DE MOTV GLOBI CIRCA AXEM OBLIQVVM QVEMCVNQVE GYRANTIS ET SVPER PLANO HORIZONTALI INCEDENTIS.

Auctore
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Cum in omnibus quae hactenus de motu globorum super planis sunt tradita alius motus gyratorius non sit consideratus, nisi qui fiat circa axem ad motus directionem normalem; quaestio superest maxime ardua: quomodo globus, cui circa axem quemcunque obliquum fuerit impressus motus gyratorius, super plano sit progressurus; quoniam principia, ex quibus huiusmodi motus determinari oportet, neutiquam adhuc satis sunt euoluta, vt ad quosuis casus, qui occurrere possunt, applicari queant. Primus equidem haec principia in Tractatu meo de motu corporum solidorum seu rigidorum ista principia in lucem produxi, indeque plurima motus Phoenomena explicaui, quae principiis mechanicis vulgaribus prorsus erant inaccessa. Quin etiam in vltimo huius libri capite hoc ipsum argumentum de globo super plano horizontali incedente, dum interea circa axem obliquum quemcunque gyratur, omnio studio sum perscrutatus. Verum quia hic liber in paucorum manibus versatur, ac ista ipsa tractatio plerisque Geometris etiamnunc videtur incognita; haud abs re fore arbitror, si totum hoc argumentum hic denuo in lucem protraxero, prouti in loco memorato est pertractatum, vbi quidem nonnullas dilucidationes, si opus fuerit visum, adiungam, quo vniuersa Theoria globorum super plano horizontali vtcunque propulsorum completa reddatur. In hac autem inuestigatione imprimis ad effectum frictionis est respiciendum, quandoquidem remota frictione globus perpetuo eundem motum tam progressiuum quam gyratorium, sine vlla alteratione, esset conseruaturus. Frictionem autem eodem modo in calculum introducam, quo hactenus a Geometris tractari est solita. Quanquam enim forte omnia frictionis symptomata nondum fuerint penitus perspecta: tamen ista tractatio inde nullam mutationem pati est censenda; quoniam praecipuum negotium hic in euolutione abstrusissimorum principiorum mechanicae sublimioris et in integratione plurium formularum differentialium alias difficillimarum absoluitur.

Problema I.

§. 1. *Si globus super plano horizontali vtcunque tam motu progressiuo quam gyratorio moueatur, determinare celeritatem et directionem, qua punctum contactus radit superficiem horizontalem.*

Solutio.

Tab. IV. Fig. 1. Sit I centrum globi, simulque eius centrum inertiae eiusque radius sit $= f$, et contactus fiat in puncto imo T, motus

motus autem globi ita ſit comparatus, vt centrum inertiae moueatur ſecundum directionem P I R celeritate $= v$, ſimul vero gyretur circa axem quemcunque I O celeritate angulari $= \varepsilon$, in eum ſenſum, vt punctum T circa O incedat per arculum Tt, ac pro poſitione puncti O ſtatuamus angulum $PTO = \theta$ et arcum $TO = s$, vbi quidem arcus ita ſumo, quaſi radius globi eſſet $= 1$. Ducatur T V ipſi P I R parallela, ac ſi motus gyratorius abeſſet, punctum contactus T raſurum eſſet planum horizontale celeritate v in directione T V. Deinde ſi globus ſolo motu gyratorio ferretur, punctum T per Tt moueretur celeritate $f\varepsilon$ ſin. $TO = f\varepsilon$ ſin. s, cuius directio cum ſit horizontalis, in plano per rectam $T\Theta$ referatur, ita vt ſit angulus $ST\Theta = PTt = \theta - 90°$, ob OTt rectum. Erit ergo $VT\theta = 270° - \theta$. Capiantur rectae $TV = v$ et $T\Theta = f\varepsilon$ ſin. s, et quia punctum T his duobus motibus coniunctim mouetur, eius verus motus fiet ſecundum rectam T F, diagonalem parallelogrammi $TVF\Theta$. Ex F ad T V ducta normali F H, erit $VH = f\varepsilon$ ſin. s ſin. θ et $FH = -f\varepsilon$ ſin. s coſ. θ, vnde fit $FH = v - f\varepsilon$ ſin. s ſin. θ atque celeritas radens

$$TF = \sqrt{(vv - 2f\varepsilon v \text{ ſin. } s \text{ ſin. } \theta + ff\varepsilon\varepsilon \text{ ſin. } s^2)} \text{ et}$$
$$\text{tang. } VTF = \frac{-f\varepsilon \text{ ſin. } s \text{ coſ. } \theta}{v - f\varepsilon \text{ ſin. } s \text{ ſin. } \theta}.$$

Ducatur ex centro I ipſi T F parallela I Q, erit arcus T Q quadrans, et angulus $RTQ = VTF$; quare ſi I Q ſit directioni, ſecundum quam punctum T radit, parallela, erit

$$\text{tang. } PTQ = \frac{f\varepsilon \text{ ſin. } s \text{ coſ. } \theta}{v - f\varepsilon \text{ ſin. } s \text{ ſin. } \theta},$$

ac poſita celeritate radente

$\sqrt{(v v - 2 f \aleph v \sin. s \sin. \theta + f f \aleph^2 \sin. s^2)} = u$, erit
$\sin. PTQ = -\frac{f \aleph \sin. s \cos. \theta}{u}$ et
$\cos. PTQ = \frac{f \aleph \sin. s \sin. \theta - v}{u}$.

Corollarium 1.

§. 2. Fieri ergo poteſt, vt celeritas radens ideoque et attritus euaneſcat, quo caſu hae duae aequationes locum habere debent: altera $\aleph \sin. s \cos. \theta = 0$, altera vero $v = f \aleph \sin. s \sin. \theta$. Vnde ſtatim patet, ſi nullus adſit motus progreſſiuus, ſeu $v = 0$, nullum attritum affore ſi $\sin. s = 0$, hoc eſt ſi globus circa axem verticalem ZT gyretur.

Corollarium 2.

§. 3. Deinde motus globi ab attritu erit liber, ſi fuerit primo $\cos. \theta = 0$, ſeu angulus PTO rectus: deinde celeritas progreſſiua v ad angularem $\aleph$ hanc relationem tenere debet, vt ſit $v = f \aleph \sin. s$, ſeu $TV = T\Theta$, et angulus $ST\Theta = 0$.

Corollarium 3.

§. 4. Quando ergo globus huiusmodi motum eſt conſecutus, quia ſublato omni attritu etiam nulla adeſt frictio, globus eundem motum conſtanter conſeruabit, ſiquidem axis gyrationis IO habeat proprietatem axis principalis.

Scholion 1.

§. 5. Quemadmodum hic frictionem conſtituimus, ea non obſtat, quominus globus ſuper plano horizontali motum

motum ſuum intemeratum conſeruari poſſit, quod tamen minime fieri obſeruamus, cum globus ſuper tabula tali motu latus mox omnem motum amittat, cuius rei cauſſa reſiſtentiae aëris tribui nequit. Verum hic primum animaduerto, experimenta Theoriae nunquam perfectiſſime congruere: veluti dum caſu hic tractato aſſumſimus, contactum vnico fieri puncto, id ſemper in praxi ſecus euenit. Interim tamen ſi arcus TO eſt quadrans et $PTO = 90°$, exiſtente $v = f\,8$, etſi contactus non fiat in vnico puncto, tamen attritus euaneſcit, ideoque haec motus extinctio frictioni neutiquam adſcribi poteſt. Ex quo concludere debemus, praeter frictionem, vti hic eam definiuimus, aliud adhuc dari motus impedimentum, dum corpora ſuper ſuperficiebus incedunt, a frictione probe diſtinguendum, cuius ratio vtcunque fuerit comparata, eius effectus potius ſeorſim inueſtigari conuenit, quam frictionis indolem hic ſtabilitam immutari. Et quemadmodum hic a reſiſtentia aëris mentem abſtrahimus ita etiam licebit hoc obſtaculum frictionem concomitans a praeſenti argumento ſeiungere.

Scholion 2.

§. 6. Corpora hic ſphaerica conſidero, in quorum centro ſitum ſit ipſorum centrum inertiae I, quod ergo ipſum etiam in plano horizontali moueatur, et puncto contactus T perpetuo verticaliter immineat; ex quo patet, preſſionem in contactu ſemper fore ponderi corporis M aequalem. Si ergo corpus ex materia vniformi conſtaret, omnes eius diametri proprietate axium principalium gauderent. Sed concipiamus materiae diſtributionem

vtcun-

vtcunque inaequabilem, ita tamen vt centrum inertiae ca-
dat in centrum figurae: hocque pacto neceſſe erit in globo
ternos axes principales conſiderari, qui ex centro I pertin-
gant in puncta A, B, C, quadrantibus a ſe inuicem dis-
tantia, quorum reſpectu ſint momenta inertiae M $a\,a$,
M $b\,b$, M $c\,c$. Quanquam autem deinceps bina vel omnia
haec momenta inter ſe aequalia ſtatuemus, tamen conue-
niet tria huiusmodi puncta in ſuperficie fixa notaſſe, quo
ex eorum relatione ad ſpatium abſolutum facilius motus
globi definiri poſſit. Conſtitutis autem in globis his tri-
bus punctis A, B, C, quoniam motus gyrationis circa O,
quem in plagam T t dirigi aſſumimus, ſenſum habet CBA,
contrarium ei quem ſupra ſtatuimus, in applicatione for-
mularum generalium ad hunc caſum celeritatem angula-
rem ϑ vt negatiuam ſpectare debemus.

Problema II.

§. 7. *Si globus ſuper plano horizontali vtcunque moueatur, definire vires, quibus ſollicitatur, earumque momenta reſpectu ternorum axium principalium globi.*

Solutio.

