COMMENTATIO HYPOTHETICA

DE PERICVLO, A NIMIA COMETAE APPROPINQVATIONE METVENDO.

Auctore

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Cum haec quaestio sine dubio maximi sit momenti, neque tamen ob summas calculi difficultates quicquam certi adhuc definiri potuerit, laborem haud ingratum me susscepturum esse arbitror, si hypothetice casum finxero, quo cometa proxime ad terram esset accessurus; atque omnes mutationes, quos tam terra quam cometa in motu forent passuri, accuratius determinavero. Quod quo facilius exsequi liceat, cometam in ipso plano eclipticae moueri assumam, vt saltem difficultates calculi ex diuersitate planorum oriundas remoueam. Hunc in finem ante omnia formulas generales pro motu tam cometae quam terrae quatenus in se inuicem agunt perpendi conueniet.

§. 1. Sit igitur S centrum solis pro fixo habendum, eiusque massa sit $= A$; tum vero ad tempus quodcunque $= t$ reperiatur centrum terrae in T, cometa vero in Z, ac ponatur massa terrae $= B$, cometae autem $= C$: vbi ex parallaxi solis nuper ac- T. XXIV. Fig. 1.

curatissime determinata compertum est esse propemodum $A:B = 360000:1$. Iam ponantur distantiae a Sole $ST = u$ et $SZ = v$; at distantiae terram inter et cometam $TZ = w$. Et assumta directione fixa $S♈$, quae in plano eclipticae ad initium arietis tendat, ad eam ex T et Z perpendicula demittantur TX et Zx, vocenturque coordinatae pro terra $SX = X$ et $XT = Y$; at pro cometa $Sx = x$ et $xZ = y$. Praeterea ponantur anguli $♈ST = \Phi$ et $♈SZ = \omega$, vt hinc obtineamus coordinatas

$$SX = X = u \cos. \Phi \text{ et } XT = Y = u \sin. \Phi.$$

Tum vero

$$Sx = x = v \cos. \omega \text{ et } xZ = y = v \sin. \omega,$$

hincque duplici modo erit

$$TZ = w = \sqrt{(x-X)^2 + (y-Y)^2};$$

deinde vero etiam

$$w = \sqrt{uu + vv - 2uv \cos. (\omega - \Phi)}.$$

§. 2. His constitutis erit vis qua sol terram attrahit $= \frac{A}{uu}$, vnde oritur vis secundum

$$XS = \frac{AX}{u^3} = \frac{A \cos. \Phi}{uu} \text{ et vis secundum } TX = \frac{AY}{u^3} = \frac{A \sin. \Phi}{uu}.$$

Deinde quia cometa ad solem vrgetur vi $\frac{A}{vv}$, hinc nascentur vires secundum

$$xS = \frac{Ax}{v^3} = \frac{A \cos. \omega}{vv} \text{ et secundum } Zx = \frac{Ay}{vv} = \frac{A \sin. \omega}{vv}.$$

Porro quia terra trahitur versus cometam in Z vi $= \frac{C}{ww}$, hinc oritur vis secundum

$$SX = \frac{C(x-X)}{w^3} = \frac{C(v \cos. \omega - u \cos. \Phi)}{w^3}, \text{ et vis secundum}$$

$$XT = \frac{C(y-Y)}{w^3} = \frac{C(v \sin. \omega - u \sin. \Phi)}{w^3}.$$

Vicis-

Viciſſim autem cometa ad terram vrgetur vi $= \frac{B}{ww}$, vnde naſcitur vis ſecundum

$$x\,S = \frac{B(x - X)}{w^3} = \frac{B(v \cos.\omega - u \cos.\Phi)}{w^3} \text{ et ſecundum}$$

$$Z\,x = \frac{B(y - Y)}{w^3} = \frac{B(v \sin.\omega - u \sin.\Phi)}{w^3}.$$

His igitur viribus coniunctis terra ſollicitabitur his viribus acceleratricibus:

$$\text{vi ſecundum } X\,S = \frac{A \cos.\Phi}{uu} - \frac{C(v \cos.\omega - u \cos.\Phi)}{w^3}$$

$$\text{vi ſecundum } T\,X = \frac{A \sin.\Phi}{uu} - \frac{C(v \sin.\omega - u \sin.\Phi)}{w^3}$$

cometa autem ſollicitabitur

$$\text{vi ſecundum } x\,S = \frac{A \cos.\omega}{vv} + \frac{B(v \cos.\omega - u \cos.\Phi)}{w^3} \text{ et}$$

$$\text{vi ſecundum } Z\,x = \frac{A \sin.\omega}{vv} + \frac{B(v \sin.\omega - u \sin.\Phi)}{w^3}.$$

§ 3. Quia vero etiam ſol ad terram trahitur vi ſecundum $S\,T = \frac{B}{uu}$, quae reſoluta dat ſecundum directionem $S\,X$ vim $\frac{B \cos.\Phi}{uu}$ et ſecundum directionem $X\,T$ vim $\frac{B \sin.\Phi}{uu}$, pari modo etiam ſol ad cometam vrgetur vi $\frac{C}{vv}$, quae reſoluta dat pro directione $S\,x$ vim $\frac{C \cos.\omega}{vv}$ et pro directione $x\,Z$ vim $\frac{C \sin.\omega}{vv}$. Quare cum ſolem in quiete ſtatuamus, has vires mutatis ſignis inſuper ad vires quibus terra et cometa ſollicitantur adiici oportet, vnde pro terra habebuntur vires ſequentes:

$$\text{ſecundum } X\,S \text{ vim } \frac{(A+B)\cos.\Phi}{uu} - \frac{C(v \cos.\omega - u \cos.\Phi)}{w^3} + \frac{C \cos.\omega}{vv}$$

$$\text{ſecundum } T\,X \text{ vim } \frac{(A+B)\sin.\Phi}{uu} - \frac{C(v \sin.\omega - u \sin.\Phi)}{w^3} + \frac{C \sin.\omega}{vv}.$$

At vero cometa vrgebitur ſequentibus viribus:

$$\text{ſecundum } x\,S \text{ vi } \frac{(A+C)\cos.\omega}{vv} + \frac{B(v \cos.\omega - u \cos.\Phi)}{w^3} + \frac{B \cos.\Phi}{uu}$$

 ſecun-

ſecundum Zx vi $\frac{(A+C)\,ſin.\,\omega}{vv} + \frac{B(v\,ſin.\,\omega - u\,ſin.\,\Phi)}{w^3} + \frac{B\,ſin.\,\Phi}{uu}$ quae vires iam ſunt acceleratrices, ita vt non opus ſit eas per maſſas in quas agunt diuidere.

§. 4. His igitur viribus inuentis pro motu terrae ſequentes binas adipiſcimur aequationes:

$$\frac{ddX}{2g\,dt^2} = -\frac{(A+B)\,coſ.\,\Phi}{uu} + \frac{C(v\,coſ.\,\omega - u\,coſ.\,\Phi)}{w^3} - \frac{C\,coſ.\,\omega}{vv}$$

$$\frac{ddY}{2g\,dt^2} = -\frac{(A+B)\,ſin.\,\Phi}{uu} + \frac{C(v\,ſin.\,\omega - u\,ſin.\,\Phi)}{w^3} - \frac{C\,ſin.\,\omega}{vv}$$

ſimili autem modo pro motu cometae habebimus ſequentes duas aequationes:

$$\frac{ddx}{2g\,dt^2} = -\frac{(A+C)\,coſ.\,\omega}{vv} - \frac{B(v\,coſ.\,\omega - u\,coſ.\,\Phi)}{w^3} - \frac{B\,coſ.\,\Phi}{uu}$$

$$\frac{ddy}{2g\,dt^2} = -\frac{(A+C)\,ſin.\,\omega}{vv} - \frac{B(v\,ſin.\,\omega - u\,ſin.\,\Phi)}{w^3} - \frac{B\,ſin.\,\Phi}{uu}.$$

In his aequationibus differentio-differentialibus elementum temporis dt aſſumtum eſt conſtans, et ſecundum regulas in Mechanica traditas g denotat altitudinem lapſus grauium vno minuto ſecundo, ſi quidem tempora in minutis ſecundis exprimere velimus; qui computus cum hic foret nimis prolixus, mox aliam temporum menſuram introducemus, ex motu ſcilicet terrae medio petitam.

§. 5. Quoniam autem hic coordinatas orthogonales X, Y et x, y ideo in calculum induximus, vt principia motus in Mechanica tradita, quae ad certas directiones fixas reſpiciunt, applicare licuerit, nunc in gratiam calculi aſtronomici eas iterum elidi conueniet; quem in finem pro terra ſequentibus combinationibus vtamur:

I,

$$\text{I. } \frac{ddX\cos.\Phi + ddY\sin.\Phi}{2g\,dt^2} = -\frac{(A+B)}{uu} + \frac{C(v\cos.(\omega-\Phi)-u)}{w^3} - \frac{C\cos.(\omega-\Phi)}{vv}$$

$$\text{II. } \frac{ddY\cos.\Phi - ddX\sin.\Phi}{2g\,dt^2} = \frac{Cv\sin.(\omega-\Phi)}{w^3} - \frac{C\sin.(\omega-\Phi)}{vv}.$$

Simili modo pro cometa adhibeamus ſequentes combinationes:

$$\text{I. } \frac{ddx\cos.\omega + ddy\sin.\omega}{2g\,dt^2} = -\frac{(A+C)}{vv} - \frac{B(v-u\cos.(\omega-\Phi))}{w^3} - \frac{B\cos.(\omega-\Phi)}{uu}$$

$$\text{II. } \frac{ddy\cos.\omega - ddx\sin.\omega}{2g\,dt^2} = -\frac{Bu\sin.(\omega-\Phi)}{w^3} - \frac{B\sin.(\omega-\Phi)}{uu}.$$

§. 6. Verum ad literas X, Y, x et y penitus extrudendas notemus eſſe

$$X\cos.\Phi + Y\sin.\Phi = u \text{ et } Y\cos.\Phi - X\sin.\Phi = 0,$$

hinc differentiando erit

$$dX\cos.\Phi + dY\sin.\Phi - Xd\Phi\sin.\Phi + Yd\Phi\cos.\Phi = du$$

hincque

$$dX\cos.\Phi + dY\sin.\Phi = du. \text{ Deinde}$$

$$dY\cos.\Phi - dX\sin.\Phi - Yd\Phi\sin.\Phi - Xd\Phi\cos.\Phi = 0, \text{ ergo}$$

$$dY\cos.\Phi - dX\sin.\Phi = +ud\Phi,$$

Ac denuo differentiando fiet

$$ddX\cos.\Phi + ddY\sin.\Phi - dXd\Phi\sin.\Phi + dYd\Phi\cos.\Phi = ddu,$$

ergo

$$ddX\cos.\Phi + ddY\sin.\Phi = ddu - ud\Phi^2. \text{ Porro}$$

$$ddY\cos.\Phi - ddX\sin.\Phi = ud\Phi + dYa\Phi\sin.\Phi + dxd\Phi\cos.\Phi$$

ſiue

$$ddY\cos.\Phi - ddX\sin.\Phi = udd\Phi + 2\,dud\Phi.$$

His igitur valoribus ſubſtitutis pro motu terrae has duas obtinebimus aequationes:

$$\text{I. } \frac{ddu - ud\Phi^2}{2g\,dt^2} = -\frac{(A+B)}{uu} + \frac{C(v\cos.(\omega-\Phi)-u)}{w^3} - \frac{C\cos.(\omega-\Phi)}{vv}$$

$$\text{II. } \frac{udd\Phi + 2dud\Phi}{2g\,dt^2} = \frac{Cv\sin.(\omega-\Phi)}{w^3} - \frac{C\sin.(\omega-\Phi)}{vv}.$$

§. 7.

§. 7. Simili modo pro cometa cum ſit
$x \operatorname{cof.} \omega + y \operatorname{fin.} \omega = v$ et $y \operatorname{cof.} \omega - x \operatorname{fin.} \omega = 0$.
erit differentiando
$dx \operatorname{cof.} \omega + dy \operatorname{fin.} \omega = dv$ et $dy \operatorname{cof.} \omega - dx \operatorname{fin.} \omega = v d\omega$
ac denuo differentiando
$ddx \operatorname{cof.} \omega + ddy \operatorname{fin.} \omega = ddv - v d\omega^2$ et
$ddy \operatorname{cof.} \omega - ddx \operatorname{fin.} \omega = v dd\omega + 2 dv d\omega$.
vnde motus cometae his ſequentibus aequationibus exprimetur:

$$\frac{ddv - v d\omega^2}{2g dt^2} = -\frac{(A+C)}{vv} - \frac{B(v - u \operatorname{cof.}(\omega - \Phi))}{w^3} - \frac{B \operatorname{cof.}(\omega - \Phi)}{uu}$$

$$\frac{v dd\omega + 2 dv d\omega}{2g dt^2} = -\frac{B u \operatorname{fin.}(\omega - \Phi)}{w^3} - \frac{B \operatorname{fin.}(\omega - \Phi)}{uu}.$$

Sicque omnia reducuntur ad binas diſtantias u et v et binos angulos Φ et ω, quae ad quoduis tempus determinari oportet.

§. 8. Quod ſi vero omnia potius per ipſas coordinatas X, Y et x, y exprimere malimus, ita vt ſit

$u = \sqrt{X^2 + Y^2}$; $v = \sqrt{xx + yy}$ et $\omega = \sqrt{(x-X)^2 + (y-Y)^2}$;

tum vero pro angulis

$\operatorname{cof.} \Phi = \frac{X}{u}$, $\operatorname{fin.} \Phi = \frac{Y}{u}$, $\operatorname{cof.} \omega = \frac{x}{v}$, $\operatorname{fin.} \omega = \frac{y}{v}$

aequationes immediate ex principiis mechanicis deductae dabunt primo pro terra

$$\frac{ddX}{2g dt^2} = -\frac{(A+B)X}{u^3} + \frac{C(x-X)}{w^3} - \frac{Cx}{v^3}$$

$$\frac{ddY}{2g dt^2} = -\frac{(A+B)Y}{u^3} + \frac{C(y-Y)}{w^3} - \frac{Cy}{v^3}$$

ſimilique modo pro motu cometae

$$\frac{ddx}{2g dt^2} = -\frac{(A+C)x}{v^3} - \frac{B(x-X)}{w^3} - \frac{BX}{u^3}$$

$$\frac{ddy}{2g dt^2} = -\frac{(A+C)y}{v^3} - \frac{B(y-Y)}{w^3} - \frac{BY}{u^3}.$$

§. 9.

