sqrt{m})}$
$Pkk'S = 2m$	$\mathfrak{E} = \frac{9\sqrt{m}+3}{1+\sqrt{m}}$	$E = \frac{-7\sqrt{m}-2}{2+6\sqrt{m}}$
$Pkk'ST = \frac{4m}{3}$	$\mathfrak{F} = \frac{16\sqrt{m}+6}{3(1+\sqrt{m})}$	$F = \frac{-10\sqrt{m}-6}{3+7\sqrt{m}}$
$Pkk'STU = m$	$\mathfrak{G} = 1$	$G = \infty.$

§. 29. In hoc telescopiorum genere tertia lens singularia suppeditat phaenomena; cum enim primo quam minimam requirat aperturam, ea commodissime vicem geret diaphragmatis, quo radii peregrini ab introitu in lentes sequentes arceantur; deinde vero quia portio confusionis ex hac lente nata est

$$= -\frac{1}{B^3\sqrt{m}}\left(\frac{\lambda(1+\vartheta)^3}{\vartheta^3} + \frac{\nu(1+\vartheta)}{\vartheta^2}\right)$$

non

non solum ob $\frac{(1+\vartheta)^3}{\vartheta^3}$ numerum satis notabilem ac vnitate semper maiorem haec quantitas vnitatem non mediocriter superabit etiamsi capiatur $\lambda'' = 1$, sed etiam quia $\frac{1}{B^3}$ sit numerus plerumque satis magnus; cum enim primo casu quo $i = 1$ sit

$$B = \frac{-\sqrt{m}+1}{2\sqrt{m}} \text{ erit } -\frac{1}{B} = \frac{2\sqrt{m}}{\sqrt{m}-1} \text{ et } -\frac{1}{B^3\sqrt{m}} = \frac{8m}{(\sqrt{m}-1)^3}.$$

Casu autem secundo quo $i = 2$ est

$$B = \frac{-4\sqrt{m}+3}{3\sqrt{m}} \text{ hinc } -\frac{1}{B} = \frac{3\sqrt{m}}{4\sqrt{m}-3} \text{ et } -\frac{1}{B^3\sqrt{m}} = \frac{27m}{(4\sqrt{m}-3)^3}$$

tertio vero casu quo est

$$B = \frac{-3\sqrt{m}+2}{5\sqrt{m}} \text{ erit } -\frac{1}{B^3\sqrt{m}} = \frac{125m}{(3\sqrt{m}-2)^3};$$

Quarto denique casu ob

$$B = \frac{-6\sqrt{m}+5}{13\sqrt{m}} \text{ erit } -\frac{1}{B^3\sqrt{m}} = \frac{2197m}{(6\sqrt{m}-5)^3} \text{ etc.}$$

hinc igitur littera nostra Ω tantum accipere poterit augmentum, vt lens obiectiua triplicata satis notabilem mutationem subire queat, quam in praxi negligere nequaquam licebit, id quod vnico exemplo ostendisse sufficiet.

EXEMPLVM TELESCOPII

ex sex lentibus compositi quod obiecta centies multiplicet.

§. 30. Hic igitur erit $i = 3$, vnde valores ex casu tertio depromere opportet; tum vero semidiameter campi erit $\Phi = 23\frac{1}{3}$ min. ideoque instar spatii circularis in coelo spectabitur cuius semidiameter erit $= 34^\circ. 10'$ ideoque diameter $68^\circ. 20'$; tum vero

ro assumamus $\vartheta = 3$, vnde sequentes prodibunt valores:

$P = 15$	$\mathfrak{B} = -\frac{14}{11}$	$B = -\frac{14}{25}$
$Pk = 10$	$\mathfrak{C} = \frac{3}{4}$	$C = 3$
$Pkk' = 300$	$\mathfrak{D} = \frac{93}{11}$	$D = -\frac{93}{82}$
$Pkk'S = 150$	$\mathfrak{E} = \frac{37}{11}$	$E = -\frac{37}{26}$
$Pkk'ST = 100$	$\mathfrak{F} = 1$	$F = \infty$

horum igitur numerorum logarithmos in subsidium calculi adponamus

logar. $P = 1,1760913$	$l\mathfrak{B} = (-)0,1047358$	$lB = (-)9,7481883$
logar. $Pk = 1,0000000$	$l\mathfrak{C} = (+)9,8750613$	$lC = (+)0,4771213$
log. $Pkk' = 2,4771213$	$l\mathfrak{D} = (+)0,9270902$	$lD = (-)0,0546690$
$lPkk'S = 2,1760913$	$l\mathfrak{E} = (+)0,5268090$	$lE = (-)0,1532284$
$lPkk'ST = 2,0000000$	$l\mathfrak{F} = (+)0,0000000$	$lF = \infty$.