Incluſus concipiatur globus Sphaerae vel fixae vel cum
Tab. IV. Fig. 2. eo parem motum progreſſiuum habenti, in qua Z ſit punc-
tum verticale eiusque oppoſitum T punctum contactus,
D E vero ſit diameter horizontalis ad certam mundi pla-
gam tendens et D P Q E circulus maximus horizontalis.
Nunc autem elapſo tempore t moueatur globus motu
progreſſiuo ſecundum directionem P I celeritate $= v$, po-
naturque arcus D P, ſeu angulus D Z P $= \varphi$; axes autem
prin-

principales nunc ſint in A, B, C. Tum vero globus iam gyretur circa axem IO, celeritate angulari $= \varepsilon$ in ſenſum ACB, ſitque pro ſitu puncti O angulus PTO ſeu $PZO = \theta$ et arcus $ZO = s$. Etſi enim ante arcum TO poſuimus $= s$, quia tantum eius ſinus in computum intrat, perinde eſt. Erit ergo angulus $DZO = \theta + \Phi$ et $EZO = 180° - \theta - \Phi$. Deinde a punctis A, B, C tam ad O quam ad Z arcus circulorum maximorum ducti concipiantur, ſintque hi arcus $AO = \alpha$, $BO = \beta$, $CO = \gamma$; $ZA = l$, $ZB = m$, $ZC = n$ et anguli $EZA = \lambda$, $EZB = \mu$, $EZC = \nu$. In praecedente autem problemate oſtendimus, punctum contactus T planum ſubiectum radere ſecundum directionem radio IQ parallelam celeritate $= \sqrt{(vv - 2f\varepsilon v \,\text{ſin.}\, s \,\text{ſin.}\, \theta + ff\varepsilon\varepsilon \,\text{ſin.}\, s^2)}$, foreque

$$\text{tang. } PTQ = \text{tang. } PZQ = \frac{f\varepsilon \,\text{ſin.}\, s \,\text{coſ.}\, \theta}{v - f\varepsilon \,\text{ſin.}\, s \,\text{ſin.}\, \theta},$$

denotante f radium globi. Cum igitur preſſio in T ſit $= M$, frictio erit $= \delta M$, quae puncto T eſt applicata ſecundum directionem ipſi QI parallelam. Hac ergo vi reſoluta ſecundum directiones axium principalium IA, IB, IC, prodibunt ternae vires, quae in puncto T applicatae ſunt concipiendae, ex quibus porro colliguntur ſequentia momenta:

Reſpectu axis IA in ſenſum BC:

$$-\delta M f \,\text{coſ.}\, CQ \,\text{coſ.}\, BT + \delta M f \,\text{coſ.}\, BQ \,\text{coſ.}\, CT = P.$$

Reſpectu axis IB in ſenſum CA:

$$-\delta M f \,\text{coſ.}\, AQ \,\text{coſ.}\, CT + \delta M f \,\text{coſ.}\, CQ \,\text{coſ.}\, AT = Q.$$

Reſpectu axis IC in ſenſum AB:

$$-\delta M f \,\text{coſ.}\, BQ \,\text{coſ.}\, AT + \delta M f \,\text{coſ.}\, AQ \,\text{coſ.}\, BT = R,$$

erunt

erunt ergo haec tria momenta:

$$P = \delta M f (\text{cos.}\, m \,\text{cos.}\, CQ - \text{cos.}\, n \,\text{cos.}\, BQ)$$
$$Q = \delta M f (\text{cos.}\, n \,\text{cos.}\, AQ - \text{cos.}\, l \,\text{cos.}\, CQ)$$
$$R = \delta M f (\text{cos.}\, l \,\text{cos.}\, BQ - \text{cos.}\, m \,\text{cos.}\, AQ).$$

Pro puncto autem Q ponamus angulum $PZQ = \xi$, vt sit

$$\text{tang.}\, \xi = \frac{f \beta \,\text{sin.}\, s \,\text{cos.}\, \theta}{v - f \beta \,\text{sin.}\, s \,\text{sin.}\, \theta}$$

et posita celeritate radente

$$\sqrt{(vv - 2 f v \beta \,\text{sin.}\, s \,\text{sin.}\, \theta + f f \beta \beta \,\text{sin.}\, \theta^2} = u, \text{ erit}$$
$$\text{sin.}\, \xi = \frac{-f \beta \,\text{sin.}\, s \,\text{cos.}\, \theta}{u} \text{ et cos.}\, \xi = \frac{f \beta \,\text{sin.}\, s \,\text{sin.}\, \theta - v}{u}.$$

Fit ergo $DZQ = \Phi + \xi$ et $EZQ = 180^\circ - \Phi - \xi$, hinc $AZQ = 180^\circ - \xi - \Phi - \lambda$; $BZQ = \mu + \xi + \Phi - 180^\circ$; ergo

$$\text{cos.}\, AQ = -\text{cos.}\, (\xi + \Phi + \lambda) \,\text{sin.}\, l$$
$$\text{cos.}\, BQ = -\text{cos.}\, (\xi + \Phi + \mu) \,\text{sin.}\, m$$
$$\text{cos.}\, CQ = -\text{cos.}\, (\xi + \Phi + \nu) \,\text{sin.}\, n.$$

Ex relatione igitur, quae inter angulos λ, μ et ν intercedit, concludemus momenta virium:

$$P = \delta M f \,\text{sin.}\, l \times \text{sin.}\, (\lambda + \Phi + \xi)$$
$$Q = \delta M f \,\text{sin.}\, m \times \text{sin.}\, (\mu + \Phi + \xi)$$
$$R = \delta M f \,\text{sin.}\, n \times \text{sin.}\, (\nu + \Phi + \xi).$$

Pro-

Problema III.

§. 8. *Si motum gyratorium ad quoduis tempus vt datum spectemus, definire motum progressiuum globi.*

Solutio.

Quia centrum globi in plano horizontali moue- Tab. IV. Fig. 3. tur, descripserit id tempore t lineam GI, quae referatur ad directionem GX superiori directioni fixae DE parallelam, ductaque IX ad GX normali, sint coordinatae GX $= X$, XY $= Y$. Per I ducatur recta DE ipsi GX parallela, quae erit ipsa diameter DE (fig. 2). Ducatur IP, ita vt sit angulus DIP $=$ EIR $= \Phi$, et centrum I per hypothesin progreditur in directione IR celeritate $= v$, ita vt sit celeritas secundum GX $= v \operatorname{cos.} \Phi$ et celeritas secundum XI $= v \operatorname{sin.} \Phi$, ideoque $dX = v\,dt \operatorname{cos.} \Phi$ et $dY = v\,dt \operatorname{sin.} \Phi$. Ducatur recta QIS, ita vt IQ sit directioni, qua punctum contactus radit, parallela, erit angulus EIQ $=$ DIS $= 180° - \xi - \Phi$; (est enim aequalis angulo EZQ in praecedente figura) vnde globus sollicitari censendus est vi $= \delta M$ in directione IS. Hinc ergo oritur vis secundum ID $= -\delta M \operatorname{cos.} (\xi + \Phi)$ et vis secundum XI $= \delta M \operatorname{sin.} (\xi + \Phi)$, ex quibus colligitur

$$\frac{d.\,v \operatorname{cos.} \Phi}{2g\,dt} = \frac{dv \operatorname{cos.} \Phi - v\,d\Phi \operatorname{sin.} \Phi}{2g\,dt} = \delta \operatorname{cos.} (\xi + \Phi)$$

$$\frac{d.\,v \operatorname{sin.} \Phi}{2g\,dt} = \frac{dv \operatorname{sin.} \Phi + v\,d\Phi \operatorname{cos.} \Phi}{2g\,dt} = \delta \operatorname{sin.} (\xi + \Phi)$$

hincque porro

$$\frac{dv}{2g\,dt} = \delta \operatorname{cos.} \xi \text{ et } \frac{v\,d\Phi}{2g\,dt} = \delta \operatorname{sin.} \xi,$$

ita

ita vt sit

$$\frac{v\,d\,\Phi}{d\,v} = \text{tang.}\,\xi = \frac{f u \,\text{sin.}\, s\, \text{sin.}\,\theta}{v - f u \,\text{sin.}\, s\, \text{sin.}\,\theta}.$$

Problema IV.

§. 9. *Definito motu progressiuo globi, determinare eius motum gyratorium.*

Solutio.