§. 9. Praemissis igitur his formulis generalibus, quibus tam motus terrae quam cometae ob actionem mutuam perturbatur, vtrumque motum seorsim neglecta actione mutua euoluamus, et cursum cometae in plano eclipticae ita instituamus, vt, vbi per orbitam terrae est transiturus, ibi simul ipsam terram offendat, et quasi contingat. Ante vero quam hoc eueniat, cum cometa iam terrae ita apropinquauerit, vt actio mutua iam satis notabilis euadat ac fortasse actionem Solis superet, tum nobis erit inuestigandum, quantam mutationem vterque motus sit subiturus, vt inde Phaenomena definire liceat, quae tam ante quam post hanc coniunctionem apparere debeant.

§. 10. Primo igitur cum inaequalitates motus terrae ex excentricitate oriundae nihil ad hanc inuestigationem conferant, ipsam terram in circulo circa solem motu vniformi circumferri assumamus, cuius circuli radius sit distantia media terrae a sole $= a$; tum vero tempore t circa solem conficiat angulum $= \vartheta$, quem pro tempore vnius diei nouimus esse $= 59'. 8''$ siue in partibus radii spatium $= 0,017204$ posito scilicet sinu toto $= 1$. Quamobrem reiectis terminis ab actione cometae ortis habebimus primo eius distantiam a sole $u = a$, tum vero pro tempore t angulum $\Phi = \vartheta$; quibus valoribus substitutis formulae (§. 6.) datae ob $dd\Phi = dd\vartheta = 0$, quia motus est vniformis, erunt

prima $- \frac{a\,d\vartheta^2}{2g\,dt^2} = - \frac{A + B}{aa}$ altera vero $0 = 0$.

Hinc igitur loco temporis t angulum seu arcum ϑ in calculum introducere poterimus, dum ex ista aequalitate obtinemus $2 g\, d t^2 = \frac{a^3 d\vartheta^2}{A + B}$; quare si hunc valorem introducamus, et aequationes generales per $2 g\, d t^2$ multiplicemus, eae sequentes formas induent:

$$\text{I. } ddu - ud\Phi^2 = -\frac{a^3 d\vartheta^2}{uu} + \frac{a^3 C d\vartheta^2 (v \operatorname{cos}.(\omega - \Phi) - u)}{w^3 (A + B)} - \frac{a^3 C d\vartheta^2 \operatorname{cos}.(\omega - \Phi)}{vv (A + B)}$$

$$\text{II. } udd\Phi + 2dud\Phi = \frac{a^3 C v\, d\vartheta^2 \operatorname{fin}.(\omega - \Phi)}{w^3 (A + B)} - \frac{C a^3 d\vartheta^2 \operatorname{fin}.(\omega - \Phi)}{vv (A + B)}.$$

Quia autem massa C prae massa solis A est quam minima, ponamus breuitatis gratia fractionem $\frac{C}{A+B} = m$, vt habeamus has aequationes:

$$\text{I. } ddu - ud\Phi^2 = -\frac{a^3 d\vartheta^2}{uu} + \frac{m a^3 d\vartheta^2 (v \operatorname{cos}.(\omega - \Phi) - u)}{w^3} - \frac{m a^3 d\vartheta^2 \operatorname{cos}.(\omega - \Phi)}{vv}$$

$$\text{II. } udd\Phi + 2dud\Phi = \frac{m a^3 d\vartheta^2 \operatorname{fin}.(\omega - \Phi)}{w^3} - \frac{m a^3 d\vartheta^2 \operatorname{fin}.(\omega - \Phi)}{vv}.$$

§. 11. At si ipsas coordinatas X et Y in calculo retinere velimus, aequationes supra §. 8. exhibitae, si etiam per $2 g\, d t^2$ multiplicentur induent has formas:

$$d d X = -\frac{a^3 X d\vartheta^2}{u^3} + \frac{m a^3 d\vartheta^2 (x - X)}{w^3} - \frac{m a^3 x\, d\vartheta^2}{v^3}$$

$$d d Y = -\frac{a^3 Y d\vartheta^2}{u^3} + \frac{m a^3 d\vartheta^2 (y - Y)}{w^3} - \frac{m a^3 y\, d\vartheta^2}{v^3}$$

hae scilicet aequationes erunt euoluendae, postquam actio mutua terram inter et cometam sensibilis fieri coeperit; ante vero hunc terminum, quamdiu motus terrae adhuc fuerit vniformis, ob $u = a$ et $\Phi = \vartheta$, erit $X = a \operatorname{cof}.\vartheta$ et $Y = a \operatorname{fin}.\vartheta$, hincque porro $\frac{dX}{d\vartheta} = -a \operatorname{fin}.\vartheta$ et $\frac{dY}{d\vartheta} = a \operatorname{cof}.\vartheta$; vbi istae formulae $\frac{dX}{d\vartheta}$ et $\frac{dY}{d\vartheta}$

denotant celeritates terrae secundum directionem coordinatarum; vnde ipso momento, quo terra per directiorectionem fixam S ♈ transit, quandoquidem in hoc loco actionem cometae incipere statuemus, erit

$$X = a,\ Y = 0 \text{ et } \frac{dX}{d\vartheta} = 0 \text{ et } \frac{dY}{d\vartheta} = a.$$

§. 12. Nunc vero pro motu cometae in plano eclipticae, quo calculus simplicior reddatur, ponamus cometam recta versus solem cursum suum tendere, quoniam obliquitas cursus ad nostrum institutum nihil confert. Hoc modo remota actione terrae angulus ♈ S Z $= \omega$ erit constans puta $= \alpha$, vnde aequationes pro cometa (§. 7.) erunt

$$\frac{ddv}{2g\,dt^2} = -\frac{A+C}{vv} \text{ et } 0 = 0,$$

vnde si loco $2g\,dt^2$ valorem supra stabilitum substituamus habebimus

$$ddv = -\frac{a^3\,d\vartheta^2\,(A+C)}{vv\,(A+B)};$$

quia vero massa A prae B et C est quasi infinita, erit

$$\frac{A+C}{A+B} = 1, \text{ ideoque } ddv = -\frac{a^3\,d\vartheta^2}{vv},$$

qua aequatione motus cometae per lineam rectam Z S exprimitur. Eam igitur per $2dv$ multiplicemus et integremus, atque ob $d\vartheta$ constans obtinebimus

$$dv^2 = \frac{2a^3\,d\vartheta^2}{v}, \text{ siue } \frac{dv^2}{d\vartheta^2} = \frac{2a^3}{v} + C,$$

vbi $\frac{dv}{d\vartheta}$ denotat celeritatem, qua cometa a sole recederet. Verum ex theoria cometarum constat, eos ita moueri ac si ex distantia infinita motum incepissent; vnde patet, constantem hanc esse $= 0$; quare cum cometam ad solem accedere assumamus eius celeritas erit $\frac{dv}{d\vartheta} = -\sqrt{\frac{2a^3}{v}}$.

T. XXIV. Fig. 2. §. 13. Pro motu autem penitus determinando ex hac aequatione deducimus $d\vartheta = -\frac{dv\sqrt{v}}{\sqrt{2a^3}}$, cuius integrale est $\vartheta = C - \frac{2v\sqrt{v}}{3\sqrt{2a^3}}$. Ad constantem autem determinandam ponamus initio, quo terra transiit per directionem fixam S♈ in puncto B (quoniam hoc tempore actionem mutuam incipere deinceps statuemus) cometam fuisse in C, existente distantia $SC = c$, ac tempus quo ex C in Z vsque perueniet erit $\vartheta = \frac{2c\sqrt{c} - 2v\sqrt{v}}{3\sqrt{2a^3}}$. Quare si iam ponatur $v = SO = a$, habebimus tempus, quo cometa ex C vsque ad eclipticam in O pertingit $= \frac{2c\sqrt{c} - 2a\sqrt{a}}{3\sqrt{2a^3}}$.

§. 14. Cum igitur angulus BOC positus sit $= \alpha$, tempus quo terra ex B ad eundem locum O perueniet erit $\vartheta = \alpha$; necesse igitur est vt fiat

$$\alpha = \frac{2c\sqrt{c} - 2a\sqrt{a}}{3\sqrt{2a^3}},$$

vnde colligimus

$$2c\sqrt{c} = 3\alpha\sqrt{2a^3} + 2a\sqrt{a} = (3\alpha\sqrt{2} + 2)a\sqrt{a}$$

hincque nanciscimur ipsam distantiam

$$SC = c = a\left(\frac{3\alpha}{\sqrt{2}} + 1\right)^{\frac{2}{3}}.$$

Quocirca si initio statuamus terram fuisse in B, existente distantia $SB = a$: simul vero cometam fuisse in C, existente angulo $BSC = \alpha$, et distantia

$$SC = c = a\left(\frac{3\alpha + \sqrt{2}}{\sqrt{2}}\right)^{\frac{2}{3}}$$

tum tam terra quam cometa ob motum proprium, neglecta scilicet mutua perturbatione, elapso tempore $= \alpha$ simul peruenient in idem punctum O, ibique

que idcirco conflictum exercerent, quem autem fortasse ob actionem mutuam euitabunt, propterea quod vtriusque motus ob mutuam actionem non mediocriter immutabitur.

§. 15. Quod si igitur assumamus, actionem mutuam terrae et cometae tum demum fieri sensibilem, cum terra versata fuerit in puncto B, cometa autem in puncto C, quod temporis momentum tanquam epocham accipiamus, vnde vtrumque motum deinceps prosequamur; hinc elapso tempore $= \vartheta$ cometa peruenerit in Z, ita vt sit distantia $SZ = v$, et angulus $\Upsilon SZ = \omega$. Vt supra ergo demisso perpendiculo Zx, positisque coordinatis $Sx = x$ et $Zx = y$, aequationes pro motu cometae inter coordinatas x et y erunt sequentes, siquidem fractionem minimam ponamus $\frac{B}{A + B} = n$

$$ddx = -\frac{a^3 x d\vartheta^2}{v^3} - \frac{na^3 d\vartheta^2 (x - X)}{w^3} - \frac{na^3 d\vartheta^2 X}{u^3} \text{ et}$$

$$ddy = -\frac{a^3 y d\vartheta^2}{v^5} - \frac{na^3 d\vartheta^2 (y - Y)}{w^3} - \frac{na^3 d\vartheta^2 Y}{u^3}$$

pro quarum aequationum resolutione notandum est, initio, quo cometa adhuc haesit in C, existente distantia $SC = c$ et angulo $\Upsilon SC = \alpha$, fore coordinatas $x = c \operatorname{cos.} \alpha$ et $y = c \operatorname{sin.} \alpha$. Deinde cum supra inuenerimus $\frac{dv}{d\vartheta} = -\frac{a\sqrt{2a}}{\sqrt{v}}$, erunt celeritates laterales pro initio

$$\frac{dx}{d\vartheta} = \frac{dv}{d\vartheta} \operatorname{cos.} \alpha = -a \operatorname{cos.} \alpha \sqrt{\frac{2a}{c}} \text{ et } \frac{dy}{d\vartheta} \operatorname{sin.} \alpha = -a \operatorname{sin.} \alpha \sqrt{\frac{2a}{c}}$$

supra vero pro eadem epocha iam pro terra dedimus

$$X = a;\ Y = 0;\ \frac{dX}{d\vartheta} = 0;\ \frac{dY}{d\vartheta} = a.$$

§. 16. Nunc angulum α tantum accipiamus, vt tum demum actio mutua terrae et cometae euadat sensibilis, id quod ob summam paruitatem terrae et cometae respectu solis non ante contingere statuamus, quam duobus circiter diebus ante coniunctionem in O, quamobrem statuamus angulum

$$BSC = \alpha = 2\,(59'.\ 8'') = 1^\circ.\ 58'.\ 17'',$$

cuius valor in partibus radii est 0, 034408; hinc igitur colligetur distantia

$$SC = c = \left(\frac{\alpha\alpha}{\sqrt{2}} + 1\right)^{\frac{2}{3}} a = 1,\ 048086.\ a$$

vnde fit $OC = 0,\ 048086.\ a$, existente angulo

$$BSC = 1^\circ.\ 58'.\ 17''.$$

Hinc igitur pro initio huius epochae habebimus

$$x = 1,\ 047472.\ a \text{ et } y = 0,\ 036054.\ a$$

porro $\frac{dx}{d\vartheta} = -1,\ 380581\ a$; $\frac{dy}{d\vartheta} = -0,\ 047520\ a$.

§. 17. Constituta igitur hac epocha, vbi terra et cometa primum in se inuicem agere concipiuntur, si ponamus hinc elapsum esse tempus $= \vartheta$, primo pro motu terrae sequentes binas nacti sumus aequationes:

$$ddX = -\frac{a^3 X\, d\vartheta^2}{u^3} + \frac{m a^3\, d\vartheta^2 (x - X)}{w^3} - \frac{m a^3 x\, d\vartheta^2}{v^3}$$

$$ddY = -\frac{a^3 Y\, d\vartheta^2}{u^3} + \frac{m a^3\, d\vartheta^2 (y - Y)}{w^3} - \frac{m a^3 y\, d\vartheta^2}{v^3}.$$

Pro motu cometae autem istas binas:

$$ddx = -\frac{a^3 x\, d\vartheta^2}{v^3} - \frac{n a^3\, d\vartheta^2 (x - X)}{w^3} - \frac{n a^3 X\, d\vartheta^2}{u^3}$$

$$ddy = -\frac{a^3 y\, d\vartheta^2}{v^3} - \frac{n a^3\, d\vartheta^2 (y - Y)}{w^3} - \frac{n a^3 Y\, d\vartheta^2}{u^3}.$$

Has

Has autem aequationes non solum nullo modo integrare licet, sed etiam solita methodus appropinquandi inde petita, quod perturbationes tanquam minimae spectari queant, hic locum habere nequit; quandoquidem hic euenire potest, vt actio mutua terrae et cometae adeo superet actionem solis, cum scilicet satis prope ad se inuicem accesserint. Quamobrem alia via non superest, nisi vt per gradus satis exiguos vtrumque motum ex ipsis formulis differentio-differentialibus prosequamur, dum elemento temporis $d\vartheta$ successiue valores satis exiguos tribuemus vt hinc nullus error sit metuendus.