§. 31. Ex his igitur valoribus definiamus singularum lentium distantias determinatrices, quas cum suis logarithmis singulas hic apponamus

$b = -0,0666\,\alpha$	$lb = (-)8,8239087$	$\beta = +0,0373.\alpha$
$c = +0,0500\,\alpha$	$lc = (+)8,7481883$	$\gamma = +0,1680.\alpha$
$d = +0,0056\,\alpha$	$ld = (+)7,7481883$	$\delta = -0,0063.\alpha$
$e = +0,0127\,\alpha$	$le = (+)8,1038873$	$\varepsilon = -0,0181.\alpha$
$f = +0,0271\,\alpha$	$lf = (+)8,4332070$	$\zeta = \infty$.

$l\beta = (+)8,5720970$
$l\gamma = (+)9,2253096$
$l\delta = (-)7,8028573$
$l\varepsilon = (-)8,2571157$
$l\zeta = \infty$.

§. 32. Ex his porro valoribus deriuemus interualla lentium, tum vero etiam distantias focales cum suis logarithmis:

$I - II = \alpha + b = 0,9333.\alpha$	$q = 0,0848.\alpha$	$lq = 8,9286445$
$II - III = \beta + c = 0,0933.\alpha$	$r = 0,0420.\alpha$	$lr = 8,6232496$
$III - IV = \gamma + d = 0,1736.\alpha$	$s = 0,0473.\alpha$	$ls = 8,6752785$
$IV - V = \delta + e = 0,0064.\alpha$	$t = 0,0427.\alpha$	$lt = 8,6306963$
$V - VI = \varepsilon + f = 0,0090.\alpha$	$u = 0,0271.\alpha$	$lu = 8,4332070$

distantia oculi ab vltima lente $= \frac{u(m+1)}{3\sqrt{m}} = 0,010\,\alpha$.

§. 33. Nunc igitur videamus quanta confusio ex tertia lente oriatur, quae, vt tam parua oriatur quam fieri potest sumamus $\lambda'' = 1$, et cum sit

$$l - \frac{1}{B^3 P k} = 9,7554351 \text{ erit } \frac{\lambda''}{B^3 C^3 P k} = 1,3497$$

vnde; si pro reliquis lentibus circiter vnam partem quartam vnitatis computemus, fiet $\Omega = 1,60$; hinc igitur, pro constructione lentis triplicatae quam supra tertio loco descripsimus, habebimus

$$\lambda = 2,37 \text{ hinc } \lambda - 1 = 1,37 \text{ et } \sqrt{\lambda - 1} = 1,17$$

quocirca pro lente obiectiua composita lentis primae coronariae

radius faciei anterioris erit $= \frac{0,43\,\Pi}{0,62}$ et posterioris $= \frac{0,43\,\Pi}{1,41}$; hinc igitur si sumamus $\Pi = \frac{m}{4} = 25$ pro hac lente reperietur

Radius faciei $\begin{cases} \text{anterioris} = 19\tfrac{1}{2} \text{ dig.} \\ \text{posterioris} = 8\tfrac{1}{2} \text{ dig.} \end{cases}$

binae reliquae lentes ad obiectiuam pertinentes maneant vti supra sunt assignatae, sumto $\Pi = \frac{m}{4} = 25$ dig. Tum vero vt reliquae lentes cum hac obiectiua debite iungantur, sumi debet

$$\alpha = 0,2558\, m = 25,58 \text{ dig.}$$

et lens obiectiua ante nostram sequentem lentem constitui debet ad distantiam

$$\Pi + b = \tfrac{1}{4} m - 0,066\,\alpha = 23,31 \text{ digit.}$$

§. 34. Cum igitur sit $\alpha = 25,58$ digit. erit $l\alpha = 1,40790$ vnde colligimus distantias focales sequentium lentium in digitis

$q = 2,170$ dig.; $r = 1,074$ dig.; $s = 1,211$ dig.; $t = 1,092$ dig.; $u = 0,693$

tum vero interualla harum lentium omisso primo quippe quod iam est assignatum erunt

$II - III = 2,39$ dig.; $III - IV = 4,45$ dig.;
$IV - V = 0,16$ dig.; $V - VI = 0,23$ dig.

quibus accedit distantia oculi ab vltima lente $= 0,23$ dig. interim tamen etiam primum interuallum quod hic erat $\alpha + b = 23,91$ ideo notari meretur: quoniam, si in hoc loco obiectum constituatur, eius imago per secundum lentem q proiecta in locum tertiae lentis cadere debet.

§. 35. Restat igitur vt adhuc constructionem sequentium lentium doceamus. Pro secunda quidem lente q supra notauimus sumi posse $\lambda' = 1$; vt autem haec lens etiam alteram partem aperturae a litera X pendentem recipere possit, statuamus $\lambda' = 2$ et eius constructio ita si habebit

Radius faciei anterioris $= \frac{q}{\sigma - \mathfrak{B}(\sigma - \varrho) - \tau} = \frac{q}{1,0614} = 1,16$ dig.