Tab. IV. Fig. 2. Spectetur nunc centrum globi I vt quiescens, et maneant omnes denominationes in Problemate II adhibitae, sintque $M\,a\,a$, $M\,b\,b$, $M\,c\,c$ momenta inertiae respectu axium principalium I A, I B, I C, quae primo vt inaequalia consideremus. Quoniam vero hic celeritatem angularem u vt negatiuam spectare debemus, quia tendit in sensum A C B, si ponamus $u\,\text{cos.}\,\alpha = x$, $u\,\text{cos.}\,\beta = y$, et $u\,\text{cos.}\,\gamma = z$, in formulis generalibus has litteras x, y, z negatiue sumi oportet, quo facto ex § 810. Theoriae meae motus corporum solidorum deducuntur hae aequationes motum determinantes:

$$d x + \frac{bb - cc}{a a} y z\, d t + \frac{2 \delta f g}{a a} d t \,\text{sin.}\, l \,\text{sin.}\, (\lambda + \Phi + \xi) = 0;$$

$$d y + \frac{cc - aa}{b b} x z\, d t + \frac{2 \delta f g}{b b} d t \,\text{sin.}\, m \,\text{sin.}\, (\mu + \Phi + \xi) = 0;$$

$$d z + \frac{aa - bb}{c c} x y\, d t + \frac{2 \delta f g}{c c} d t \,\text{sin.}\, n \,\text{sin.}\, (\nu + \Phi + \xi) = 0;$$

$$d l \,\text{sin.}\, l = d t\, (z \,\text{cos.}\, m - y \,\text{cos.}\, n);$$

$$d m \,\text{sin.}\, m = d t\, (x \,\text{cos.}\, n - z \,\text{cos.}\, l);$$

$$d n \,\text{sin.}\, n = d t\, (y \,\text{cos.}\, l - x \,\text{cos.}\, m);$$

$$d \lambda \,\text{sin.}\, l^2 = d t\, (y \,\text{cos.}\, m + z \,\text{cos.}\, n);$$

$$d \mu \,\text{sin.}\, m^2 = d t\, (z \,\text{cos.}\, n + x \,\text{cos.}\, l);$$

$$d \nu \,\text{sin.}\, n^2 = d t\, (x \,\text{cos.}\, l + y \,\text{cos.}\, m).$$

Tum

Tum vero ex motu progreſſiuo habemus

$$d v = 2\delta g\, d t \operatorname{cof.} \xi,$$
$$v\, d \Phi = 2\delta g\, d t \operatorname{fin.} \xi \text{ et}$$
$$\operatorname{tang.} \xi = \frac{f s \operatorname{fin.} s \operatorname{cof.} \theta}{v - f s \operatorname{fin.} s \operatorname{cof.} \theta},$$

vbi eſt $PZO = \theta$ et $ZO = s$. Cum ergo ſit angulus $EZO = 180° - \theta - \Phi$, erit $AZO = 180° - \lambda - \theta - \Phi$: hincque

$$\operatorname{cof.} \alpha = \operatorname{cof.} l \operatorname{cof.} s - \operatorname{fin.} l \operatorname{fin.} s \operatorname{cof.} (\lambda + \theta + \Phi)$$
$$\operatorname{cof.} \beta = \operatorname{cof.} m \operatorname{cof.} s - \operatorname{fin.} m \operatorname{fin.} s \operatorname{cof.} (\mu + \theta + \Phi)$$
$$\operatorname{cof.} \gamma = \operatorname{cof.} n \operatorname{cof.} s - \operatorname{fin.} n \operatorname{fin.} s \operatorname{cof.} (\nu + \theta + \Phi),$$

exiſtente

$$\operatorname{cof.} s = \operatorname{cof.} l \operatorname{cof.} \alpha + \operatorname{cof.} m \operatorname{cof.} \beta + \operatorname{cof.} n \operatorname{cof.} \gamma,$$

vnde ſequitur fore

$$\left.\begin{array}{l} + \operatorname{fin.} l \operatorname{cof.} l \operatorname{cof.} (\lambda + \theta + \Phi) \\ + \operatorname{fin.} m \operatorname{cof.} m \operatorname{cof.} (\mu + \theta + \Phi) \\ + \operatorname{fin.} n \operatorname{cof.} n \operatorname{cof.} (\nu + \theta + \Phi) \end{array}\right\} = 0.$$

Ponamus $8 \operatorname{cof.} s = p$ et $8 \operatorname{fin.} s = q$ ita vt ſit

$$\operatorname{tang.} \xi = \frac{f q \operatorname{cof.} \theta}{v - f q \operatorname{fin.} \theta} = \frac{v\, d \Phi}{d v},$$

eritque

$$x = p \operatorname{cof.} l - q \operatorname{fin.} l \operatorname{cof.} (\lambda + \theta + \Phi),$$
$$y = p \operatorname{cof.} m - q \operatorname{fin.} m \operatorname{cof.} (\mu + \theta + \Phi),$$
$$z = p \operatorname{cof.} n - q \operatorname{fin.} n \operatorname{cof.} (\nu + \theta + \Phi),$$

ex quibus valoribus fit

$$d l = q\, d t \operatorname{fin.} (\lambda + \theta + \Phi);$$
$$d m = q\, d t \operatorname{fin.} (\mu + \theta + \Phi);$$
$$d n = q\, d t \operatorname{fin.} (\nu + \theta + \Phi);$$
$$d \lambda = p\, d t + q\, d t \operatorname{cot.} l \operatorname{cof.} (\lambda + \theta + \Phi);$$
$$d \mu = p\, d t + q\, d t \operatorname{cot.} m \operatorname{cof.} (\mu + \theta + \Phi);$$
$$d \nu = p\, d t + q\, d t \operatorname{cot.} n \operatorname{cof.} (\nu + \theta + \Phi);$$

 inde-

indeque porro

$$d x = d p \cos. l - d q \sin. l \cos. (\lambda + \theta + \Phi) + q (d \theta + d \Phi) \sin. l \sin. (\lambda + \theta + \Phi),$$

$$d y = d p \cos. m - d q \sin. m \cos. (\mu + \theta + \Phi) + q (d \theta + d \Phi) \sin. m \sin. (\mu + \theta + \Phi),$$

$$d z = d p \cos. n - d q \sin. n \cos. (\nu + \theta + \Phi) + q (d \theta + d \Phi) \sin. n \sin. (\nu + \theta + \Phi),$$

At sine subsidio harum substitutionum ex aequationibus ternis primis cum in genere sit

$$\sin. l \cos. l \sin. (\lambda + A) + \sin. m \cos. m (\mu + A) + \sin. n \cos. n \sin. (\nu + A) = 0,$$

elicimus hanc aequationem:

$$\left. \begin{array}{l} a a d x \cos. l + b b d y \cos. m + c c d z \cos. n \\ - a a x d l \sin. \lambda - b b y d m \sin. m - c c z d n \sin. n \end{array} \right\} = 0,$$

cuius integrale est

$$a a x \cos. l + b b y \cos. m + c c z \cos. n = C,$$

quae aequatio, adhibitis substitutionibus, abit in hanc:

$$\left. \begin{array}{l} p (a a \cos. l^2 + b b \cos. m^2 + c c \cos. n^2) \\ - q a a \sin. l \cos. l \cos. (A + \theta + \Phi) \\ - q b b \sin. m \cos. m \cos. (\mu + \theta + \Phi) \\ - q c c \sin. n \cos. n \cos. (\nu + \theta + \Phi) \end{array} \right\} = \text{Const.}$$

Deinde etiam per reductiones §. 934. Theoriae meae traditas pro vi viua colligitur haec aequatio differentialis:

$$a a x d x + b b y d y + c c z d z = 2 \delta f g q d t \sin. (\xi - \theta).$$

Scholion.

§. 10. Ad reductiones hic factas intelligendas ex formulis traditis, vbi angulos μ et ν per l, λ, m, n exprespreſſimus, notari conuenit fieri

$$\operatorname{coſ.}(\mu+\theta+\Phi) = -\frac{\operatorname{coſ.} l \operatorname{coſ.} m \operatorname{coſ.}(\lambda+\theta+\Phi) + \operatorname{coſ.} n \operatorname{ſin.}(\lambda+\theta+\Phi)}{\operatorname{ſin.} l \operatorname{ſin.} m},$$

$$\operatorname{coſ.}(\nu+\theta+\Phi) = -\frac{\operatorname{coſ.} l \operatorname{coſ.} n \operatorname{coſ.}(\lambda+\theta+\Phi) - \operatorname{coſ.} m \operatorname{ſin.}(\lambda+\theta+\Phi)}{\operatorname{ſin.} l \operatorname{ſin.} m},$$

$$\operatorname{ſin.}(\mu+\theta+\Phi) = -\frac{\operatorname{coſ.} l \operatorname{coſ.} m \operatorname{ſin.}(\lambda+\theta+\Phi) - \operatorname{coſ.} n \operatorname{coſ.}(\lambda+\theta+\Phi)}{\operatorname{ſin.} l \operatorname{ſin.} m},$$

$$\operatorname{ſin.}(\nu+\theta+\Phi) = -\frac{\operatorname{coſ.} l \operatorname{coſ.} n \operatorname{ſin.}(\lambda+\theta+\Phi) + \operatorname{coſ.} m \operatorname{coſ.}(\lambda+\theta+\Phi)}{\operatorname{ſin.} l \operatorname{ſin.} n}.$$

Ac ſimili modo anguli $\mu+\Phi+\xi$ et $\nu+\Phi+\xi$ ad angulum $\lambda+\Phi+\xi$ reuocari poſſunt. Deinde etiam pro ſequentibus reductionibus haec forma imprimis eſt notanda:

$$\operatorname{ſin.}(\mu+B)\operatorname{coſ.}(\nu+C) - \operatorname{ſin.}(\nu+B)\operatorname{coſ.}(\mu+C),$$

quae ob

$$\operatorname{ſin.} M \operatorname{coſ.} N = \tfrac{1}{2}\operatorname{ſin.}(M+N) + \tfrac{1}{2}\operatorname{ſin.}(M-N),$$

reducitur ad

$$\operatorname{ſin.}(\mu-\nu)\operatorname{coſ.}(B-C);$$

hocque modo reductionem pro aliis formulis inſtituendo, reperiemus:

$$\operatorname{ſin}(\mu+B)\operatorname{coſ}(\nu+C) - \operatorname{ſin.}(\nu+B)\operatorname{coſ.}(\mu+C) = \operatorname{ſin.}(\mu-\nu)\operatorname{coſ.}(B-C),$$

$$\operatorname{ſin.}(\mu+B)\operatorname{ſin.}(\nu+C) - \operatorname{ſin.}(\nu+B)\operatorname{ſin.}(\mu+C) = -\operatorname{ſin.}(\mu-\nu)\operatorname{ſin.}(B-C),$$

$$\operatorname{coſ.}(\mu+B)\operatorname{coſ}(\nu+C) - \operatorname{coſ}(\nu+B)\operatorname{coſ.}(\mu+C) = -\operatorname{ſin.}(\mu-\nu)\operatorname{ſin}(B-C),$$

vbi ſin $(\mu-\nu)$ per formulas vſurpatas datur: eſt enim $\operatorname{ſin.}(\mu-\nu) = \frac{\operatorname{coſ.} e}{\operatorname{ſin.} m \operatorname{ſin.} n}$.