Praeparatio harum aequationum ad calculum sequentem.

§. 18. Primum hic necesse est omnia elementa quae in has formulas ingrediuntur ad mensuras determinatas magisque cognitas reduci. Primo igitur distantiam terrae mediam $= a$ per semidiametros terrestres exprimamus, quoniam haec mensura maxime idonea videtur ad distantiam cometae a terra definiendam. Hinc ex parallaxi solis nuper inuenta statuamus $a = 24000$ semidiametris terrae. Deinde cum fractio $n = \frac{B}{A+B}$ sit satis exacte $\frac{1}{360000}$, tribuamus cometae etiam massam terrae aequalem, vt sit quoque $m = \frac{1}{360000}$, atque hinc per logarithmos habimus

$$l a^3 = 13,1406336, \; l m a^2 = l n a^3 = 7,5843311.$$

Porro autem mensura temporis, quam hic introduximus

mus per angulum ϑ, qui respondet motui medio terrae, nimis est incommoda et non satis clara; eius ergo loco potius conueniet tempus ab epocha nostra elapsum per dies naturales exprimere, quorum numerum ponamus $= \tau$. Tum igitur ob motum diurnum $= 59'.8'' = 0,017204$ erit $\vartheta = 0,017204\,\tau$ ideoque

$$d\vartheta = 0,017204\, d\tau.$$

Quod si nunc breuitatis gratia ponamus

$$a^3 d\vartheta^2 = \Delta\, d\tau^2 \text{ et } m a^3 d\vartheta^2 = n a^3 d\vartheta^2 = \delta\, d\tau$$

habebimus

$$l\Delta = 9,6118924 \text{ et } l\delta = 4,0555899:$$

quibus valoribus substitutis quatuor nostrae aequationes erunt:

$$ddX = -\frac{\Delta X d\tau^2}{u^3} + \frac{\delta(x-X)d\tau^2}{w^3} - \frac{\delta x d\tau^2}{v^3}$$

$$ddY = -\frac{\Delta Y d\tau^2}{u^3} + \frac{\delta(y-Y)d\tau^2}{w^3} - \frac{\delta x d\tau^2}{v^3}$$

$$ddx = -\frac{\Delta x d\tau^2}{v^3} - \frac{\delta(x-X)d\tau^2}{w^3} - \frac{\delta X d\tau^2}{u^3}$$

$$ddy = -\frac{\Delta Y d\tau^2}{v^3} - \frac{\delta(y-Y)d\tau^2}{w^3} - \frac{\delta Y d\tau^2}{u^3}$$

vbi membra postrema ob nimiam paruitatem tuto omitti possunt: media enim membra eatenus tantum in sensum veniunt, quatenus distantia continuo diminuitur.

§. 19. Haec etiam mensura temporis nobis multo clariorem ideam et mensuram celeritatum suppeditat quam ante, vbi verbi gratia $\frac{dX}{d\vartheta}$ exprimebat spatium quod hac celeritate percurri posset tempo-

tempore $\vartheta = 1$, siue per angulum $\vartheta = 57^\circ. 17'$. cui respondet tempus circiter 60 dierum. Nunc igitur easdem celeritates exprimemus per formulas

$$\frac{dX}{d\tau},\ \frac{dY}{d\tau};\ \frac{dx}{d\tau} \text{ et } \frac{dy}{d\tau},$$

quae formulae exprimunt spatia, quae his celeritatibus tempore vnius diei percurrentur, haecque spatia in semidiametris terrae dabuntur. His igitur nouis mensuris introductis, pro initio nostrae epochae nanciscemur sequentes mensuras penitus determinatas in semidiametris terrae expressas

$$X = 24000;\ Y = 0;\ \frac{dX}{d\tau} = 0,\ \frac{dY}{d\tau} = 412,896$$

$$x = 25139,328;\ y = 865,296$$

$$\frac{dx}{d\tau} = -570,036 \text{ et } \frac{dy}{d\tau} = -19,621$$

his igitur praemissis sequens problema principale perpendamus.

Problema.

§. 20. *Si ad tempus τ dierum ab epocha elapsum dentur distantiae X et Y pro terra, et x, y pro cometa; tum vero etiam celeritates $\frac{dX}{d\tau}$ et $\frac{dY}{d\tau}$ pro terra, atque $\frac{dx}{d\tau}$ et $\frac{dy}{d\tau}$ pro cometa, inuenire pro tempore $\tau + d\tau$ dierum ab epocha elapso valores earundem litterarum qui sint X' Y' x' y', vna cum celeritatibus* T. XXIV. Fig. 3.

$$\frac{dX'}{d\tau},\ \frac{dY'}{d\tau},\ \frac{dx'}{d\tau},\ \frac{dy'}{d\tau}.$$

Solutio.

Cum $d\tau$ sit elementum quod pro minimo haberi queat, erit ex natura differentialium

 $X' =$

$$X' = X + dX + \tfrac{1}{2}ddX;\quad Y' = Y + dY + \tfrac{1}{2}ddY$$
$$x' = x + dx + \tfrac{1}{2}ddx;\quad y' = y + dy + \tfrac{1}{2}ddy$$

deinde vero pro celeritatibus

$$\frac{dX'}{d\tau} = \frac{dX}{d\tau} + \frac{ddX}{d\tau};\quad \frac{dY'}{d\tau} = \frac{dY}{d\tau} + \frac{ddY}{d\tau}$$
$$\frac{dx'}{d\tau} = \frac{dx}{d\tau} + \frac{ddx}{d\tau};\quad \frac{dy'}{d\tau} = \frac{dy}{d\tau} + \frac{ddy}{d\tau}$$

vbi valores differentiales fecundi gradus ex noftris aequationibus fupra datis elici debent. Primo igitur ex elementis datis quaeri debent diftantiae $ST = u$ et $SZ = v$, quod commodiffime fiet per angulos $BST = \Phi$ et $BSZ = \omega$, quos quidem per fe nosfe iuuabit: hic autem reperientur ex his formulis

tang. $\Phi = \frac{Y}{X}$ et tang. $\omega = \frac{y}{x}$, quibus inuentis erit

$$u = \frac{X}{\operatorname{cof.}\Phi} = \frac{Y}{\operatorname{fin.}\Phi} \text{ et } v = \frac{x}{\operatorname{cof.}\omega} = \frac{y}{\operatorname{fin.}\omega}.$$

Deinde pro diftantia $TZ = w$ inuenienda ducatur axi parallela TV, quae erit $x - X$, et $VZ = y - Y$; tum vero ponatur angulus $VTZ = \psi$, qui defignabit longitudinem cometae ex terra vifi eritque tang. $\psi = \frac{y - Y}{x - X}$; atque hinc obtinebitur ipfa diftantia

$$TZ = w = \frac{x - X}{\operatorname{cof.}\psi} = \frac{y - Y}{\operatorname{fin.}\psi};$$

His autem valoribus definitis ex fuperioribus aequationibus habebimus

$$ddX = -\frac{\Delta X d\tau^2}{u^3} + \frac{\delta(x - X)d\tau^2}{w^3}$$
$$ddY = -\frac{\Delta Y d\tau^2}{u^3} + \frac{\delta(y - Y)d\tau^2}{w^3}$$
$$ddx = -\frac{\Delta x d\tau^2}{v^3} - \frac{\delta(x - X)d\tau^2}{w^3}$$
$$ddy = -\frac{\Delta y d\tau^2}{v^3} - \frac{\delta(y - Y)d\tau^2}{w^3}.$$

§. 21.

§. 21. Hic autem non amplius elementum $d\tau$ pro infinite paruo habemus, sed potius ei valorem satis exiguum dabimus, quem tam paruum sufficit assumi, vt interea membra nostrarum aequationum nullam mutationem sensibilem subeant. Atque hic facile intelligitur, statim ab initio pro $d\tau$ integri vnius diei interuallum tuto assumi posse, ita vt sit $d\tau = 1$; deinceps vero, cum cometa multo propius ad terram accesserit, interualla $d\tau$ paullatim diminui conueniet, prouti ex circumstantiis facile erit diiudicare.

§. 22. Hoc igitur modo ab ipsa nostra epocha successiue per gradus progrediamur, atque pro primo quidem spatium vnius diei assumere licebit. Atque hac ratione ad quoduis tempus ab epocha nostra elapsum poterimus tam situm terrae et cometae quam vtriusque motum assignare; quam ob rem calculum pro istis temporis interuallis hic apponamus.

Calculus pro ipsa epocha, vbi $\tau = 0$.

§. 23. Elementa igitur pro hoc calculo erunt:

$X = 24000,000 \qquad Y = 0,000$

$x = 25139,328 \qquad y = 865,296$

$x - X = 1139,328 \qquad y - Y = 865,296$

$dX = 0\, d\tau \qquad dY = 412,896.\, d\tau$

$dx = -570,036.\, d\tau; \quad dy = -19,621.\, d\tau$

 vnde

vnde ſtatim habemus $\Phi = 0$ et $u = a = 24000$; tum vero $\omega = 1^\circ.\ 58'.\ 17'$ et $v = c = 25154,08$. Primo igitur tantum quaeratur angulus ψ cum diſtantia w hoc modo:

$a\,l(y-Y) = 2,9371647$	ad $l(x-X) = 3,0566489$
ſubtr. $l(x-X) = 3,0566489$	add. l ſec. $\psi = 10,0988986$
l tang. $\psi = 9,8805158$	$l\,w = 3,1555475$
ideoque $\psi = 37^\circ.\ 13\tfrac{1}{4}'$	ideoque $w = 1430,697$

Nuncm embra noſtrarum aequationum ita computentur

$l\,\Delta = 9,6118924$	$l\,\Delta = 9,6118924$	$l\,\delta = 4,0555899$
$l\,X = 4,3802112$	$l\,x = 4,4003537$	$l(x-X) = 3,0566489$
$= 3,9921036$	$4,0122461$	$7,1122388$
$l\,u^3 = 3,1406336$	$l\,v^3 = 3,2018252$	$l\,w^3 = 9,4666425$
$l\,\frac{\Delta X}{u^3} = 0,8514700$	$l\,\frac{\Delta x}{v^3} = 0,8104209$	$l\,\frac{\delta(x-X)}{w^3} = 7,6455963$
	l tang. $\omega = 8,5368175$	l tang. $\psi = 9,8805158$
hinc $\frac{\Delta X}{u^3} = 7,103$	$l\,\frac{\Delta y}{v^3} = 9,3472384$	$l\,\frac{\delta(y-Y)}{w^3} = 7,5261121$
et $\frac{\Delta Y}{u^3} = 0$	ergo $\frac{\Delta x}{v^3} = 6,463$	ergo $\frac{\delta(x-X)}{w^3} = 0,004$
	et $\frac{\Delta y}{v^3} = 0,222$	et $\frac{\delta(y-Y)}{w^3} = 0,003$

hinc igitur colligitur

$ddX = -7,099\,d\tau^2$	$ddx = -6,467.\,d\tau^2$
$ddY = +0,003\,d\tau^2$	$ddy = -0,225.\,d\tau^2$

quocirca habebimus

$$X' = 24000 + 0.\,d\tau - 3,549.\,d\tau^2$$
$$Y' = 0 + 412,896.\,d\tau + 0,001.\,d\tau^2$$
$$x' = 25139,328 - 570,036.\,d\tau - 3,233.\,d\tau^2$$
$$y' = 865,296 - 19,621.\,d\tau - 0,112.\,d\tau^2$$

deinde

deinde

$$dX' = 0,00.\,d\tau - 7,099.\,d\tau^2$$
$$dY' = 412.896.\,d\tau + 0,003.\,d\tau^2$$
$$dx' = -570,036.\,d\tau - 6,467.\,d\tau^2$$
$$dy' = -19,621.\,d\tau - 0,225.\,d\tau^2.$$

Calculus pro tempore $\tau = 1\,d$ poſt epocham.