Radius faciei posterioris $= \frac{q}{\varrho + \mathfrak{B}(\sigma - \varrho) + \tau} = \frac{q}{0,0029} = 869,71$ d.

vbi est $\mathfrak{B}(\sigma - \varrho) = -1,1264$.

Patet ergo hanc lentem conuexo-planam confici posse, dummodo distantiam focalem assignatam obtineat: tanto maiorem autem industriam adhibere opportet in formatione tertiae lentis pro qua sumsimus $\lambda'' = 1$ vnde hinc erit pro ista lente

Radius faciei anterioris $= \frac{r}{\sigma - \mathfrak{C}(\sigma - \varrho)} = \frac{r}{0,5050} = 1,836$ dig.

Radius faciei posterioris $= \frac{r}{\varrho + \mathfrak{C}(\sigma - \varrho)} = \frac{r}{1,3019} = 0,825$ dig.

vbi est $\mathfrak{C}(\sigma - \varrho) = 1,07520$.

Reliquae tres lentes debent esse aequaliter conuexae vnde erit

Radius vtriusque faciei

Pro lente quarta $s = 1,283$ dig.

Pro lente quinta $t = 1,157$ dig.

Pro lente sexta $u = 0,734$ dig.

CON-

CONSTRVCTIO TELESCOPII pro multiplicatione $m = 100$.

§. 36. Constabit igitur hoc telescopium ex sex lentibus, quarum prima autem est triplicata: constructio sequentibus articulis continetur.

I°. Lentis obiectiuae distantia focalis est $= 25$ dig. et semidiameter aperturae $= 2$ dig.

1°. Eius primae lentis coronariae conuexae distantia focalis est $= 11,137$ dig. et

Radius faciei $\begin{cases} \text{anterioris} = 19\tfrac{1}{3} \text{ dig.} \\ \text{posterioris} = 8\tfrac{1}{3} \text{ dig.} \end{cases}$

2°. A medio huius lentis ad medium sequentis, interuallum est $= 0,566$ dig.

3°. Secundae lentis conuexae et ex vitro crystallino parandae distantia focalis est $= -6,296$ dig. et

Radius vtriusque faciei $= -7,883$ dig.

4°. A medio huius ad medium tertiae, interuallum esto $= 0,566$ dig.

5°. Tertiae lentis coronariae distantia focalis est $= 11,010$ digit. et

Radius vtriusque faciei $= 11,671$.

II. A lente obiectiua vsque ad secundam lentem statuatur distantia $= 23,31$.

III. Secundae lentis ex vitro coronario parandae distantia focalis $= 2,170$ dig. et

 Radius

Radius faciei $\begin{cases} \text{anterioris} = 1,160 \text{ dig.} \\ \text{posterioris} = 869,710 \text{ dig.} \end{cases}$

Tum vero eius semidiameter aperturae $= 0,163 + \frac{2}{11}$ dig.

IV. Ab hac lente vsque ad tertiam statuatur interuallum $= 2,39$ dig.

V. Tertiae lentis itidem coronariae distantia focalis est $= 1,074\frac{1}{4}$ dig.

Radius faciei $\begin{cases} \text{anterioris} = 1,836 \text{ dig.} \\ \text{posterioris} = 0,825 \text{ dig.} \end{cases}$

Et semidiameter aperturae $= 0,4 + \frac{1}{4}$ dig.

VI. A lente tertia vsque ad quartam statuatur interuallum $= 4,45$ dig.

VII. Quartae lentis coronariae distantia focalis $= 1,211$ dig.

Radius vtriusque faciei $= 1,283$ dig.

Et semidiameter aperturae $= 0,303 + \frac{1}{10}$ dig.

VIII. Ab hac lente vsque ad quintam interuallum $= 0,16$ dig.

IX. Quintae lentis coronariae distantia focalis $= 1,092$ dig.

Radius vtriusque faciei $= 1,157$ dig. et

Semidiameter aperturae $= 0,273 + \frac{1}{11}$ digit.

X. Ab hac lente ad vltimam statuatur interuallum $= 0,23$ dig.

XI. Vltimae lentis distantia focalis est $= 0,693$

Radius vtriusque faciei $= 0,734$ et

Semi-

Semidiameter aperturae $= 0,173 + \frac{11}{30}$ dig.

XII. Ab hac lente vsque ad oculum distantia sit $= 0,23$ dig.

XIII. Tum vero semidiameter campi apparentis erit $= 23\frac{1}{3}$ min. qui instar spatii circularis in coelo spectabitur cuius semidiameter est $= 34$ gr. 10 min.

XIV. Tandem longitudo totius tubi erit $32,47$ dig.

Quia tres lentes vltimae proprie lentem ocularem constituunt, eae simul capsulae mobili inserantur, vt pro indole oculi paulisper vel admoueri vel remoueri possint.

ADDI-