Problema V.

§. 11. *Si globus ex materia vniformi conſtet, vel ſaltem ita fuerit comparatus, vt omnia momenta inertiae ſint*

inter

inter se aequalia, eique initio impressus fuerit motus quicunque, determinare eius continuationem.

Solutio.

Cum hic sit $aa = bb = cc$, seu momentum inertiae respectu omnium diametrorum $= Maa$, prima aequatio integrata praebet $aap =$ Const. vnde p erit quantitas constans. Statuatur ergo $p = b$, et ternae aequationes priores hanc induent formam:

$$\text{I. } -dq \cos.(\lambda+\theta+\varphi) + q(d\theta+d\varphi)\sin.(\lambda+\theta+\varphi) + \tfrac{2\delta fg}{aa}dt\sin.(\lambda+\varphi+\xi) = 0,$$

$$\text{II. } -dq \cos.(\mu+\theta+\varphi) + q(d\theta+d\varphi)\sin.(\mu+\theta+\varphi) + \tfrac{2\delta fg}{aa}dt\sin.(\mu+\varphi+\xi) = 0,$$

$$\text{III. } -dq \cos.(\nu+\theta+\varphi) + q(d\theta+d\varphi)\sin.(\nu+\theta+\varphi) + \tfrac{2\delta fg}{aa}dt\sin.(\nu+\varphi+\xi) = 0,$$

quarum autem sufficit binas consideraſſe, quia iam inde nata est conclusio $p = b$. Iam per superiores reductiones binae posteriores aequationes ita combinentur:

$$\text{II. } \cos.(\nu+\theta+\varphi) - \text{III. } \cos.(\mu+\theta+\varphi)$$

quae combinatio praebet

$$q(d\theta+d\varphi)\sin.(\mu-\nu) + \tfrac{2\delta fg}{aa}dt\sin.(\mu-\nu)\cos.(\xi-\theta) = 0,$$

seu

$$q(d\theta+d\varphi) + \tfrac{2\delta fg}{aa}dt\cos.(\xi-\theta) = 0.$$

Deinde combinatio

$$\text{II. } \sin.(\nu+\theta+\varphi) - \text{III. } \sin.(\mu+\theta+\varphi) \text{ dat}$$

dq

seu

$$d q \text{ fin.} (\mu - \nu) - \tfrac{2 \delta f g}{a a} d t \text{ fin.} (\mu - \nu) \text{ fin.} (\xi - \theta) = 0$$

$$d q = \tfrac{2 \delta f g}{a a} d t \text{ fin.} (\xi - \theta),$$

qui valor in vltima aequatione pro viribus viuis substitutus praebet

$$x d x + y d y + z d z = q d q, \text{ hincque}$$

$$x x + y y + z z = 88 = \text{const.} + q q = \text{const.} + 88 \text{ fin.} s^2$$

ita vt sit $88 \text{ cos.} s^2$ quantitas constans, vti iam inuenimus, ob $8 \text{ cos} s = p = h$. Hinc istas habemus aequationes a litteris l, m, n, λ, μ, ν immunes:

I. $q (d \theta + d \varphi) + \tfrac{2 \delta f g}{a a} d t \text{ cos.} (\xi - \theta) = 0,$

II. $d q = \tfrac{2 \delta f g}{a a} d t \text{ fin.} (\xi - \theta) = 0,$

III. $d v = 2 \delta g d t \text{ cos.} \xi,$

IV. $v d \varphi = 2 \delta g d t \text{ fin.} \xi,$

quibus adiungatur haec finita: $\text{tang.} \xi = \frac{f q \text{ cos.} \theta}{v - f q \text{ fin.} \theta}$, quae in hanc transformata:

$$v \text{ fin.} \xi - f q \text{ cos.} (\xi - \theta) = 0,$$

differentietur, prodibitque

$$d v \text{ fin.} \xi + v d \xi \text{ cos.} \xi - f d q \text{ cos.} (\xi - \theta) + f q d \xi \text{ fin.} (\xi - \theta)$$
$$- f q d \theta \text{ fin.} (\xi - \theta) = 0.$$

Iam combinatio:

I. $\text{fin.} (\xi - \theta)$ + II. $\text{cos.} (\xi - \theta)$ dat

$$q (d \theta + d \varphi) \text{ fin.} \xi - \theta) + d q \text{ cos.} (\xi - \theta) = 0,$$

quae aequatio per f multiplicata illi addatur, fietque

$$d v \text{ fin.} \xi + v d \xi \text{ cos.} \xi + f q (d \xi + d \varphi) \text{ fin.} (\xi - \theta) = 0.$$

 Porro

Porro ob $\frac{dv}{v\,d\phi} = \frac{\operatorname{cof.}\xi}{\operatorname{fin.}\xi}$ erit

$$v\,(d\phi + d\xi)\operatorname{cof}\xi + fq\,(d\phi + d\xi)\operatorname{fin.}(\xi - \theta) = 0, \text{ feu}$$
$$(d\phi + d\xi)\,(v\operatorname{cof.}\xi + fq\operatorname{fin.}(\xi - \theta)) = 0,$$

quorum factorum finitus: $v\operatorname{cof.}\xi + fq\operatorname{fin.}(\xi - \theta)$, euanescere nequit ob

$$v\operatorname{fin.}\xi - fq\operatorname{cof.}(\xi - \theta) = 0,$$

fequeretur enim inde $v\operatorname{cof.}\theta = 0$ et $fq\operatorname{cof.}\theta = 0$, quod non nifi cafu $\theta = 90^\circ$ locum habet. Relinquitur ergo vt fit $d\phi + d\xi = 0$, ideoque $\phi + \xi$ conftans. Hoc impetrato reliqua non difficulter expedientur. Ad integrationes autem determinandas pro ftatu initiali $t = 0$, ponamus fuiffe celeritatem progreffiuam $v = e$, $\phi = 0$, $PZO = \theta = \mathfrak{h}$; $ZO = s = \mathfrak{f}$ et celeritatem angularem $\varepsilon = \varepsilon$ in fenfum ACB; hinc erit $p = b = \varepsilon\operatorname{cof.}\mathfrak{f}$ et $q = \varepsilon\operatorname{fin.}\mathfrak{f}$; porro

$$\operatorname{tang}\xi = \frac{\varepsilon f\operatorname{fin.}\mathfrak{f}\operatorname{cof.}\mathfrak{h}}{e - \varepsilon f\operatorname{fin.}\mathfrak{f}\operatorname{fin.}\mathfrak{h}}.$$

Statuatur

$$\frac{\varepsilon f\operatorname{fin.}\mathfrak{f}\operatorname{cof.}\mathfrak{h}}{e - \varepsilon f\operatorname{fin.}\mathfrak{f}\operatorname{fin.}\mathfrak{h}} = \operatorname{tang.}\zeta,$$

vt fuerit initio $\xi = \zeta$, ac perpetuo erit $\xi + \phi = \zeta$, ita vt angulus $DZQ = \zeta$ maneat conftans. Quare cum fit $\xi = \zeta - \phi$ erit

$$v\operatorname{fin.}(\zeta - \phi) = fq\operatorname{cof.}(\zeta - \theta - \phi).$$

Supra autem inuenimus:

$$\frac{d.\,v\operatorname{cof.}\phi}{2g\,dt} = \delta\operatorname{cof.}(\xi + \phi) = \delta\operatorname{cof.}\zeta \text{ et}$$
$$\frac{d.\,v\operatorname{fin.}\phi}{2g\,dt} = \delta\operatorname{fin.}(\xi + \phi) = \delta\operatorname{fin.}\zeta,$$

vnde integrando colligimus:

$$v\operatorname{cof.}\phi = e + 2\delta g t\operatorname{cof.}\zeta \text{ et } v\operatorname{fin.}\phi = 2\delta g t\operatorname{fin.}\zeta,$$

hinc-

hincque

$$v = \sqrt{(ee + 4\delta e g t \operatorname{cof.}\zeta + 4\delta\delta g g t t)},$$

$$\operatorname{tang.}\phi = \frac{2\delta g t \operatorname{fin.}\zeta}{e + 2\delta g t \operatorname{cof.}\zeta}, \text{ atque}$$

$$\operatorname{tang.}(\zeta - \phi) = \frac{e \operatorname{fin.}\zeta}{e \operatorname{cof.}\zeta + 2\delta g t} = \frac{f q \operatorname{cof.}\theta}{v - f q \operatorname{fin.}\theta} = \operatorname{tang.}\xi.$$

Deinde ob $d\Phi = -d\xi$ binae priores aequationes abeunt in

$$\text{I. } q(d\xi - d\theta) = \frac{2\delta f g}{aa} dt \operatorname{cof.}(\xi - \theta).$$

$$\text{II. } dq = \frac{2\delta f g}{aa} dt \operatorname{fin.}(\xi - \theta),$$

quarum haec per illam diuifa dat

$$\frac{dq}{q(d\xi - d\theta)} = \frac{\operatorname{fin.}(\xi - \theta)}{\operatorname{cof.}(\xi - \theta)},$$

qua integrata prodit $q \operatorname{cof.}(\xi - \theta) = \mathbf{C}$, ideoque

$$q \operatorname{cof.}(\xi - \theta) = \varepsilon \operatorname{fin.}\mathfrak{f} \operatorname{cof.}(\zeta - \mathfrak{h}),$$

vnde valor ipfius q in prima fubftitutus praebet:

$$\frac{\varepsilon(d\xi - d\theta) \operatorname{fin.}\mathfrak{f} \operatorname{cof.}(\zeta - \mathfrak{h})}{\operatorname{cof.}(\xi - \theta)^2} = \frac{2\delta f g}{aa} dt,$$

et integrando

$$\varepsilon \operatorname{fin.}\mathfrak{f} \operatorname{cof.}(\zeta - \mathfrak{h}) \operatorname{tang.}(\xi - \theta) = \mathbf{C} + \frac{2\delta f g}{aa} t,$$

vbi $\mathbf{C} = \varepsilon \operatorname{fin.}\mathfrak{f} \operatorname{fin.}(\theta - \mathfrak{h})$, at

$$\operatorname{tang.}(\xi - \theta) = \operatorname{tang.}(\zeta - \phi - \theta) = \frac{\operatorname{tang.}(\zeta - \Phi) - \operatorname{tang.}\theta}{1 + \operatorname{tang.}(\zeta - \Phi)\operatorname{tang.}\theta} \text{ et}$$

$$\operatorname{tang.}\theta = \frac{\operatorname{tang.}\xi - \operatorname{tang.}(\xi - \theta)}{1 + \operatorname{tang.}\xi \operatorname{tang.}(\xi - \theta)}.$$