§. 24. Sumto igitur $d\tau = 1$ elementa huius calculi erunt

$X = 23996,451$	$Y = 412,897$
$x = 24566,059$	$y = 845,563$
$x - X = 569,608$	$y - Y = 432,666$
$dX = -7,099\,d\tau$	$dY = 412,899\,d\tau$
$dx = -576,503\,d\tau$	$dy = -19,846\,d\tau$

ſuper his igitur elementis calculus ita inſtituatur

$a\,l\,\dot{Y} = 2,6158417$	$a\,l\,y = 2,9271460$	$a\,l(y-Y) = 2,6361527$
ſubtr $lX = 4,3801469$	ſ. $l\,x = 4,3903354$	ſ. $l\,(x-X) = 2,7555761$
$l$tang. $\phi = 8,2356948$	$l$tang. $\omega = 8.5368106$	l tang. $\psi = 9,8805766$
ergo $\phi = 59'.\ 8''$	ergo $\omega = 1^\circ.\ 58'.\ 17$	ergo $\psi = 37^\circ.\ 13\frac{1}{5}'$
ad $l\ X = 4,3801469$	ad $l\,x = 4,3903354$	ad $l(x-X) = 2,7555761$
l ſec. $\phi = 10,0000642$	l ſec. $\omega = 10,0002571$	l ſec. $\psi = 10,0989130$
$l\,u = 4,3802111$	$l\,v = 4,3905925$	$l\,w = 2,8544891$
ergo $u = 24000$	ergo $v = 24580,600$	ergo $w = 715,302$

ad $l\,\Delta = 9{,}6118924$	ad $l\,\Delta = 9{,}6118924$	ad $l\,\delta = 4{,}0555899$
$l\,X = 4{,}3801469$	$l\,x = 4{,}3903354$	$l\,(x-X) = 2{,}7555761$
$3{,}9920393$	$4{,}0022278$	$6{,}8111660$
f $l\,u^3 = 3{,}1406333$	f. $l\,v^3 = 3{,}1717775$	$l\,w^3 = 8{,}5634673$
$l\,\frac{\Delta X}{u^3} = 0{,}8514060$	$l\,\frac{\Delta x}{v^3} = 0{,}8304503$	$l\,\frac{\delta(x-X)}{w^3} = 8{,}2476987$
l tang. $\Phi = 8{,}2356948$	l tang. $\omega = 8{,}5368106$	l tang. $\psi = 9{,}8805766$
$l\,\frac{\Delta Y}{u^3} = 9{,}0871008$	$l\,\frac{\Delta y}{v^3} = 9{,}3672609$	$l\,\frac{\delta(y-Y)}{w^3} = 8{,}1282753$
ergo $\frac{\Delta X}{u^3} = 7{,}102$	ergo $\frac{\Delta x}{v^3} = 6{,}768$	ergo $\frac{\delta(x-X)}{w^3} = 0{,}018$
et $\frac{\Delta Y}{u^3} = 0{,}122$	et $\frac{\Delta y}{v^3} = 0{,}233$	et $\frac{\delta(y-Y)}{w^3} = 0{,}013.$

Annotatio. Membra poſtrema litera δ affecta continent perturbationem ex actione mutua ortam, quatenus ſcilicet naſcitur ex diſtantia cometae a terra tempore $\tau = 1$ diei. Quod ſi ergo hinc progrediamur per tempus $\frac{1}{2}d$, ſtatuendo $d\tau = \frac{1}{2}$, in fine huius temporis diſtantia illa ad ſemiſſem reducetur, vnde quadruplo maior perturbatio naſceretur. Quamobrem cum labente hoc interuallo $d\tau = \frac{1}{2}d$ perturbatio continuo fiat maior, conuenit medium ſumere, quod quadruplo maius erit quam inuentum, ſicque ſtatuamus

$$\frac{\delta(x-X)}{w^3} = 0{,}036 \quad \text{et} \quad \frac{\delta(y-Y)}{w^3} = 0{,}026$$

quocirca habebimus

$$ddX = -7{,}066.\,d\tau^2 \qquad ddx = -6{,}804.\,d\tau^2$$
$$ddY = -0{,}096.\,d\tau^2 \qquad ddy = -0{,}259.\,d\tau^2$$

hincque oriuntur ſequentes valores:

$$X' = 23996{,}451 - 7{,}099.\,d\tau - 3{,}533.\,d\tau^2$$
$$Y' = 412{,}897 + 412{,}899.\,d\tau - 0{,}048.\,d\tau^2$$
$$x' = 24566{,}059 - 576{,}503.\,d\tau - 3{,}402.\,d\tau^2$$
$$y' = 845{,}563 - 19{,}846.\,d\tau - 0{,}129.\,d\tau^2$$

porro

porro

$$dX' = -7,099.\, d\tau - 7,066.\, d\tau^2$$
$$dY' = +412,899.\, d\tau - 0,096.\, d\tau^2$$
$$dx' = -576,503.\, d\tau - 6,804.\, d\tau^2$$
$$dy' = -19,846.\, d\tau - 0,259.\, d\tau^2$$

hinc autem sumi debet $d\tau = \frac{1}{2}$, vnde producitur.

Calculus pro tempore $\tau = 1\frac{1}{2}\, d$ post epocham.

§. 25. Sumto igitur $d\tau = \frac{1}{2}$ et $d\tau^2 = \frac{1}{4}$, sequentia habebuntur elementa:

$X = 23992,019$	$Y = 619,334$
$x = 24276,957$	$y = 835,608$
$x - X = 284,938$	$y - Y = 216,274$
$dX = -10,032.\, d\tau$	$dY = +412,850.\, d\tau$
$dx = -579,904.\, d\tau$	$dy = -19,976.\, d\tau$

et hinc calculus vti supra sequenti modo instituitur

$a\, l\, Y = 2,7919249$	$a\, l\, y = 2,9220026$	$a\, l\, (y-Y) = 2,3350043$
$l\, X = 4,3800669$	subtr. $l\, x = 4,3851942$	s. $l\, (x-X) = 2,4547504$
l tang $\Phi = 8,4118580$	l tang $\omega = 8,5368084$	l tang. $\psi = 9,8802539$
ergo $\Phi = 1^\circ.\, 28'.\, 43$	ergo $\omega = 1^\circ.\, 58'.\, 17''$	ergo $\psi = 37^\circ.\, 12'$
ad $l\, X = 4,3800669$	ad $l\, x = 4,3851942$	ad $l\, (x\, X) = 2,4547504$
l sec. $\Phi = 10,0001429$	l sec. $\omega = 10,0002569$	l sec. $\psi = 10,0987979$
$l\, u = 4,3802098$	$l\, v = 4,3854511$	$l\, \omega = 2,5535483$
ergo $u = 24000$	ergo $v = 24291,320$	ideoque $w = 357,724$

ad $l\, \Delta$

ad $l\Delta = 9,6118924$	ad $l\Delta = 9,6118924$	ad $l\delta = 4,0555899$
$lX = 4,3800969$	$lx = 4,3854511$	$l(x-X) = 2,4544700$
$3,9919593$	$3,9973435$	$6,5100599$
ſubtr. $lu^3 = 3,1406294$	ſ. $lv^3 = 3,1563533$	ſ. $lu^3 = 7,6606449$
$l\frac{\Delta X}{u^3} = 0,8513299$	$l\frac{\Delta x}{v^3} = 0,8409902$	$l\frac{\delta(x-X)}{w^3} = 8,8494150$
l tang. $\Phi = 8,4118588$	l tang. $\omega = 8,5368084$	l tang. $\psi = 9,8802539$
$l\frac{\Delta Y}{u^3} = 9,2631887$	$l\frac{\Delta Y}{v^3} = 9,3777986$	$l\frac{\delta(y-Y)}{w^3} = 8,7296689$
ergo $\frac{\Delta X}{u^3} = 7,101$	ergo $\frac{\Delta x}{v^3} = 6,934$	ergo $\frac{\delta(x-X)}{w^3} = 0,071$
et $\frac{\Delta Y}{u^3} = 0,183$	et $\frac{\Delta y}{v^3} = 0,239$	et $\frac{\delta(y-Y)}{w^3} = 0,054$

iſta ergo perturbatio reſpondet tempori $\tau = 1\frac{1}{2}d$: at vero tempori $\tau = 1\frac{3}{4}d$ ſumendo $d\tau = \frac{1}{4}$, perturbatio fiet quadruplo maior, vnde medium ſumendo iſtos valores duplicemus ſtatuendo

$$\frac{\delta(x-X)}{w^3} = 0,142 \text{ et } \frac{\delta(y-Y)}{w^3} = 0,108$$

vnde colligimus

$$ddX = -6,957.d\tau^2 \quad ddx = -7,076.d\tau^2$$
$$ddY = -0,075.d\tau^2 \quad ddy = -0,347.d\tau^2$$

quocirca oriuntur

$$X' = 23992,019 - 10,632.d\tau - 3,478.d\tau^2$$
$$Y' = 619,334 + 412,850.d\tau - 0,037.d\tau^2$$
$$x' = 24276,957 - 579,904.d\tau - 3,538.d\tau^2$$
$$y' = 835,608 - 19,976.d\tau - 0,173.d\tau^2$$

deinde

$$dX' = -10,632.d\tau - 6,957.d\tau^2$$
$$dY' = +412,850.d\tau - 0,075.d\tau^2$$
$$dx' = -579,904.d\tau - 7,076.d\tau^2$$
$$dy' = -19,976.d\tau - 0,347.d\tau^2$$

vbi

vbi iam ſumi debet $d\tau = \frac{1}{4}$, vnde oritur

Calculus pro tempore $\tau = 1\frac{3}{4}\,d$ poſt epocham.

§. 26. Sumto igitur $d\tau = \frac{1}{4}$ et $d\tau^2 = \frac{1}{16}$ ſequentia habebuntur elementa:

$X = 23989,144$	$Y = 722,544$
$x = 24131,760$	$y = 830,603$
$x - X = 142,616$	$y - Y = 108,059$
$dX = -\ 12,371 . d\tau$	$dY = +\ 412,831 . d\tau$
$dx = -581,673\ d\tau$	$dy = -\ 20,063 . d\tau$

hincque ſequens calculus

		$a\,l(y-Y) = 2,0336609$
$a\,l\,y = 2,8588643$	$a\,l\,y = 2,9193935$	$l\,(x-X) = 2,1541683$
$l\,X = 4,3800147$	$l\,x = 4,3825890$	
$l\,\text{tang.}\,\Phi = 8,4788496$	$l\,\text{tang.}\,\omega = 8,5368045$	$l\,\text{tang.}\,\psi = 9,8794926$
ergo $\Phi = 1^\circ.\,43'.\,31''$	ergo $\omega = 1^\circ.\,58'.\,17''$	ergo $\psi = 37^\circ.\,9'$
ad $l\,X = 4,3800147$	ad $l\,x = 4,3825890$	ad $l(x-X) = 2,1541683$
$l\,\text{ſec.}\,\Phi = 10,0001976$	$l\,\text{ſec.}\,\omega = 10,0002569$	$l\,\text{ſec.}\,\psi = 10,0985105$
$l\,u = 4,3802123$	$l\,v = 4,3828459$	$l\,w = 2,2526788$
hinc $u = 24000$	hinc $v = 24146,00$	hinc $w = 178,928$
ad $l\,\Delta = 9,6118924$	ad $l\,\Delta = 9,6118924$	ad $l\,\delta = 4,0555899$
$l\,X = 4,3800147$	$l\,x = 4,3825890$	$l\,(x-X) = 2,1541683$
$3,9919071$	$3,9944814$	$6,2097582$
ſ. $l\,u^3 = 3,1406369$	ſ. $l\,v^3 = 3,1485377$	ſ. $l\,w^3 = 6,7580364$
$l\,\frac{\Delta X}{u^3} = 0,8512702$	$l\,\frac{\Delta x}{v^3} = 0,8459437$	$l\,\frac{\delta(x-X)}{w^3} = 9,4517218$
$l\,\text{tang.}\,\Phi = 8,4788496$	$l\,\text{tang.}\,\omega = 8,5368045$	$l\,\text{tang.}\,\psi = 9,8794926$
$l\,\frac{\Delta Y}{u^3} = 9,3301198$	$l\,\frac{\Delta y}{v^3} = 9,3827482$	$l\,\delta(y-Y) = 9,3312144$

ergo $\frac{\Delta X}{u^3} = 7,100$	ergo $\frac{\Delta x}{v^3} = 7,014$	ergo $\frac{\delta (x - X)}{w^3} = 0,283$
et $\frac{\Delta Y}{u^3} = 0,214$	et $\frac{\Delta y}{v^3} = 0,241$	et $\frac{\delta (y - Y)}{w^3} = 0,214$.

Nunc iterum valores tertiae columnae $\frac{\delta (x - X)}{w^3}$ et $\frac{\delta (y - Y)}{w^3}$ duplicentur, siquidem ponatur $d\tau = \frac{1}{2}$, hincque habebimus

$$ddX = -6,534 . d\tau^2 \qquad ddx = -7,666 . d\tau^2$$
$$ddY = +0,213 . d\tau^2 \qquad ddy = -0,669 . d\tau^2$$

hincque nanciscimur

$$X' = 23989,144 - 12,371 . d\tau - 3,267 . d\tau^2$$
$$Y' = 722,544 - 412,831 . d\tau + 0,106 . d\tau^2$$
$$x' = 24131,760 - 581,673 . d\tau - 3,833 . d\tau^2$$
$$y' = 830,603 - 20,063 . d\tau - 0,334 . d\tau^2$$

deinde

$$dX' = -12,371 . d\tau - 6,534 . d\tau^2$$
$$dY' = +412,831 . d\tau - 0,213 . d\tau^2$$
$$dx' = -581,673 . d\tau - 7,666 . d\tau^2$$
$$dy' = -20,063 . d\tau - 0,669 . d\tau^2$$

vnde iam aggrediamur sequentem calculum.

Calculus pro tempore $\tau = 1\frac{1}{2}d$ post epocham.