Sed per hypothefin eft $\varepsilon \operatorname{fin.}\mathfrak{f} = \frac{e \operatorname{fin.}\zeta}{f \operatorname{cof.}(\zeta - \mathfrak{h})}$, vnde fit

$$\operatorname{tang.}(\xi - \theta) = \operatorname{tang.}(\zeta - \mathfrak{h}) + \frac{2\delta f f g t}{e a a \operatorname{fin.}\zeta} \text{ et}$$

$$\operatorname{tang.}\xi = \frac{e \operatorname{fin.}\zeta}{e \operatorname{cof.}\zeta + 2\delta g t},$$

hincque angulus θ facile determinatur: indeque $q = \frac{e \operatorname{fin.}\zeta}{f \operatorname{cof.}(\xi - \theta)}$.

Verum

Verum hic notari oportet, cum ſit

$$\text{tang.}\ \zeta = \frac{e f\ \text{ſin.}\ \mathfrak{f}\ \text{coſ.}\ \mathfrak{h}}{e - \varepsilon f\ \text{ſin.}\ \mathfrak{f}\ \text{ſin.}\ \mathfrak{h}},$$

eſſe vt ſupra de angulo ξ oſtendimus

$$\text{ſin.}\ \zeta = \frac{-e f\ \text{ſin.}\ \mathfrak{f}\ \text{coſ.}\ \mathfrak{h}}{\sqrt{(e e - 2 e \varepsilon f\ \text{ſin.}\ \mathfrak{f}\ \text{ſin.}\ \mathfrak{h} + \varepsilon \varepsilon f f\ \text{ſin.}\ \mathfrak{f}^2)}}\ \text{et}$$

$$\text{coſ.}\ \zeta = \frac{-e + \varepsilon f\ \text{ſin.}\ \mathfrak{f}\ \text{ſin.}\ \mathfrak{h}}{\sqrt{(e e - 2 \varepsilon e f\ \text{ſin.}\ \mathfrak{f}\ \text{ſin.}\ \mathfrak{h} + \varepsilon \varepsilon f f\ \text{ſin.}\ \mathfrak{f}^2)}},\ \text{vnde}$$

$$\text{coſ.}\ (\zeta - \mathfrak{h}) = \frac{-e\ \text{coſ.}\ \mathfrak{h}}{\sqrt{(e e - 2 e \varepsilon f\ \text{ſin.}\ \mathfrak{f}\ \text{ſin.}\ \mathfrak{h} + \varepsilon \varepsilon f f\ \text{ſin.}\ \mathfrak{f}^2)}}.$$

His inuentis cum ſit $\mathfrak{s}\ \text{coſ.}\ s = \varepsilon\ \text{coſ.}\ \mathfrak{f}$ et $\mathfrak{s}\ \text{ſin.}\ s = q$, erit

$$\mathfrak{s} = \sqrt{(q q + \varepsilon \varepsilon\ \text{coſ.}\ \mathfrak{f}^2)}\ \text{et tang.}\ s = \frac{q}{\varepsilon\ \text{coſ.}\ \mathfrak{f}}.$$

Sicque tam motus progreſſiuus, quam ad quoduis tempus axis gyrationis O cum celeritate angulari $\mathfrak{s}$ poterit aſſignari, id quod ad motus cognitionem ſufficit. Determinatio autem ſitus punctorum A, B, C ad quoduis tempus nimis eſt ardua, quam vt eam perficere liceat.

Corollarium 1.

§. 12. Cum ſit celeritas angularis $\mathfrak{s} = \frac{\varepsilon\ \text{coſ.}\ \mathfrak{f}}{\text{coſ.}\ s}$, ſeu coſinui arcus S O reciproce proportionalis; ſequitur ſi polus gyrationis O initio fuerit in ſuperiori hemiſphaerio DZE, eum nunquam in inferius peruenire poſſe; in tranſitu enim per circulum horizontalem D E prodiret celeritas angularis $\mathfrak{s}$ infinita.

Corollarium 2.

§. 13. Ob eandem rationem, ſi polus gyrationis O initio fuerit in hemiſphaerio inferiori D T E, is nunquam in ſuperius aſcendet; ſin autem initio fuerit in ipſo

ipso circulo horizontali DE, perpetuo in eodem manebit: scilicet si initio axis gyrationis fuerit horizontalis, perpetuo horizontalis manebit.

Corollarium 3.

§. 14. Si fuerit initio angulus $DZO = \mathfrak{h}$ rectus, fiet $\sin. \zeta = 0$ et ob

$$\tang. (\zeta - \mathfrak{h}) = \frac{e f \sin. \mathfrak{f} - e \sin. \mathfrak{h}}{e \cos. \mathfrak{h}},$$

erit etiam $\xi - \theta$ rectus. Sed ob $\tang. \xi = \frac{e \sin. \zeta}{e \cos. \zeta + 2 \delta g t}$ angulus ξ euenescit, vnde angulus $\theta = PZO$ prodit rectus. Simulatque igitur angulus PZO factus fuerit rectus, perpetuo rectus manebit.

Corollarium 4.

§. 15. Memorabilis est etiam proprietas, quod angulus $\xi + \varphi$, seu DZQ, et in fig. 3 angulus DIQ fit constans. Recta enim QIS sibi perpetuo manebit parallela, et quia globus in motu progressiuo sollicitatur vi constante δM secundum eandem directionem IS, curua ab eo descripta GI parabola sit necesse est.

Scholion 1.

§. 16. Hic autem motus globi, vti nostris formulis est definitus, diutius non durat, quam reuera frictio adest, seu planum horizontale in puncto contactus T raditur. Si enim eueniat vt rasio cesset, seu celeritas radens in T euanescat, subito frictio euanescit, formulaeque inuentae non amplius locum habent. Tum igitur globus motu tam progressiuo quam gyratorio vniformiter in directum progredietur, neque axis gyrationis vllam amplius mutationem patietur. Ac si statim initio motus globo impressus ita fuerit comparatus, vt frictio fuerit nulla, quod euenit si tam εf sin. $\mathfrak{f}$ cos. $\mathfrak{h} = 0$, quam $e = \varepsilon f$ sin. $\mathfrak{f}$ sin. $\mathfrak{h}$, tum etiam globus nullam frictionem sentiet, et statim ab initio motum progressiuum uniformiter in directum prosequetur, simulque vniformiter circa eundem axem gyrabitur. Verum si corpori ab initio alius motus quicunque fuerit impressus, semper aliquo tempore elapso eo reducetur, vt frictio euanescat, indeque motum suum vniformiter prosequetur, quod memorabile temporis punctum in sequenti problemate inuestigabimus.

Scholion 2.

Tab. IV. Fig. 2. §. 17. Quae in solutione problematis elicuimus, huc redeunt: ex motu primum impresso habemus celeritatem motus progressiui $= e$, secundum directionem DI: ac si gyretur circa axem IO celeritate angulari ε in sensum ACB, seu ZETD, qui sensus *antrorsum tendens* dici solet, fueritque arcus $ZO = \mathfrak{f}$ et angulus $DZO = \mathfrak{h}$: tum vero radius globi sit $= f$ eiusque momentum inertiae $= Maa$ respectu omnium diametrorum, existente M eius massa: ex his datis colligitur celeritas radens in puncto contactus $= \sqrt{(ee - 2\varepsilon ef \text{ sin. } \mathfrak{f} \text{ sin. } \mathfrak{h} + \varepsilon\varepsilon ff \text{ sin. } \mathfrak{f}^2)}$, quae si ponatur $= k$, quaeratur angulus $DZQ = \zeta$, vt sit

$$\text{sin. } \zeta = \frac{-\varepsilon f \text{ sin. } \mathfrak{f} \text{ cos. } \mathfrak{h}}{k} \text{ et cos. } \zeta = \frac{\varepsilon f \text{ sin. } \mathfrak{f} \text{ sin. } \mathfrak{h} - e}{k},$$

eritque IQ directio motus radentis. Tum si elapso tempore t globi centrum proferatur celeritate v secundum directionem PI, et gyret celeritate angulari $= \nu$ in sensum

ſum ZETD circa polum O, ponaturque $DZP = \phi$, $PZO = \theta$ et $ZO = s$: inuenimus primo

$$\text{tang.}\, \phi = \frac{2\delta g t \,\text{ſin.}\, \zeta}{e + 2\delta g t \,\text{coſ.}\, \zeta}$$

et celeritatem centri $= \sqrt{(ee + 4\delta e g t \,\text{coſ.}\, \zeta + 4\delta\delta g g t t)}$, at celeritas radens etiamnunc fiet in directione IQ, exiſtente $DZQ = \zeta$; vnde poſito $PZQ = \xi$ erit

$$\text{tang.}\, \xi = \frac{e \,\text{ſin.}\, \zeta}{e \,\text{coſ.}\, \zeta + 2\delta g t}.$$ Porro eſt

$$\text{tang.}\, (\xi - \theta) = \text{tang.}\, (\zeta - \mathfrak{h}) + \frac{2\delta f f g t}{e a a \,\text{ſin.}\, \zeta},$$

exiſtente

$$\text{tang.}\, (\zeta - \mathfrak{h}) = \frac{\varepsilon f \,\text{ſin.}\, \mathfrak{f} - e \,\text{ſin.}\, \mathfrak{h}}{e \,\text{coſ.}\, \mathfrak{h}},$$

vnde angulus θ innoteſcit, hincque ob $DZO = \phi + \theta = \zeta - \xi + \theta$, concluditur

$$\text{tang.}\, DZO = \text{tang.}\, (\phi + \theta) = \frac{\varepsilon a a k \,\text{ſin.}\, \mathfrak{f} \,\text{ſin.}\, \mathfrak{h} + 2\delta f g t (e - \varepsilon) f \,\text{ſin.}\, \mathfrak{f} \,\text{ſin.}\, \mathfrak{h}}{\varepsilon a a k \,\text{ſin.}\, \mathfrak{f} \,\text{coſ.}\, \mathfrak{h} - 2\delta \varepsilon f f g t \,\text{ſin.}\, \mathfrak{f} \,\text{coſ.}\, \mathfrak{h}}.$$

Atque ex his tandem nacti ſumus $\vartheta \,\text{coſ.}\, s = \varepsilon \,\text{coſ.}\, \mathfrak{f}$ et $\vartheta \,\text{ſin.}\, s = \frac{e \,\text{ſin.}\, \zeta}{f \,\text{coſ.}\, (\xi - \theta)}$. Denique pro celeritate radente ſecundum IQ, ea eſt $\sqrt{(vv - 2\vartheta v f \,\text{ſin.}\, s \,\text{ſin.}\, \theta + \vartheta\vartheta f f \,\text{ſin.}\, s^2)}$; quae ſi vocetur $= w$, ſupra oſtendimus eſſe

$$\text{ſin.}\, \xi = -\frac{\vartheta f \,\text{ſin.}\, s \,\text{coſ.}\, \theta}{w} \quad \text{et} \quad \text{coſ.}\, \xi = \frac{\vartheta f \,\text{ſin.}\, s \,\text{ſin.}\, \theta - v}{w},$$

vnde ϑ et s definiuntur. Sed pro ſitu punctorum A, B, C in globo fixorum ad quoduis tempus determinando formulae adeo fiunt intricatae, vt nihil inde concludi queat. Interim ſi pro puncto A vocetur $ZA = l$ et $EZA = \lambda$, ad has binas aequationes totum negotium reducitur:

I. dl

$$\text{I. } dl = dt\left(\varepsilon \sin. \mathfrak{f} \sin. (\mathfrak{h} + \lambda) - \frac{2\delta f g t}{aa} \cos. (\zeta + \lambda)\right),$$

$$\text{II. } d\lambda \sin. l = \varepsilon dt \cos. \mathfrak{f} \sin. l + \varepsilon dt \cos. l \sin. \mathfrak{f} \cos. (\mathfrak{h} + \lambda) + \frac{2\delta f g t}{aa} \sin. (\zeta + \lambda),$$

quarum resolutio vereor ne frustra suscipiatur. Cum autem ad quoduis tempus axem gyrationis cum celeritate angulari assignare valeamus, quod ad motus cognitionem, qualis vulgo desideratur, sufficere potest, eo magis mirum videtur, quod motus singulorum globi punctorum quasi vires analyseos superet. Multo minus igitur de motu globorum, in quibus momenta inertiae non sunt aequalia, quicquam definire licebit.

Problema VI.

§. 18. *Si globo, cuius omnia momenta inertiae sunt inter se aequalia, motus quicunque fuerit impressus, assignare temporis punctum, vbi celeritas radens, ideoque et frictio euanescit, indeque globus motu vniformi progredi pergit.*

Solutio.

Supra §. 2. vidimus, vt attritus euanescat, has duas conditiones requiri: alteram $\mathfrak{u} \sin. s \cos. \theta = 0$, alteram $v = f \mathfrak{u} \sin. s \sin. \theta$, seu in expressione

$$\text{tang. } \xi = \frac{f \mathfrak{u} \sin. s \cos. \theta}{v - f \mathfrak{u} \sin. s \sin. \theta},$$

tam numeratorem quam denominatorem simul euanescere debere. Cum autem inuenerimus

$$\text{tang. } \xi = \frac{e \sin. \zeta}{e \cos. \zeta + 2\delta g t},$$

vbi numerator $e \sin. \zeta$ est constans, si in illa forma numerator euanescat, positio $\cos. \theta = 0$ tempus quaesitum declarabit.

Verum

Verum idem luculentius determinabimus, si ad quoduis tempus elapsum t celeritatem radentem w inuestigemus. Cum igitur ex valore §. 17. inuento: fin. $\xi = -\frac{\text{ʚ} f \text{ fin. } s \text{ cof. } \theta}{w}$ habeamus

$$w = \frac{-\text{ʚ} f \text{ fin. } s \text{ cof. } \theta}{\text{fin. } \xi},$$

quae expressio ob ʚ fin. $s = \frac{e \text{ fin. } \zeta}{f \text{ cof. } (\xi - \theta)}$ abit in hanc:

$$w = \frac{-e \text{ fin. } \zeta \text{ cof. } \theta}{\text{fin. } \xi \text{ cof. } (\xi - \theta)}:$$

atque ob $\theta = \xi - (\xi - \theta)$ in hanc:

$$w = -e \text{ fin. } \zeta (\text{cot. } \xi + \text{tang. } (\xi - \theta));$$

si hic pro tang. ξ et tang. $(\xi - \theta)$ valores supra inuentos substituamus, reperiemus:

$$w = -(e \text{ cof. } \xi + 2\delta g t + e \text{ fin. } \zeta \text{ tang. } (\zeta - \mathfrak{h}) + \frac{2\delta ffgt}{aa}).$$

At vero est

$$\text{cof. } \zeta + \text{fin. } \zeta \text{ tang. } (\zeta - \mathfrak{h}) = \frac{\text{cof. } \mathfrak{h}}{\text{cof. } (\zeta - \mathfrak{h})} \text{ et}$$
$$\text{cof. } (\zeta - \mathfrak{h}) = -\frac{e \text{ cof. } \mathfrak{h}}{k}; \text{ vnde fit}$$
$$e \text{ cof. } \zeta + e \text{ fin. } \zeta \text{ tang. } (\zeta - \mathfrak{h}) = -k,$$

vbi k denotat celeritatem radentem initialem. Quamobrem elapso tempore t habebimus celeritatem radentem

$$w = k - 2\delta g (1 + \frac{ff}{aa}) t,$$

ita vt ea labente tempore vniformiter decrescat, tandem ergo certe euanescat, id quod eueniet elapso tempore $t = \frac{aak}{2\delta g (aa + ff)}$, eritque tum cof. $\theta = 0$ et $\theta = 90° = PZO$. Quod ergo cum euenerit, videamus quomodo reliquae motus determinationes se sint habiturae, et quoniam

$2\delta g t = \frac{aak}{aa+ff}$ erit

$$\text{tang.}\,\Phi = \frac{aak\,\text{fin.}\,\zeta}{e(aa+ff)+aak\,\text{cof.}\,\zeta} \text{ et}$$

$$\text{tang.}\,\xi = \frac{e(aa+ff)\,\text{tang.}\,\zeta}{e(aa+ff)+aak},$$

hinc fit $\mathfrak{v}\,\text{fin.}\,s = \frac{e\,\text{fin.}\,\zeta}{f\,\text{fin.}\,\xi}$. Cum autem fit

$$v = \sqrt{\left(ee + \frac{2aaek\,\text{cof.}\,\zeta}{aa+ff} + \frac{a^4kk}{(aa+ff)^2}\right)} \text{ erit}$$

$$\text{fin.}\,\Phi = \frac{aak\,\text{fin.}\,\zeta}{(aa+ff)v}, \quad \text{cof.}\,\Phi = \frac{e(aa+ff)+aak\,\text{cof.}\,\zeta}{(aa+ff)v},$$

atque $\text{fin.}\,\xi = \frac{e\,\text{fin.}\,\zeta}{v}$ ideoque $\mathfrak{v}\,\text{fin.}\,s = \frac{v}{f}$. Porro quia eft $\mathfrak{v}\,\text{cof.}\,s = \varepsilon\,\text{cof.}\,\mathfrak{f}$, erit $\text{tang.}\,s = \frac{v}{\varepsilon f\,\text{cof.}\,\mathfrak{f}}$ et

$$\mathfrak{v} = \sqrt{\left(\frac{vv}{ff} + \varepsilon\varepsilon\,\text{cof.}\,\mathfrak{f}^2\right)},$$

fiue fubftituto valore v:

$$\mathfrak{v} = \frac{\sqrt{(eeff + 2\varepsilon eaaf\,\text{fin.}\,\mathfrak{f}\,\text{fin.}\,\mathfrak{h} + \varepsilon\varepsilon a^4\,\text{fin.}\,\mathfrak{f}^2 + \varepsilon\varepsilon(aa+ff)^2\,\text{cof.}\,\mathfrak{f}^2)}}{aa+ff}$$

ob $kk = ee - 2\varepsilon ef\,\text{fin.}\,\mathfrak{f}\,\text{fin.}\,\mathfrak{h} + \varepsilon\varepsilon ff\,\text{fin.}\,\mathfrak{f}^2$.

Corollarium 1.