§. 27. Sumto igitur $d\tau = \frac{1}{2}$ et $d\tau^2 = \frac{1}{4}$ sequentia elementa prodibunt:

$X = 23987,546$	$Y = 774,146$
$x = 24058,992$	$y = 828,090$
$x - X = 71,446$	$y - Y = 53,944$
$dX = -13,188 . d\tau$	$dY = +412,804 . d\tau$
$dx = -582,631 . d\tau$	$dy = -20,147 . d\tau$

calcu-

calculus igitur ad modum praecedentis ita procedit

$a\,l\,Y = 2,8888129$	$a\,l\,y = 2,9180775$	$a\,l(y-Y) = 1,7319431$
$l\,X = 4,3799857$	$l\,x = 4,3812774$	s. $l\,(x-X) = 1,8539799$
l tang. $\Phi = 8,5088372$	l tang. $\omega = 8,5368001$	l tang. $\psi = 9,8779632$
ideoque $\Phi = 1^\circ.50'.54''$	ideoque $\omega = 1^\circ.58'.17''$	ergo $\psi = 37^\circ.3'.9''$
ad $l\,X = 4,3799857$	ad $l\,x = 4,3812774$	ad $l(x-X) = 1,8539799$
l sec. $\Phi = 10,0002260$	l sec. $\omega = 10,0002569$	l sec. $\psi = 10,0979467$
$l\,u = 4,3802117$	$l\,v = 4,3815343$	$l\,w = 1,9519266$
hinc $u = 24000,03$	hinc $v = 24073,23$	hinc $w = 89,521$
ad $l\,\Delta = 9,6118924$	ad $l\,\Delta = 9,6118924$	ad $l\,\delta = 4,0555899$
add. $l\,X = 4,3799857$	$l\,x = 4,3812774$	$l\,(x-X) = 1,8539799$
$3,9918781$	$3,9931698$	$5,9095698$
s. $l\,u^3 = 3,1406351$	s. $l\,v^3 = 3,1446029$	s. $l\,w^3 = 5,8557798$
$l\,\frac{\Delta X}{u^3} = 0,8512430$	$l\,\frac{\Delta x}{v^3} = 0,8485669$	$l\,\frac{\delta(x-X)}{w^3} = 0,0537900$
l tang. $\Phi = 8,5088372$	l tang. $\omega = 8,5368001$	l tang. $\psi = 9,8779632$
$l\,\frac{\Delta Y}{u^3} = 9,3600802$	$l\,\frac{\Delta y}{v^3} = 9,3853670$	$l\,\frac{\delta(y-Y)}{w^3} = 9,9317532$
ergo $\frac{\Delta X}{u^3} = 7,100$	ergo $\frac{\Delta x}{v^3} = 7,056$	ergo $\frac{\delta(x-X)}{w^3} = 1,132$
et $\frac{\Delta Y}{u^3} = 0,229$	et $\frac{\Delta y}{v^3} = 0,243$	et $\frac{\delta(y-Y)}{w^3} = 0,854$

Nunc adhuc duplicentur valores $\frac{\delta(x-X)}{w^3}$ et $\frac{\delta(y-Y)}{w^3}$ vt sit

$ddX = -4,836.\,d\tau^2$	$ddx = -9,320\,d\tau^2$
$ddY = +1,479.\,d\tau^2$	$ddy = -1,951\,d\tau^2$

hincque colligimus

$$X' = 23987,546 - 13,188.\,d\tau - 2,418.\,d\tau^2$$
$$Y' = 774,146 + 412,804.\,d\tau + 0,739.\,d\tau^2$$
$$x' = 24058,992 - 582,631.\,d\tau - 4,660.\,d\tau^2$$
$$y' = 828,090 - 20,147.\,d\tau - 0,975.\,d\tau^2$$

Vvv 2 deinde

deinde

$$dX' = -13,188.\,d\tau - 4,836.\,d\tau^2$$
$$dY' = +412,804.\,d\tau + 1,479.\,d\tau^2$$
$$dx' = -582,631.\,d\tau - 9,320.\,d\tau^2$$
$$dy' = -20,147.\,d\tau - 1,951.\,d\tau^2$$

vbi pro calculo fequente fumi debet $d\tau = \frac{1}{16}$.

Calculus pro tempore $\tau = 1\frac{15}{16}\,d$ poft epocham.

§. 28. Elementa igitur huius calculi ita fe habebunt

$X = 23986,713$	$Y = 799,943$
$x = 24022,560$	$y = 826,837$
$x - X = 35,847$	$y - Y = 26,884$
$dX = -13,490.\,d\tau$	$dY = +412,896.\,d\tau$
$dx = -583,213.\,d\tau$	$dy = -20,269.\,d\tau$

quibus inuentis calculum fequenti modo profequamur

$a\,l\,Y = 2,9030590$	$a\,l\,y = 2,9174147$
fubtr. $l\,X = 4,3796707$	fubtr. $l\,x = 4,3806092$
l tang. $\Phi = 8,5230883$	l tang. $\omega = 8,5368055$
ideoque $\Phi = 1^\circ\,54'.36''$	ideoque $\omega = 1^\circ.\,58'.17''$
l fec. $\Phi = 10,0002413$	l fec. $\omega = 10,0002569$
ad $l\,X = 4,3799707$	ad $l\,x = 4,3806092$
$l\,u = 4,3802120$	$l\,v = 4,380866$[illegible]
hinc $u = 24000,04$	hinc $v = 24036,22.$

ad

ad $l\Delta$	$= 9,6118924$	ad $l\Delta$	$= 9,6118924$
add. lX	$= 4,3799707$	add. lX	$= 4,3806092$
	$3,9918631$		$3,9925016$
f. lu^3	$= 3,1406360$	f. lv^3	$= 3,1425983$
$l\frac{\Delta X}{u^3}$	$= 0,8512271$	$l\frac{\Delta x}{v^3}$	$= 0,8499033$
l tang. Φ	$= 8,5230883$	l tang. ω	$= 8,5368055$
$l\frac{\Delta Y}{u^3}$	$= 9,3743154$	$l\frac{\Delta y}{v^3}$	$= 9,3867088$
ergo $\frac{\Delta X}{u^3}$	$= 7,099$	ergo $\frac{\Delta x}{v^3}$	$= 7,078$
et $\frac{\Delta Y}{u^3}$	$= 0,237$	et $\frac{\Delta y}{v^3}$	$= 0,243.$

Si hinc vlterius progredientes ſumeremus $d\tau = \frac{1}{32}$, termini litera δ affecti fierent quadruplo maiores; at ſi ſumeremus $d\tau = \frac{1}{16}$ hi termini adeo in immenſum excreſcerent. Verum ſi minimum $d\tau$ vlterius augeamus, hi termini adeo euaderent negatiui, et effectus in contrarium vergeret; vnde ſi ſumamus $d\tau = \frac{1}{8}$, vt fiat $\tau + d\tau = 2\frac{1}{16}$, iſti termini modo inuentis aequales prodibunt, ſed ſigno contrario affecti. Quamobrem poſito $d\tau = \frac{1}{8}$ totus effectus actionis mutuae ad nihilum redigetur, vnde pro ſequente calculo hos terminos penitus negligere opportebit, quam ob cauſam in hoc calculo tertiam columnamne quidem adiecimus; erit autem

$ddX = -7,099.\,d\tau^2$	$ddx = -7,078.\,d\tau^2$
$ddY = -0,237.\,d\tau^2$	$ddy = -0,243.\,d\tau^2$

ex quibus valoribus eliciuntur ſequentes:

$$X' = 23986,713 - 13,490.\,d\tau - 3,549.\,d\tau^2$$
$$Y' = 799,943 + 412,896.\,d\tau - 0,118.\,d\tau^2$$

 $x' = 2$

$$x' = 24022,560 - 583,213.d\tau - 3,539.d\tau^2$$
$$y' = 826,827 - 20,269.d\tau - 0,121.d\tau^2$$
$$dX' = - 13,490.d\tau - 7,099.d\tau^2$$
$$dY' = + 412,896.d\tau - 0,237.d\tau^2$$
$$dx = - 583,213.d\tau - 7,078.d\tau^2$$
$$dy = - 20,269.d\tau - 0,243.d\tau^2.$$

Calculus pro tempore $\tau = 2\frac{1}{10}d$ post epocham.

§. 29. Nunc igitur in praecedentibus valoribus statui debet $d\tau = \frac{1}{2}$ et prodibunt sequentia elementa:

$X = 23984,972$	$Y = 851,552$
$x = 23949,603$	$y = 824,291$
$x - X = -35,369$	$y - Y = -27,261$
$dX = -14,377.d\tau$	$dY = +412,867.d\tau$
$dx = -584,098.d\tau$	$dy = -20,298.d\tau$

hinc sequens calculus

a $ly = 2,9302112$	ad $ly = 2,9160805$	$l(y-Y) = (-)1,4355418$
s. $lX = 4,3799391$	s. $lx = 4,3792983$	s. $l(x-X) = (-)1,5548622$
$l$tang. $\phi = 8,5502721$	$l$tang. $\omega = 8,5367822$	$l$tang. $\psi = (+)9,8869190$
ergo $\phi = 2°.2'.0''$	ergo $\omega = 1°.58'.17''$	ergo $\psi = 37°.37'.23''$
ad $lX = 4,3799391$	ad $lx = 4,3792983$	ad $l(x-X) = (-)1,5486229$
$l$sec. $\phi = 10,0002735$	$l$sec. $\omega = 10,0002569$	$l$sec. $\psi = (+)10,1012458$
$lu = 4,3802126$	$lv = 4,3795552$	$lw = (-)1,64[illegible]8686$
hinc $u = 24000,07$	hinc $v = 23963,77$	hinc $w = 44,654$

ad $l\Delta$

ad $l\Delta = 9,6118924$	ad $l\Delta = 9,6118924$	ad $l\delta = (+)\,4,0555899$
$lX = 4,8799391$	$lx = 4,8792983$	$l(x-X) = (-)\,1,5486228$
$3,9918315$	$3,9911907$	$(-)\,5,6042127$
f. $lu^3 = 3,1406378$	f. $lv^3 = 3,1386656$	f. $lw^3 = (+)\,4,9496058$
$l\frac{\Delta X}{u^3} = 0,8$[illegible]$11937$	$l\frac{\Delta x}{v^3} = 0,8525251$	$l\frac{(x-X)}{w^3}\ (-)\ 0,6546069$
$l$tang. $\Phi = 8,55$[illegible]2771	$l$tang. $\omega = 8,5367822$	$l$tang. $\psi = 9,8869190$
$l\frac{\Delta Y}{u^3} = 9,4014708$	$l\frac{\Delta y}{v^3} = 9,3893073$	$l\frac{(y-Y)}{w^3} = (-)\,0,5415259$
ergo $\frac{\Delta X}{u^3} = 7,099$	ergo $\frac{\Delta x}{v^3} = 7,121$	ergo $\frac{\delta(x-X)}{w^3} = -4,514$
et $\frac{\Delta Y}{u^3} = 0,252$	et $\frac{\Delta y}{v^3} = 0,245$	et $\frac{\delta(y-Y)}{w^3} = -3,479.$

Vti hic effectus in tertia columna inuenti labente tempore continuo fiunt minores, et fumto interuallo $d\tau = \frac{1}{10}$ adeo fiunt quadruplo minores, pro hoc interuallo hos effectus ad dimidium reduci conueniet, ita vt fiat

$$\frac{\delta(x-X)}{w^3} = -2,257 \quad \text{et} \quad \frac{\delta(y-Y)}{w^3} = -1,739$$

vnde nanciscimur

$$ddX = -9,356\,d\tau^2 \qquad ddx = -4,864.\,d\tau^2$$
$$ddY = -1,991\,d\tau^2 \qquad ddy = +1,494.\,d\tau^2$$

vnde deducuntur sequentes

$$X' = 23984,972 - 14,377.\,d\tau - 4,678\,d\tau^2$$
$$Y' = 851,552 + 412,857.\,d\tau - 0,996.\,d\tau^2$$
$$x' = 23949,603 - 584,098.\,d\tau - 2,423.\,d\tau^2$$
$$y' = 824,291 - 20,298.\,d\tau + 0,747.\,d\tau^2$$

dX'

$$dX' = -14,377\ d\tau - 9,356.\ d\tau^2$$
$$dY' = +412.867.\ d\tau + 1,991.\ d\tau^2$$
$$dx' = -584,089.\ d\tau - 4,864.\ d\tau^2$$
$$dy' = -20,298.\ d\tau - 1,494.\ d\tau^2.$$

Calculus pro tempore $\tau = 2\frac{1}{8}\ d$ poſt epocham.

§. 30. In vltimis igitur formulis inuentis ſumamus $d\tau = \frac{1}{16}$ et obtinebimus

$X = 23984,055$	$Y = 877,352$
$x = 23913,088$	$y = 823,026$
$x-X = -70,967$	$y-Y = -54,326$
$dX = -14,961\ d\tau$	$dY = +412,743\ d\tau$
$dx = -584,402\ d\tau$	$dy = -20,205\ d\tau$

hinc ſequens calculus

$a\,l\,Y = 2,9431739$	$a\,l\,y = 2,9154135$	$a\,l(y-Y) = 1,7340077$
ſubtr. $l\,X = 4,3799226$	ſ. $l\,x = 4,3786356$	ſ. $l\,(x-X) = 1,8510564$
$l$tang. $\Phi = 8,5632513$	$l$tang. $\omega = 8,5367779$	$l$tang. $\psi = 9,8829513$
ergo $\Phi = 2°. 5'. 42''$	ergo $\omega = 1°. 58'. 17$	ergo $\psi = 37°. 22'. 16''$
ad $l\,X = 4,3799226$	ad $l\,x = 4,3786356$	ad $l(x-X) = 1,8510564$
l ſec. $\Phi = 10,0002904$	l ſec. $\omega = 10,0002569$	l ſec. $\psi = 10,0997838$
$l\,u = 4,3802130$	$l\,v = 4,3788925$	$l\,w = 1,9508402$
hinc $u = 24000,10$	hinc $v = 32927,24$	hinc $w = 89,298$

ad $l\,\Delta$

ad $l\,\Delta = 9,6118924$	ad $l\,\Delta = 9,6118924$	ad $l\,\delta = 4,0555899$
add. $l\,X = 4,3799226$	add. $l\,x = 4,3786356$	$l\,(x - X) = 1,8510564$
$3,9918150$	$3,9905280$	$5,9066463$
f. $l\,u^3 = 3,1406390$	f. $l\,v^3 = 3,1366775$	$l\,w^3 = 5,8525206$
$l\,\frac{\Delta X}{u^3} = 0,8511760$	$l\,\frac{\Delta x}{v^3} = 0,8538505$	$l\,\frac{\delta(x - X)}{w^3} = 0,0541257$
l tang. $\Phi = 8,563[illegible]513$	l tang. $\omega = 8,5367779$	l tang. $\psi = 9,8829513$
$l\,\frac{\Delta Y}{u^3} = 9,4144273$	$l\,\frac{\Delta y}{v^3} = 9,3906284$	$l\,\frac{\delta(y - Y)}{w^3} = 9,9370770$
ergo $\frac{\Delta X}{u^3} = 7,099$	ergo $\frac{\Delta x}{v^3} = 7,142$	ergo $\frac{\delta(x - X)}{w^3} = -1,133$
et $\frac{\Delta Y}{u^3} = 0,260$	et $\frac{\Delta y}{v^3} = 0,246$	et $\frac{\delta(y - Y)}{w^3} = -0,865$

quod si iam sumamus interuallum $d\tau = \frac{1}{8}$, effectus tertiae columnae ad semissem redigi opportet, vnde fiet;

$$ddX = -7,665.\,d\tau^2 \quad ddx = -6,576.\,d\tau^2$$
$$ddY = -0,692.\,d\tau^2 \quad ddy = +0,186.\,d\tau^2$$

vnde colliguntur sequentes valores:

$$X' = 23984,055 - 14,961.\,d\tau - 3,832.\,d\tau^2$$
$$Y' = 877,352 + 412,743.\,d\tau - 0,346.\,d\tau^2$$
$$x' = 23913,088 - 584,402.\,d\tau - 3,288.\,d\tau^2$$
$$y' = 823,026 - 20,205.\,d\tau + 0,093.\,d\tau^2$$
$$dX' = -14,961.\,d\tau - 7,665.\,d\tau^2$$
$$dY' = +412,743.\,d\tau - 0,692.\,d\tau^2$$
$$dx' = -584,402.\,d\tau - 6,576.\,d\tau^2$$
$$dy' = -20,205.\,d\tau + 0,186.\,d\tau^2$$

Calculus pro tempore $\tau = 2\frac{1}{4}\,d$ post epocham.