§. 19. Quo maior ergo initio fuerit celeritas radens k, eo diutius motus durat, antequam ceffante frictione ad vniformitatem redigatur. Ac fi globus conftet ex materia homogenea, fit $aa = \frac{2}{5}ff$, ideoque motus vniformitas incipit elapfo tempore $t = \frac{k}{7\delta g}$ min. fec., hinc in hypothefi $\delta = \frac{1}{3}$ fit $t = \frac{3k}{7g}$, exiftente $g = 15\frac{5}{8}$ pedum Rhenanorum.

Corollarium 2.

§. 20. Vt centrum globi eodem tempore ad quietem redigatur, flatus initialis ita comparatus effe debet, vt fit $\text{cof.}\,\zeta = -1$ et $e = \frac{aak}{aa+ff}$. Fit ergo $\text{fin.}\,\mathfrak{h} = 1$ et $k = e - \varepsilon f\,\text{fin.}\,\mathfrak{f}\,\text{fin.}\,\mathfrak{h} = e - \varepsilon f\,\text{fin.}\,\mathfrak{f}$

hinc-

hincque ε fin. $\mathfrak{f} = \frac{ef}{aa}$. Porro ob $v = 0$, fit $s = 0$ et $\mathfrak{s} = \varepsilon$ cof. $\mathfrak{f}$, qua celeritate angulari iam globus circa axem verticalem quiefcentem gyrabitur, elapfo ab initio tempore $t = \frac{e}{2\delta g}$ min. fec.

Corollarium 3.

§. 21. Hoc autem cafu, quo initio eft $\mathfrak{h} = 90°$ et $\varepsilon = \frac{-ef}{aa\,\text{fin.}\,\mathfrak{f}}$ fit $\zeta = 180°$; $\Phi = 0$; $\xi = 180°$; $\theta = 90°$; $v = e - 2\delta g t$; tum vero habebimus $\mathfrak{s}$ cof. $s = \frac{-ef\,\text{cof.}\,\mathfrak{f}}{aa\,\text{fin.}\,\mathfrak{f}}$; $\mathfrak{s}$ fin. $s = -\frac{ef}{aa}(1 - \frac{2\delta g t}{e})$; hincque

$$\text{tang.}\, s = (1 - \tfrac{2\delta g t}{e})\,\text{tang.}\,\mathfrak{f} \text{ et}$$

$$\mathfrak{s} = \frac{-e}{aa\,\text{fin.}\,\mathfrak{f}} \sqrt{(1 - \tfrac{4\delta g t}{e}\,\text{fin.}\,\mathfrak{f} + \tfrac{4\delta\delta g g t t}{ee}\,\text{fin.}\,\mathfrak{f}^2)}.$$

At initio erat celeritas radens $k = e(1 + \frac{ff}{aa})$, elapfo autem tempore t ea eft $w = (1 + \frac{ff}{aa})(e - 2\delta g t)$, ficque pofito $t = \frac{e}{2\delta g}$ fimul fit $w = 0$, $v = 0$ et $s = 0$, vt ante.

Corollarium 4.

§. 22. Ne valor $\mathfrak{s}$ fin. $s = \frac{e\,\text{fin.}\,\zeta}{f\,\text{cof.}\,(\xi - \theta)}$ indefinitus videatur, quod fit fi numerator ac denominator euanefcant, feu $\zeta = 0$, conueniet loco fin. ζ et cof. $(\xi - \theta)$ valores ex fuperioribus fubftitui, atque hinc reperietur:

$$\mathfrak{s}\,\text{fin.}\, s = \sqrt{(\varepsilon\varepsilon\,\text{fin.}\,\mathfrak{f}^2 - \tfrac{4\delta\varepsilon f g t\,\text{fin.}\,\mathfrak{f}(\varepsilon f\,\text{fin.}\,\mathfrak{f} - e\,\text{fin.}\,\mathfrak{h})}{aak} + \tfrac{4\delta\delta ff gg tt}{a^4})},$$

vnde ob $\mathfrak{s}$ cof. $s = \varepsilon$ cof. $\mathfrak{f}$ prodit

$$\mathfrak{s}\mathfrak{s} = \varepsilon\varepsilon - \frac{4\delta\varepsilon f g t\,\text{fin.}\,\mathfrak{f}\,(\varepsilon f\,\text{fin.}\,\mathfrak{f} - e\,\text{fin.}\,\mathfrak{h})}{aak} + \frac{4\delta\delta ff gg tt}{a^4}.$$

Corol-

Corollarium 5.

§. 23. Cum ſit vis viua globi $= M(vv + aa\vartheta\vartheta)$, erat ea initio $= M(ee + \varepsilon\varepsilon aa)$; elapſo autem tempore t ea erit $= M(ee + \varepsilon\varepsilon aa - 4\delta gkt + 4(1 + \frac{ff}{aa})\delta\delta ggtt)$. At elapſo tempore $t = \frac{aak}{2\delta g(aa + ff)}$, vis viua fiet

$$\frac{M(eeff + 2\varepsilon e aaf \sin \mathfrak{f} \sin . \mathfrak{h} + \varepsilon\varepsilon aa(aa + ff \cos . \mathfrak{f}^2))}{aa + ff},$$

cuius defectus ab initiali eſt

$$\frac{M aa(ee - 2\varepsilon ef \sin . \mathfrak{f} \sin . \mathfrak{h} + \varepsilon\varepsilon ff \sin . \mathfrak{f}^2)}{aa + ff} = \frac{M aakk}{aa + ff},$$

ita vt iſta vis viua ſit

$$M(ee + \varepsilon\varepsilon aa - \frac{aakk}{aa + ff}).$$

Scholion.

Tab. IV. Fig. 2. §. 24. Ex his ergo formulis totus globi motus aſſignari poteſt, quicunque motus ei initio fuerit impreſſus. Interim tamen hae formulae non parum ſunt complexae, vnde ad clariorem explicationem haud abs re erit caſus quosdam magis notabiles euoluere. Cuiusmodi ſunt, vti iam ſupra innuimus, duo potiſſimum: alter quo arcus ZO initio erat quadrans; alter vero quo angulus $DZO = \mathfrak{h}$ erat rectus: vtrumque igitur ſeorſim explicemus.

Problema VII.

§. 25. *Si globo, in quo omnia momenta inertiae ſunt aequalia, initio motus gyratorius circa axem horizontalem fuerit impreſſus, praeter motum progreſſiuum definire continuationem motus.*

Solutio.

Solutio.

Cum initio axis gyrationis fuerit horizontalis, erit $f = ZO = 90°$. Denotante ergo e celeritatem progressiuam secundum directionem DIE, et ε celeritatem angularem circa axem IO, in sensum ZETD, sit pro puncto O angulus $DZO = \mathfrak{h}$, manente f radio globi et Maa momento inertiae. Ex his erat initio celeritas radens

$$k = \sqrt{(ee - 2\varepsilon ef \sin.\mathfrak{h} + \varepsilon\varepsilon ff)}$$

et pro eius directione IQ angulus $DZQ = \zeta$, vt sit

$$\sin.\zeta = \frac{-\varepsilon f \cos.\mathfrak{h}}{k} \text{ et } \cos.\zeta = \frac{\varepsilon f \sin.\mathfrak{h} - e}{k}.$$

His pro statu initiali constitutis, elapso tempore t centrum globi descripserit viam GI, vt iam sit in I, vbi eius celeritas secundum IR erit

$$v = \sqrt{(ee + \frac{4\delta egt(\varepsilon f \sin.\mathfrak{h} - e)}{k} + 4\delta\delta ggtt)},$$

Tab IV. Fig. 3.

vnde positis coordinatis $GX = X$, et $XI = Y$, ob

$$\tang.EIR = \tang.\Phi = \frac{-2\delta\varepsilon fgt \cos.\mathfrak{h}}{ek + 2\delta gt(\varepsilon f \sin.\mathfrak{h} - e)}$$

erit

$$dX = \varepsilon dt + \frac{2\delta gtdt}{k}(\varepsilon f \sin.\mathfrak{h} - e) \text{ et}$$

$$dY = \frac{-2\delta\varepsilon fgtdt \cos.\mathfrak{h}}{k}, \text{ ideoque}$$

$$GX = X = \varepsilon t + \frac{\delta gtt}{k}(\varepsilon f \sin.\mathfrak{h} - e) \text{ et}$$

$$XI = Y = -\frac{\delta\varepsilon fgtt}{k} \cos.\mathfrak{h}.$$

Tum vero pro motu gyratorio, qui iam fiat in sensum ZETD celeritate angulari $= \varepsilon$ circa polum O, existente $ZO = s$, $PZO = \theta$ et $DZQ = \Phi + \xi$, vbi IQ refert directionem celeritatis radentis, quia constanter est $\Phi + \xi = \zeta$, seu directio IQ constans, erit Fig. 2.

 tang.

$$\text{tang.}\,\xi = \frac{-\varepsilon f \,\text{cof.}\,\mathfrak{h}}{\varepsilon f \,\text{fin.}\,\mathfrak{h} - ee + 2\delta gkt} \text{ et}$$

$$\text{tang.}\,(\xi - \theta) = \frac{2f - e\,\text{fin.}\,\mathfrak{h}}{e\,\text{cof.}\,\mathfrak{h}} - \frac{2\delta fgkt}{\varepsilon eaa\,\text{cof.}\,\mathfrak{h}},$$

vnde ambo anguli ξ et θ definiuntur. Vel erit

$$\text{tang.}\,(\Phi + \theta) = \frac{\varepsilon aak\,\text{fin.}\,\mathfrak{h} + 2\delta fgt(e - \varepsilon f\,\text{fin.}\,\mathfrak{h})}{\varepsilon aak\,\text{cof.}\,\mathfrak{h} - 2\delta\varepsilon ffgt\,\text{cof.}\,\mathfrak{h}}.$$