§ 31. Posito igitur $d\tau = \frac{1}{8}$ sequentia elementa prodibunt:

 X =

$X = 23982,125$	$Y = 928,940$
$x = 23839,987$	$y = 820,501$
$x - X = -142,138$	$y - Y = -108,439$
$dX = -15,919 . d\tau$	$dx = +585,224 . d\tau$
$dY = +412,656 . d\tau$	$dy = -20,182 . d\tau$

quibus inuentis ſequens calculus inſtituatur

$a\,l\,Y = 2,9679877$	$a\,l\,y = 2,9140791$	$a\,l(y-Y) = 2,0351856$
ſ. $l\,X = 4,3798876$	ſubtr. $l\,x = 4,3773059$	ſ. $l(x-X) = 2,1527102$
l tang. $\Phi = 8,5881001$	l tang. $\omega = 8,5367732$	l tang. $\psi = 9,8824754$
ergo $\Phi = 2°.13'.6''$	ergo $\omega = 1°.58'.17''$	ergo $\psi = 37°.20'.25''$
ad $l\,X = 4,3798876$	ad $l\,x = 4,3773059$	ad $l(x-X) = 2,1527102$
l ſec. $\Phi = 10,0003256$	l ſec. $\omega = 10,0002569$	l ſec. $\psi = 10,0996069$
$l\,u = 4,3802132$	$l\,v = 4,3775628$	$l\,w = 2,2523171$
hinc $u = 24000,11$	hinc $v = 23854,04$	hinc $w = -178,780$
ad $l\,\Delta = 9,6118924$	ad $l\,\Delta = 9,6118924$	ad $l\,\delta = 4,0555899$
$l\,X = 4,3798876$	$l\,x = 4,3773059$	$l(x-X) = 2,1527102$
$3,9917800$	$3,9891983$	$6,2083001$
ſubtr. $l\,u^3 = 3,1406396$	ſ. $l\,v^3 = 3,1326884$	ſ. $l\,w^3 = 6,7569513$
$l\,\frac{\Delta X}{u^3} = 0,8511404$	$l\,\frac{\Delta x}{v^3} = 0,8565099$	$l\,\frac{\delta(x-X)}{w^3} = 9,4513488$
l tang. $\Phi = 8,5881001$	l tang. $\omega = 8,5367732$	l tang. $\psi = 9,8824754$
$l\,\frac{\Delta Y}{u^3} = 9,4392405$	$l\,\frac{\Delta Y}{v^3} = 9,3932861$	$l\,\frac{\delta(y-Y)}{w^3} = 9,3338242$
ergo $\frac{\Delta X}{u^3} = 7,098$	ergo $\frac{\Delta x}{v^3} = 7,186$	ergo $\frac{\delta(x-X)}{w^3} = -0,283$
et $\frac{\Delta Y}{u^3} = 0,275$	et $\frac{\Delta y}{v^3} = 0,247$	$\frac{\delta(y-Y)}{w^3} = -0,215.$

Effectus

Effectus tertiae columnae iam iterum ad semissem reducantur, siquidem deinceps ponere velimus $d\tau = \frac{1}{4}$, sicque obtinebimus

$$ddX = -7,239.\, d\tau^2 \qquad ddx = -7,045.\, d\tau^2$$
$$ddY = -0,382.\, d\tau^2 \qquad ddy = -0,140.\, d\tau^2$$

vnde obtinebimus sequentes valores:

$$X' = 23982,125 - 15,919.\, d\tau - 3,619\, d\tau^2$$
$$Y' = 928,940 + 412,656.\, d\tau - 0,191.\, d\tau^2$$
$$x' = 23839,987 - 585,224.\, d\tau - 3,522\, d\tau^2$$
$$y' = 820,501 - 20,182.\, d\tau - 0,070.\, d\tau^2$$
$$dX' = -15,919.\, d\tau - 7,239.\, d\tau^2$$
$$dY' = +412,656.\, d\tau - 0,382.\, d\tau^2$$
$$dx' = -585,224.\, d\tau - 7,045.\, d\tau^2$$
$$dy' = -20,182.\, d\tau - 0,140.\, d\tau^2.$$

Calculus pro tempore $\tau = 2\frac{1}{2}d$ post epocham.

§. 32. Ponendo in formulis modo inuentis $d\tau = \frac{1}{4}$ hosce obtinebimus valores:

$X = 23977,919$	$Y = 1032,092$
$x = 23693,461$	$y = 815,452$
$(x-X) = -284,458$	$y - Y = -216,640$
$dX = -17,792.\, d\tau$	$dY = +412,561.\, d\tau$
$dx = -586,685.\, d\tau$	$dy = -20,216.\, d\tau$

super his igitur elementis calculus ita instituatur

$a\, l\, Y = 3,0137383$	$a\, l\, y = 2,9113985$	$a\, l(y-Y) = (-)2,3357386$
$s.\, l\, X = 4,3798113$	$s.\, l\, x = 4,3746285$	$s.\, l(x-X) = (-)2,4540181$
$l\,\text{tang.}\, \Phi = 8,6339270$	$l\,\text{tang.}\, \omega = 8,5367700$	$l\,\text{tang.}\, \psi = 9,8817205$
ergo $\Phi = 2^\circ.\, 27'.\, 52''$	ergo $\omega = 1^\circ.\, 58'.\, 17''$	ergo $\psi = 37^\circ.\, 17'.\, 10''$

ad $l X = 4{,}3798113$	ad $l x = 4{,}3746285$	$l(x-X) = 2{,}4546181$
l sec. $\Phi = 10{,}0004025$	l sec. $\omega = 10{,}0002569$	l sec. $\psi = 10{,}0089420$
$l u = 4{,}3802138$	$l v = 4{,}3748854$	$l w = 2{,}5529601$
ergo $u = 24000{,}14$	ergo $v = 23707{,}511$	ergo $w = -357{,}240$
ad $l \Delta = 9{,}6118924$	ad $l \Delta = 9{,}6118924$	ad $l \delta = 4{,}0554899$
$l X = 4{,}3798113$	$l x = 4{,}3746285$	$l(x-X) = 2{,}4546181$
$3{,}9917037$	$3{,}9865209$	$6{,}5095080$
f. $l u^3 = 3{,}1406414$	$l v^3 = 3{,}1246562$	$l w^3 = 7{,}6588803$
$l \frac{\Delta X}{u^3} = 0{,}8510623$	$l \frac{\Delta x}{v^3} = 0{,}8618647$	$l \frac{\delta(x-X)}{w^3} = 8{,}8506277$
l tang. $\Phi = 8{,}6339270$	l tang. $\omega = 8{,}5367700$	l tang. $\psi = 9{,}8817205$
$l \frac{\Delta Y}{u^3} = 9{,}4849893$	$l \frac{\Delta y}{v^3} = 9{,}3986347$	$l \frac{\delta(y-Y)}{w^3} = 8{,}7323482$
ergo $\frac{\Delta X}{u^3} = 7{,}097$	ergo $\frac{\Delta x}{v^3} = 7{,}276$	ergo $\frac{\delta(x-X)}{w^3} = 0{,}071$
et $\frac{\Delta Y}{u^3} = 0{,}305$	et $\frac{\Delta y}{v^3} = 0{,}250$	et $\frac{\delta(y-Y)}{w^3} = 0{,}054$

hinc ſumto dimidio valorum $\frac{\delta(x-X)}{w^3}$ et $\frac{\delta(y-Y)}{w^3}$ obtinebimus

$$ddX = -7{,}132 . d\tau^2 \quad\Big|\quad ddx = -7{,}241 . d\tau^2$$
$$ddY = -0{,}332 . d\tau^2 \quad\Big|\quad ddy = -0{,}223 . d\tau^2$$

Vnde etiam ſequentes deducuntur

$$X' = 23977{,}919 - 17{,}792 . d\tau - 3{,}566 . d\tau^2$$
$$Y' = 1032{,}092 + 412{,}561 . d\tau - 0{,}166 . d\tau^2$$
$$x' = 23693{,}461 - 586{,}685 . d\tau - 3{,}620 . d\tau^2$$
$$y' = 815{,}452 - 20{,}216 . d\tau - 0{,}111 . d\tau^2$$
$$dX' = -17{,}792 . d\tau - 7{,}132 . d\tau^2$$
$$dY' = +412{,}561 . d\tau - 0{,}332 . d\tau^2$$
$$dx' = -586{,}685 . d\tau - 7{,}241 . d\tau^2$$
$$dy' = -20{,}216 . d\tau - 0{,}223 . d\tau^2 .$$

Calcu-

Calculus pro tempore $\tau = 3\,d$ poſt epocham.

§. 33. Poſito $d\tau = \frac{1}{2}$ huius calculi elementa ita ſe habebunt

$X = 23968,132$	$Y = 1238,331$
$x = 23399,214$	$y = 805,316$
$x - X = -568,9$[illegible]8	$y - Y = -433,015$
$dX = -21,358.\,d\tau$	$dY = +412,395.\,d\tau$
$dx = -590,305.\,d\tau$	$dy = -20,372.\,d\tau$

quibus inuentis calculum ita proſequamur

a $lY = 3,0928368$	a $ly = 2,9059663$	$l(y-Y) = (-)2,6365020$
ſ. $lX = 4,3796341$	ſ. $lx = 4,3692012$	ſ. $l(x-X) = (-)2,7550497$
$l$tang. $\Phi = 8,7132027$	$l$tang. $\omega = 8,5367651$	l tang. $\psi = 9,8814523$
ergo $\Phi = 2^\circ.\,57'.\,27''$	ergo $\omega = 1^\circ.\,58'.\,16''$	ergo $\psi = 37^\circ.\,16'.\,32''$
ad $lX = 4,3796341$	ad $lx = 4,3692012$	ad $l(x-X) = 2,7550497$
lſec $\Phi = 10,0005790$	l ſec. $\omega = 10,0002569$	l ſec. $\psi = 10,0992300$
$lu = 4,3802131$	$lv = 4,3694581$	$lw = 2,8542797$
ergo $u = 24000,11$	ergo $v = 23413,05$	ergo $w = -714,957$
ad $l\Delta = 9,6118924$	ad $l\Delta = 9,6118924$	ad $l\delta = 4,0555899$
$lX = 4,3796341$	$lx = 4,3692012$	$l(x-X) = 2,7550497$
$3,9915265$	$3,9810936$	$6,8106396$
ſ. $lu^3 = 3,1406511$	ſ. $lv^3 = 3,1083743$	ſubtr. $lw^3 = 8,5628391$
$l\,\frac{\Delta X}{u^3} = 0,8508754$	$l\,\frac{\Delta x}{v^3} = 0,8727193$	$l\,\frac{(x-X)}{w^3} = 8,2478005$
$l$tang. $\Phi = 8,7132027$	$l$tang $\omega = 8,5367651$	$l$tang. $\psi = 9,8814528$
$l\,\frac{\Delta Y}{u^3} = 9,5640781$	$l\,\frac{\Delta y}{v^3} = 9,4094844$	$l\,\frac{(y-Y)}{w^3} = 8,1292533$
ergo $\frac{\Delta X}{u^3} = 7,094$	ergo $\frac{\Delta x}{v^3} = 7,460$	ergo $\frac{\delta(x-X)}{w^3} = -0,017$
et $\frac{\Delta Y}{u^3} = 0,366$	et $\frac{\Delta y}{v^3} = 0,257$	et $\frac{\delta(y-Y)}{w^3} = -0,013$

 vbi

vbi iterum effectus actionis mutuae ad femiffem reducuntur, quo obferuato fit

$$ddX = -7,102\, d\tau^2 \qquad ddx = -7,452.\, d\tau^2$$
$$ddY = -0,372\, d\tau^2 \qquad ddy = -0,251.\, d\tau^2$$

vnde fequentes nancifcimur valores

$$X' = 23968,132 - 21,358.\, d\tau - 3,551\, d\tau^2$$
$$Y' = 1238,331 + 412,395.\, d\tau - 0,186.\, d\tau^2$$
$$x' = 23399,214 - 590,305.\, d\tau - 3,726.\, d\tau^2$$
$$y' = 805,316 - 20,372.\, d\tau - 0,125.\, d\tau^2$$
$$dX' = -21,358.\, d\tau - 7,102.\, d\tau^2$$
$$dY' = +412,395.\, d\tau - 0,372.\, d\tau^2$$
$$dx' = -590,305.\, d\tau - 7,452.\, d\tau^2$$
$$dy' = -20,372.\, d\tau - 0,251.\, d\tau^2.$$

Calculus pro tempore $\tau = 4\, d$ poft epocham.

§. 34. Calculi huius elementa pofito $d\tau = 1$ ita erunt comparata

$X = 23943,223$	$Y = 1650,540$
$x = 22805,183$	$y = 784,819$
$x - X = -1138,040$	$y - Y = -865,721$
$dX = -28,460.\, d\tau$	$dY = +412,023.\, d\tau$
$dx = -597,757.\, d\tau$	$dy = -20,623.\, d\tau$

hincque calculum fequenti modo profequamur. Et quia actio mutua penitus ceffare eft cenfenda, tantum fupereft, vt valores Φ, u, ω, v, ψ, w definiamus

$a\, l\, Y$

a $lY = 3,2176260$	a $ly = 2,8947694$	a $l(y-Y) = 2,9373780$
subtr $lX = 4,3791826$	s. $lx = 4,3580335$	s. $l(x-X) = 3,0561576$
$l$tang. $\Phi = 8,8384434$	$l$tang. $\omega = 8,5367359$	$l$tang. $\psi = 9,8812204$
ergo $\Phi = 3^{\circ}.\,56'.\,37''$	ergo $\omega = 1^{\circ}.\,58'.\,16$	ergo $\psi = 37^{\circ}.\,15'.\,40''$
ad $l\,X = 4,3791826$	ad $l\,x = 4,3580335$	ad $l(x-X) = 3,0561576$
l sec. $\Phi = 10,0010287$	l sec. $\omega = 10,0002581$	l sec. $\psi = 10,0991489$
$l\,u = 4,3802113$	$l\,v = 4,3582916$	$l\,w = 3,1553065$
hinc $u = 24000,01$	hinc $v = 22818,74$	hinc $w = -1429,903$.

§. 35. Quo haec quae his calculis inuenimus clarius ob oculos ponamus, omnia in sequenti tabella referamus in septem distributa columnas. I^{a}. Columna continet tempora ab epocha elapsa in diebus et horis expressa, scilicet valores literae τ. II^{a}. Longitudinem terrae ex sole visam, seu angulum Φ. III. Distantiam terrae a sole in semidiametris terrae expressam, seu literam u. IV. Longitudinem cometae heliocentricam, seu angulam ω. V. Distantiam cometae a sole, seu literam v. VI. longitudinem cometae geocentricam, seu angulum ψ. VII. Distantiam cometae a terra itidem in semidiametris terrae seu literam w.

Tabula

Tabula

motum tam terrae quam cometae exhibens

τ	Φ	u	ω	v	Ψ	w
D. h.	g. m. s.		g. m. s.		S. g. m. ſ.	
0. 0	0. 0. 0	24000,00	1.58.17	25154, 08	1. 7.12.57	1430,697
1. 0	0.59. 8	24000,01	1.58.17	24580, 60	1. 7.13.11	715,302
1.12	1.28.43	24000,02	1.58.17	24291, 32	1. 7.11.57	357,724
1.18	1.43.31	24000,03	1.58.17	24146, 00	1. 7. 9. 3	178,928
1.21	1.50.54	24000,04	1.58.17	24073, 23	1. 7. 3. 9	89,521
1.22½	1.54.36	24000,04	1.58.17	24036, 22	1. 7. 0. 3	44,808
2. 1½	2. 2. 0	24000,c6	1.58.17	23963, 77	7. 7.37.23	44,657
2. 3	2. 5.42	24000,10	1.58.17	23927, 24	7.17.22.16	89,298
2. 6	2.13. 6	24000,12	1 58.17	23854, 04	7. 7.20.25	178,780
2.12	2.27.52	24000,14	1.58.17	23707, 51	7. 7.17.10	357,240
3. 0	2.57.27	24000,10	1.58.16	23413, 05	7. 7.16.32	714,957
4. 0	3.56.37	24000,01	1.58.16	22818, 74	7. 7.15.40	1429,903

§. 36. Non obſtantibus leuiuſculis erroribus, quos in talibus calculis euitare non licet, concluſiones maximi momenti hinc tuto deducere poſſumus. Primo enim motus cometae reſpectu terrae manifeſto diſtingui debet in acceſſum et receſſum. Acceſſus durat vsque ad dies 2 quo cometa continuo propius ad terram accedit, ac fortaſſe vsque ad contactum appropinquaret, quo caſu vtique colliſio contingeret effectum maxime funeſtum producens. Verum aſſumamus cometam non prorſus ad contactum vsque appropinquaſſe, id quod quam minima facta mutatione in noſtra hypotheſi eueniſſet. Semota igitur colliſione intelligimus, cometam fere pari motu tu iterum a terra recedere quo accesserat, neque adeo in cursu suo multum turbari. Vnde statim eorum opinio manifesto reuellitur, qui putarunt, talem cometam ad terram proxime accedentem in satellitem vel lunam abire posse. Quin potius euidens est, neque terram neque cometam in motu suo hinc enormem perturbationem perpeti, sed potius vtrumque cursum suum sine admodum notabili mutatione esse prosecuturum. Interim tamen nullum est dubium, quin ob maximam vicinitatem phaenomena satis notabilia tam in aestu maris quam in statu Atmosphaerae se sint oblatura. Sed quoniam ista vicinitas quasi tantum per momentum durat, vix vllum inde periculum metuendum videtur.

§. 37. Deinde etiam ex hoc calculo patet, omnem effectum qui in accessu cometae ad terram fuerit productus in recessu fere maximam partem iterum destrui; quandoquidem tam situs quam motus terrae et cometae postquam actio mutua cessavit non multum discrepat ab eo qui remota actione mutua locum habuisset. Quo autem hanc ipsam mutationem accuratius determinemus, comparemus vtriusque statum, quo tam terra quam cometa quarto die vbi actio mutua cessasse est censenda versabantur cum eo statu in quo remota actione mutua fuissent reperti, vt hinc deinceps motum vtriusque futurum determinare queamus.

§. 38. Primo igitur si terra motum suum sine vlla alteratione continuasset, etiamnunc foret $u = 24000$, et elapſo tempore $\tau = 4$ foret angulos $\Phi = 3^\circ.\, 56'.\, 33'$ vnde prodit

$$X = 24000 \operatorname{cos}\Phi \quad \text{et} \quad Y = 24000 \operatorname{sin}\Phi \ \text{h. e.}$$

$$X = 23943,733 \quad \text{et} \quad Y = 1650,239.$$

Pro celeritatibus vero habebimus

$$dX = -24000\, d\Phi \operatorname{sin}\Phi \ \text{et}\ dY = +24000\, d\Phi \operatorname{cos}\Phi.$$

Supra autem vidimus eſſe $d\Phi = 0,017204\, d\tau$, ſicque erit

$$\frac{dX}{d\tau} = -0,017204 \cdot 24000 \operatorname{sin}\Phi = -28,391$$

$$\frac{dY}{d\tau} = +0,017204 \cdot 24000 \operatorname{cos}\Phi = +411,928$$

qui valores quo facilius cum iis quos vltimus calculus ſuppeditauit conferri queant hic coniunctim repraeſentemus

Remota actione mutua	Accedente actione mutua
$X = 23943,733$	$X = 23943,223$
$Y = 1650,239$	$Y = 1650,540$
$\frac{dX}{d\tau} = -28,391$	$\frac{dX}{d\tau} = -28,460$
$\frac{dY}{d\tau} = +411,928$	$\frac{dY}{d\tau} = +412,023.$

§. 39. Pro motu autem cometae ſupra inuenimus hanc aequationem

$$\frac{dv}{d\vartheta} = -\sqrt{\frac{2a^3}{v}} \quad \text{et} \quad \vartheta = \frac{2c\sqrt{c} - 2v\sqrt{v}}{3\sqrt{2a^3}}$$

vbi inuenimus eſſe

$$c\sqrt{c} = \frac{(3\ \text{[illegible]} + \sqrt{2})}{\sqrt{2}}\, a\sqrt{a}.$$

Quare cum ſumſerimus

$$a = 24000 \quad \text{et} \quad \alpha = 1^\circ.\, 58'.\, 17'' = 0,034408$$

nunc

nunc vero fit

$$2v\sqrt{v} = (3\alpha\sqrt{2}+2)a\sqrt{a} - 3\vartheta\sqrt{2a^3}$$

hic capiamus pro quatuor diebus

$$\vartheta = 2\omega \text{ erit } 2v\sqrt{v} = 2a\sqrt{a} - 2\alpha a\sqrt{2a}$$

vnde in numeris fit $v\sqrt{v} = 3446680$, hincque porro

$l\sqrt{v} = 2,1791337$ et $lv = 4,3582674$ hincque

$v = 22817,470$ vnde colligitur

$x = v\,\text{cof.}\,\alpha = 22803,963$ et $y = v\,\text{fin.}\,\alpha = 784,925$.

Tum vero pro celeritatibus ob

$$\frac{dv}{d\vartheta} = \sqrt{\frac{2b^3}{v}} = 34809,532 \text{ erit } l\frac{dv}{d\tau} = 2,7773275$$

fietque

$$\frac{dx}{d\tau} = \frac{dv}{d\tau}\,\text{cof.}\,\alpha = -598,512$$

$$\frac{dy}{d\tau} = \frac{dv}{d\tau}\,\text{fin.}\,\alpha = -20,601.$$

Quod fi igitur iftos valores comparemus cum iis quos cometa habuiffet eodem tempore fublata actione mutua, comparatio ita fe habebit.

Sublata actione mutua	Accedente actione mutua
$x = 22803,963$	$x = 22805,180$
$y = 784,925$	$y = 784,819$
$\frac{dx}{d\tau} = -598,512$	$\frac{dx}{d\tau} = -597,757$
$\frac{dy}{d\tau} = -20,601$	$\frac{dy}{d\tau} = -20,623.$

§. 40. Nunc igitur quaeftio huc redit, quanam lege tam terra quam cometa motum fuum deinceps fint profecuturi, poftquam actio mutua ceffauit. Quae quaeftio cum latiffime pateat, eam generatim in fequenti problemate complectamur.

Problema.

Si ad datum tempus cognitus fuerit tam locus quam motus siue planetae siue cometae, determinare eius orbitam et motum quo deinceps circa solem reuoluetur.

Solutio.

T. XXIV. Fig 4. §. 41. Elapso tempore τ dierum planeta siue cometa versatur in Y, vnde ad rectam S♈ ex sole ad initium arietis ductam demittatur perpendiculum Y X et vocentur coordinatae

$$SX = x \text{ et } XY = y.$$

Praeterea vero ponatur distantia a Sole $SY = u$ et angulus ♈$SY = \Phi$, ita vt sit $uu = xx + yy$ et tang. $\Phi = \frac{y}{x}$; tum vero viciſſim $x = u \cos \Phi$, $y = u \sin. \Phi$. Quibus positis principia motus sequentes suppeditant aequationes:

$$\text{I. } \frac{ddx}{d\tau^2} = -\frac{\Delta x}{u^3} \text{ et II. } \frac{ddy}{d\tau^2} = -\frac{\Delta y}{u^3}$$

vbi si omnes distantiae in semidiametris terrae exprimantur inuenitur litera Δ ita vt sit $l\Delta = 9,6118924$ siquidem distantia media terrae a sole assumatur 24000 semidiametris terrae.

§. 42. Iam pro statu planetae initiali qui ad datum tempus vt cognitus spectatur fuerit $x = a$, $y = b$; tum vero pro motu $\frac{dx}{d\tau} = \alpha$, $\frac{dy}{d\tau} = \beta$; vnde pro eodem initio erat $u = \sqrt{aa + bb}$ et tang. $\Phi = \frac{b}{a}$. Statuamus autem breuitatis gratia pro initio $u = f$ et $\Phi = \vartheta$, ita vt sit $f = \sqrt{aa + bb}$ et tang. $\vartheta = \frac{b}{a}$. Hinc porro fiat

$$\frac{dx^2 + dy^2}{d\tau^2} = \alpha\alpha + \beta\beta = \zeta\zeta.$$

Dein-

Deinde cum sit

$$u\,du = x\,dx + y\,dy \text{ erit } \frac{u\,du}{d\tau} = a\alpha + b\beta = g;$$

vbi notetur esse

$$dx^2 + dy^2 = du^2 + uu\,d\Phi^2.$$

§. 43. Nunc aggrediamur aequationes nostras differentiales secundi gradus et haec combinatio I. $2\,dx$ + II. $2\,dy$ dat

$$\frac{2\,dx\,ddx + 2\,dy\,ddy}{d\tau^2} = -\frac{2\Delta(x\,dx + y\,dy)}{u^3} = -\frac{2\Delta\,du}{uu}$$

cuius integrale est

$$\frac{dx^2 + dy^2}{d\tau^2} = \frac{2\Delta}{u} + C.$$

Pro quae constante determinanda quia in statu initiali sit

$$\frac{dx^2 + dy^2}{d\tau^2} = \zeta\zeta \text{ et } u = f \text{ erit } C = \zeta\zeta - \frac{2\Delta}{u}$$

ita vt habeamus

$$\frac{dx^2 + dy^2}{d\tau^2} = \zeta\zeta - \frac{2\Delta}{f} + \frac{2\Delta}{u}.$$

§. 44. Consideretur nunc ista combinatio: I. x + II. y, quae dat

$$\frac{x\,ddx + y\,ddy}{d\tau^2} = -\frac{\Delta(xx + yy)}{u^3} = -\frac{\Delta}{u};$$

cui addatur aequatio modo inventa integralis, eritque

$$\frac{x\,ddx + dx^2 + y\,ddy + dy^2}{d\tau^2} = \zeta\zeta - \frac{2\Delta}{f} + \frac{\Delta}{u}.$$

Cum autem sit

$$x\,ddx + dx^2 = d.\,x\,dx \text{ et } y\,ddy + dy^2 = d.\,y\,dy$$

habebimus

$$\frac{d.\,x\,dx + d.\,y\,dy}{d\tau^2} = \frac{d.\,u\,du}{d\tau^2} = \zeta\zeta - \frac{2\Delta}{f} + \frac{\Delta}{u}$$

 quae

quae multiplicata per $2u\,du$ et integrata praebet

$$\frac{uu\,du^2}{d\tau^2} = C + 2\Delta u - \frac{2\Delta uu}{f} + \zeta\zeta uu$$

vbi cum initio fuerit $u = f$ et $\frac{u\,du}{d\tau} = g$, constans ita definitur vt sit $C = gg - \zeta\zeta ff$, vnde obtinebimus

$$\frac{uu\,du^2}{d\tau^2} = gg - \zeta\zeta ff + 2\Delta u - \frac{2\Delta uu}{f} + \zeta\zeta uu$$

quae est altera aequatio integralis duas tantum continens variabiles.

§. 45. In hac formula notetur fore

$$gg - \zeta\zeta ff = -(a\beta - b\alpha)^2$$

vnde si statuamus $a\beta - b\alpha = b$, aequatio inuenta, posito adhuc breuitatis ergo $\frac{2\Delta}{f} - \zeta\zeta = F$, hanc induet formam:

$$\frac{uu\,du^2}{d\tau^2} = -bb + 2\Delta u - Fuu$$

vnde radice extracta reperitur

$$d\tau = \frac{\pm u\,du}{\sqrt{-bb + 2\Delta u - Fuu}}.$$

Vbi notari oportet, signorum ambiguorum valere superius si planeta a sole remoueatur, siue si motus a perihelio computetur: sin autem ad solem accedat, siue si motus ab aphelio computetur vti moris est pro Planetis, inferius signum capi debebit. Notum autem est huius formulae integrale partim algebraice partem per arcum circuli exprimi posse, ita vt hinc ad quoduis tempus τ distantia u per notas tabulas astronomicas assignari possit.

§. 46. Quoniam ex hac aequatione ratio $\frac{du}{d\tau}$ constat ex aequatione integrali primum inuenta, ob

$$dx^2 + dy^2 = du^2 + uu\,d\Phi^2$$

etiam

etiam ratio $\frac{d\Phi}{d\tau}$ colligi poſſet. Verum hoc facilius ex iſta combinatione I y — II x fieri poteſt, ex qua fit

$$\frac{y\,d\,dx - x\,d\,dy}{d\tau^2} = 0$$ cuius integrale eſt $\frac{y\,dx - x\,dy}{d\tau} = C$

ex ſtatu autem initiali concluditur $= b\alpha - a\beta = -b$ ita vt ſit $\frac{x\,dy - y\,dx}{d\tau} = b$. Cum vero ſit

$$x = u\,\text{coſ.}\,\Phi \text{ et } y = u\,\text{ſin.}\,\Phi$$

hincque

$$dx = du\,\text{coſ.}\,\Phi - u\,d\Phi\,\text{ſin.}\,\Phi \text{ et } dy = du\,\text{ſin.}\,\Phi + u\,d\Phi\,\text{coſ.}\,\Phi$$

erit $x\,dy - y\,dx = u\,u\,d\Phi$

ſicque aequatio noſtra erit

$$\frac{u\,u\,d\Phi}{d\tau} = b \text{ ſiue } d\Phi = \frac{b\,d\tau}{u\,u}$$

quare loco $d\tau$ valore ſubſtituto habebimus

$$d\Phi = \frac{\pm\,b\,du}{u\sqrt{-bb + 2\Delta u - F\,u\,u}}.$$

§. 47. Ad hanc formulam ſimpliciorem reddendam ponamus $u = \frac{1}{z}$ vt fiat $\frac{du}{u} = -\frac{dz}{z}$ ac reperietur

$$d\Phi = \frac{\mp\,b\,dz}{\sqrt{-b\,b\,z\,z + 2\Delta z - F}}.$$

Nunc poſt ſignum radicale ſecundum membrum $2\Delta z$ elidamus, ponendo $z = s + \frac{\Delta}{b\,b}$ ac prodibit

$$d\Phi = \frac{\mp\,b\,ds}{\sqrt{-b\,b\,s\,s + \frac{\Delta\Delta}{b\,b} - F}}$$

vbi ponatur

$$\frac{\Delta\Delta}{b\,b} - F = n\,n\,b\,b \text{ ita vt}$$

$$n = \sqrt{\frac{\Delta\Delta}{b^4} - \frac{F}{b\,b}} = \frac{1}{b\,b}\sqrt{\Delta\Delta - F\,b\,b}$$

et

et impetremus

$$d\Phi = \frac{\mp ds}{\sqrt{nn - ss}} \text{ cuius integrale manifesto est}$$

$$\Phi = C \mp A \text{ sin.} \frac{s}{n}, \text{ vel etiam } \Phi = C \pm A \text{ cos.} \frac{s}{n}$$

vbi iterum constans ex statu initiali determinari debet, pro quo fit $\Phi = \vartheta$. Deinde ob $u = f$ erit $z = \frac{1}{f}$ et $s = \frac{1}{f} - \frac{\Delta}{bb}$, vnde pro initio fit

$$\vartheta = C \pm A \text{cos.} \frac{bb - \Delta f}{nfbb}, \text{ ideoque } C = \vartheta \mp A \text{cos.} \frac{bb - \Delta f}{nfbb}.$$

Quare si iste arcus cuius cosinus $\frac{bb - \Delta f}{nfbb}$ ponatur $= \eta$ erit

$$C = \vartheta \mp \eta, \text{ ideoque } \Phi = \vartheta \mp \eta \pm A \text{ cos.} \frac{s}{n}.$$

Quare cum fit $z = \frac{1}{u}$ et $s = \frac{1}{u} - \frac{\Delta}{bb}$ erit

$$\Phi = \vartheta \mp \eta \pm A \text{ cos.} \frac{bb - \Delta u}{nbbu}.$$

§. 48. Sumamus hic signum superius valere, quia casu contrario mutatio facile instituitur, et cum sit

$$\Phi - \vartheta + \eta = A \text{ cos.} \frac{bb - \Delta u}{nbbu}$$

ponamus breuitatis gratia

$$\Phi - \vartheta + \eta = \omega \text{ eritque cos.} \omega = \frac{bb - \Delta u}{nbbu}$$

vnde colligitur $u = \frac{bb}{nbb \text{ cos.} \omega + \Delta}$. Fiat nunc $\frac{bb}{\Delta} = c$ vt prodeat $u = \frac{c}{1 + nc \text{ cos.} \omega}$; fiat porro $nc = -e$, vt sit $e = -\frac{1}{\Delta} \sqrt{\Delta^2 - Fbb}$, sicque habebitur $u = \frac{c}{1 - e \text{ cos.} \omega}$, ex qua formula intelligitur orbitam esse ellipsin cuius semiparameter $= c$ et excentricitas $= e$, angulus vero ω anomalia vera.

§. 49.

§. 49. Quodsi ergo sumamus $\omega = 0$, reperietur distantia aphelii a sole $= \frac{c}{1-e}$: at sumto $\omega = 180°$ fit distantia perihelii a sole $= \frac{c}{1+e}$, vnde axis transversus orbitae colligitur $= \frac{2c}{1-ee}$, et semiaxis transversus $= \frac{c}{1-ee} = \frac{\Delta}{F}$. Tum vero erit $\Phi = \vartheta + \eta - \omega$, vbi est $\cos. \omega = \frac{u-c}{eu}$; vnde cum initio vbi $u = f$ fiebat $\omega = \eta$, hic angulus η commodius ita definietur, vt sit

$$\cos. \eta = \frac{f-c}{ef}, \text{ existente } c = \frac{bb}{\Delta} \text{ et } e = -\frac{1}{\Delta}\sqrt{(\Delta^2 - Fbb)}.$$

§. 50. Superest autem adhuc vt positionem lineae absidum respectu axis S♈ determinemus. Hunc in finem statuamus planetam in suo aphelio, vbi vt vidimus fit $u = \frac{c}{1-e}$, et angulus Φ ipsam dabit inclinationem lineae absidum ad rectam S♈. Posito autem $u = \frac{c}{1-e}$ fiet $\cos. \omega = 1$, ideoque $\omega = 0$, vnde fit $\Phi = \vartheta - \eta$.

§. 51. Colligamus nunc breuiter omnia quae ad determinationem orbitae sunt inuenta, et ex datis quantitatibus principalibus a, b et α, β, quaeramus primo angulum ϑ, vt sit $\tang. \vartheta = \frac{b}{a}$, hincque porro distantia

$$f = \sqrt{aa + bb} = a \sec. \vartheta.$$ Praeterea capiatur

$$h = a\beta - b\alpha \text{ et } \zeta\zeta = \alpha\alpha + \beta\beta$$

vnde definiatur $F = \frac{2\Delta}{f} - \zeta\zeta$. Quibus elementis constitutis erit orbitae ellipticae semi-axis transuersus $= \frac{\Delta}{F}$, semiparameter $c = \frac{hh}{\Delta}$ et excentricitas

$$e = -\frac{1}{\Delta}\sqrt{\Delta^2 - Fhh}.$$

Deinde quaeratur angulus η, vt fit cof. $\eta = \frac{f-c}{e f}$, hincque erit longitudo lineae abfidum fiue angulus fub quo ea ad rectam fixam ♈ S inclinatur $= \vartheta - \eta$; ficque omnia innotefcunt quae ad nouam orbitam determinandam requiruntur.

Determinatio orbitae terrae poft cometae actionem.

§. 52. Ex §. 38. poftquam actio cometae ceffauit habebimus

$$a = 23943, 223 \qquad \alpha = - \ 28, 460$$
$$b = \ 1650, 540 \qquad \beta = + \ 412, 023$$

vnde fupra iam inuenimus

$$\vartheta = 3^\circ. 56'. 37'' \text{ et } f = 24000, 01$$

Hinc erit

$$b = \alpha \beta - b \alpha = 9912134, 37 \text{ ideoque}$$
$$l b = 6, 9961672 \text{ et } l b b = 13, 9923344.$$

Porro quaeratur

$$\zeta \zeta = \alpha \alpha + \beta \beta = 170572, 97 \text{ denique}$$
$$\frac{\Delta}{f} = 170483, 04 \text{ vnde colligitur}$$
$$F = \frac{2 \Delta}{f} - \zeta \zeta = 170393, 11 \text{ et } l F = 5, 2314519.$$

Praeterea

$$c = \frac{b b}{\Delta} = 24012, 76 \text{ et } e = + \sqrt{1 - \frac{F b b}{\Delta \Delta}} = +0, 0044$$

fiue excentricitas tam eft parua, vt ob errores calculi ineuitabiles ne definiri quidem queat; ita vt terra etiamnunc in circulo moueri fit cenfenda, vnde etiam

etiam nulla datur linea abfidum, cuius pofitionem inueftigari oportet. Poftremo autem erit femiaxis transuerfus $\frac{\Delta}{F} = 24012, 70$.

§. 53. Poftquam igitur actio mutua ceffauit, terra adhuc in circulo reuoluetur, cuius radius $= \frac{\Delta}{F} = 24012, 70$, cum ante affumferimus circulum cuius radius $= 24000$ femidiametris terrae. Nunc igitur tempus periodicum aliquantillum augebitur in ratione fesquiplicata axium transuerforum: hoc eft in ratione $1 : \frac{24012}{24000}$ feu vt $1 : 1\frac{30}{24000}$. Augebitur igitur tempus periodicum fui parte $\frac{1}{1151}$, quod eft augmentum circiter 7 horar., quod difcrimen profecto fatis eft exiguum, dum ex tali occurfu fubuerfio totalis metuenda videretur.

Determinatio orbitae cometae poft actionem mutuam.

§. 54. Applicemus nunc etiam noftrum problema generale ad determinationem motus cometae, quo ceffante actione mutua deinceps feretur, atque ex §. 39. habebimus pro hoc cafu

$$a = 22805, 180 : b = 784, 819$$
$$\alpha = -597, 757 : \beta = -20, 623$$

vnde iam in poftremo calculo deduximus $f = 22818,74$ et angulum $\vartheta = 1^\circ. 58'. 16''$. Porro vero reperimus

$h = a\beta - b\alpha = -1180, 33$, ideoque $lh = (-)3,0720034$ et $lhh = 6,1440068$. Porro $\zeta\zeta = \alpha\alpha + \beta\beta = 357738,64$ et $\frac{\Delta}{f} = 179308,45$, vnde fit $F = \frac{2\Delta}{f} - \zeta\zeta = 878,26$

 hinc-

hincque fit $\frac{\Delta}{F} = 4658750,00$. Praeterea vero

$$c = \frac{hh}{\Delta} = 0,0003405 \text{ et } e = + \sqrt{(1 - \frac{Fhh}{\Delta\Delta})} = 1$$

tam parum enim ab vnitate discrepat, vt error vltra decimam figuram fractionis decimalis demum occurrat. Denique fiet

$$\text{cos. } \eta = \frac{f-c}{ef} = 1 \text{ ideoque } \eta = 0.$$

§. 55. Hinc igitur patet, post actionem mutuam orbitae cometae semiaxem transuersum fore

$$\frac{\Delta}{F} = 4658750$$

cum ante fuisset infinitus. Sicque cometa nunc habebit tempus periodicum, quod reperietur diuidendo istum semiaxem transuersum per 24000 vnde prodit 194; quocirca periodus cometae erit $= 194\sqrt{194}$ annis $= 2716$. Deinde cum sit semiparameter $c = 0,0003405$ patet, hanc orbitam a linea recta vix discrepare, id quod etiam inde intelligitur quod sit excentricitas $e = 1$. Denique cum prodierit $\eta = 0$ linea absidum cometae inclinabitur ad directionem fixam S ♈ sub angulo $\vartheta = 1°. 58'. 17''$ prorsus vt ante actionem mutuam. Secundum hypothesin autem, quam fecimus hic cometa recta in Solem se esset immersurus rediturus igitur nunquam inde foret. Hoc igitur modo omnia sunt expedita quae super casu proposito desiderarii possunt. Hinc igitur manifestum est id quod iam supra innuimus, ambos effectus actionis mutuae cum in accessu tum in recessu ortos se mutuo fere penitus destruere.

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