Celeritas autem radens fecundum directionem IQ eft

$$w = k - 2\delta g\left(1 + \frac{ff}{aa}\right)t.$$

Tum vero ob $\vartheta\,\text{cof.}\,s = 0$ erit arcus $ZO = s$ quadrans et

$$\vartheta = \sqrt{\left(\varepsilon\varepsilon - \frac{4\delta\varepsilon fgt(\varepsilon f - e\,\text{fin.}\,\mathfrak{h})}{aak} + \frac{4\delta\delta ffggtt}{a^4}\right)}.$$

Hic autem motus inaequabilis tantum durabit per tempus $t = \frac{aak}{2\delta g(aa + ff)}$, quo elapfo eft $s = 90^\circ$,

$$\text{tang.}\,\Phi = \frac{-\varepsilon aaf\,\text{cof.}\,\mathfrak{h}}{e(aa + ff) + aa(\varepsilon f\,\text{fin.}\,\mathfrak{h} - e)} = \frac{-\varepsilon aa\,\text{cof.}\,\mathfrak{h}}{ef + \varepsilon aa\,\text{fin.}\,\mathfrak{h}}$$

$$= \sqrt{\left(ee + \frac{2aae(\varepsilon f\,\text{fin.}\,\mathfrak{h} - e)}{aa + ff} + \frac{a^4kk}{(aa + ff)^2}\right)};$$

$$\vartheta = \frac{v}{f} = \frac{\sqrt{(eeff + 2\varepsilon eaaf\,\text{fin.}\,\mathfrak{h} + \varepsilon\varepsilon a^4)}}{aa + ff}$$

fubftituto pro kk valore. Tum autem fit angulus $\theta = 90^\circ$ et $\text{fin.}\,\xi = \frac{e\,\text{fin.}\,\zeta}{v}$.

Corollarium 1.

§. 26. Si initio fuerit angulus $DZO = f = 0$, erit $k = \sqrt{(ee + \varepsilon\varepsilon ff)}$: pro angulo $DZQ = \zeta$ fit $\text{fin.}\,\zeta = -\frac{\varepsilon f}{k}$ et $\text{cof.}\,\zeta = \frac{-e}{k}$; tum vero poft tempus t prodit

$$v = \sqrt{\left(ee - \frac{4\delta eegt}{k} + 4\delta\delta ggtt\right)}$$

$$\text{tang.}\,\Phi = \frac{-2\delta\varepsilon ft}{e(k - 2\delta gt)};$$

$$X = t\left(1 - \frac{\delta gt}{k}\right)\varepsilon;$$

$$Y = \frac{-\delta\varepsilon fgtt}{k}.$$

tang.

$$\text{tang.}\, \xi = \frac{\varepsilon e f}{ee - 2\delta g k t};$$
$$\text{tang.}\, (\xi - \theta) = \frac{\varepsilon f}{e} - \frac{2\delta f g k t}{\varepsilon e a a};$$
$$\text{tang.}\, (\Phi = \theta) = \frac{2\delta e f g t}{\varepsilon a a k - 2\delta e f f g t}:$$
$$\vartheta = \sqrt{\left(\varepsilon\varepsilon - \frac{4\delta\varepsilon\varepsilon f f g t}{a a k} + \frac{4\delta\delta f f g g t t}{a^4}\right)} \text{ et}$$
$$w = k - 2\delta g \left(1 + \frac{ff}{aa}\right) t.$$

Elapso autem tempore $t = \frac{a a k}{2\delta g (a a + f f)}$ erit tang. $\Phi = \frac{\varepsilon a a}{e f}$; $v = \frac{f\sqrt{(e e f f + \varepsilon\varepsilon a^4)}}{aa + ff} = f\vartheta$; $\theta = 90^\circ$ et

$$\text{tang.}\, \xi = \frac{\varepsilon e f (a a + f f)}{ee(aa + ff) - (aakk)} = \frac{\varepsilon e (a a + f f)}{f(e e - \varepsilon\varepsilon a a)}.$$

Corollarium 2.

§. 27. Si angulus DZO $=$ ſ eſſet $= 180^\circ$, eaedem formulae motum indicabunt ſumta celeritate angulari ϑ negatiua, ſeu motu gyratorio in contrarium verſo. At ſi ſit $\varepsilon = 0$, ſeu globo ſolus motus progreſſiuus fuerit impreſſus, fit $k = e$, $\zeta = 180^\circ$, $v = e - 2\delta g t$; $\Phi = 0$, $X = t(e - \delta g t)$, $Y = 0$, $\xi = 180^\circ$; $\theta = 90^\circ$; $\vartheta = \frac{2\delta f g t}{a a}$, et elapſo tempore $t = \frac{a a e}{2\delta g (a a + f f)}$ fit $v = \frac{e f f}{a a + f f}$, $\vartheta = \frac{e f}{a a + f f}$ et

$$X = \frac{\varepsilon t (a a + 2 f f)}{2 (a a + f f)} = \frac{a a e e (a a + 2 f f)}{4\delta g (a a + f f)}.$$

Scholion.

§. 27. Caſus hic, quo globus initio nullum motum gyratorium eſt adeptus, in genere valet, neque ad vllam hypotheſin angulorum ſ et ꝓ eſt adſtrictus. Tum igitur globus in directum progreditur motu progreſſiuo retardato, motumque paullatim gyratorium accipiet, donec elapſo tempore $t = \frac{a a e}{2\delta g (a a + f f)}$ motum vniformem acquirat, quo deinceps continuo progrediatur. Hinc deducimur

mur ad casum, quo globus initio motum tantum gyratorium acceperit, sine vllo motu progressiuo, cuius euolutio est facilis. Posito enim $e = 0$ erit $k = \varepsilon f \sin. \mathfrak{f}$, hincque fit $\sin. \zeta = -\cos. \mathfrak{h}$ et $\cos. \zeta = \sin. \mathfrak{h}$, ergo $\zeta = \mathfrak{h} - 90°$, vbi pro axe gyrationis initio impressae IO est $ZO = \mathfrak{f}$ et $DZO = \mathfrak{h}$, existente celeritate angulari in sensum ZETD $= \varepsilon$. Elapso ergo tempore t fit $\Phi = \zeta$, scilicet sublato ab angulo $DZO = \mathfrak{h}$ angulo recto PZO, erit PI directio motus progressiui, quem globus acquiret, cuius celeritas erit $v = 2\delta g t$, ideoque tempori proportionalis. Tum vero erit $\tang. \xi = 0$ et $\tang. (\xi - \theta) = \infty$, ergo ob $\Phi + \xi = \zeta = \mathfrak{h} - 90°$ erit $\xi = 0$ et $\theta = 90°$, hinc $DZO = \zeta + 90° = \mathfrak{h}$, ita vt polus gyrationis O in eodem perpetuo circulo verticali reperiatur. Denique ex § 22. est

$$\mathfrak{s} \sin. s = \sqrt{(ee \sin. \mathfrak{f}^2 - \frac{4\delta \varepsilon f g t \sin. \mathfrak{f}}{aa} + \frac{4\delta\delta ff gg tt}{a^4}} = \varepsilon \sin \mathfrak{f} - \frac{2\delta f g t}{aa})$$

et $\mathfrak{s} \cos. s = \varepsilon \cos. \mathfrak{f}$, vnde fit

$$\tang. s = \tang. \mathfrak{f} - \frac{2\delta f g t}{\varepsilon aa \cos. \mathfrak{f}},$$

ita vt arcus ZO diminuatur, nisi fuerit quadrans vel eo maior, et

$$\mathfrak{s} = \sqrt{(\varepsilon\varepsilon - \frac{4\delta \varepsilon f g t \sin. \mathfrak{f}}{aa} + \frac{4\delta\delta ff gg tt}{a^4})}.$$

Motus autem ad vniformitatem reducetur elapso tempore

$t = \frac{\varepsilon aa f \sin. \mathfrak{f}}{2\delta g (aa + ff)}$; fitque tum

$$\mathfrak{s} = \frac{\varepsilon \sqrt{(a^4 \sin. \mathfrak{f}^2 + (aa + ff)^2 \cos. \mathfrak{f}^2)}}{aa + ff};$$

$$v = \frac{\varepsilon aa f \sin. \mathfrak{f}}{aa + ff} \quad \text{et} \quad \tang. s = \frac{aa \tang. \mathfrak{f}}{aa + ff}.$$

Si ergo fuisset $\mathfrak{f} = 0$, seu globo motus gyratorius circa axem verticalem impressus esset, sine vllo motu progressiuo, eundem motum sine vlla mutatione esset conseruaturus.

Problema

Problema VIII.

§. 28. *Si globo, in quo omnia momenta inertiae sunt aequalia, motus gyratorius fuerit impressus circa axem ad motus progressiui directionem normalem; definire continuationem motus.*

Solutio.

Cum motus progressiui initio impressi directio sit recta DIE, et celeritas $= e$, angulus DZO $= \mathfrak{h}$ est rectus, et sumto ZO $= \mathfrak{f}$ erat O polus circa quem initio globus accepit celeritatem angularem $= \varepsilon$ in sensum ZETD. Habemus ergo $k = \pm (e - f\varepsilon \sin. \mathfrak{f})$, vbi valorem positiuum pro k sumi oportet, ita vt hic duo prodeant casus seorsim euoluendi.

Casus primus

Sit $e > \varepsilon f \sin. \mathfrak{f}$, erit $k = e - \varepsilon f \sin. \mathfrak{f}$, quae est celeritas radens initio, eiusque directio IQ, ita vt sit $\sin. DQ = 0$ et $\cos. DQ = -1$, ideoque $DQ = \zeta = 180°$, et Q cadat in E globusque a frictione δM secundum directionem ID constanter retrahatur; vnde statim concluditur globi centrum I in eadem recta DE esse mansurum. Elapso ergo tempore t, ob $\cos. \zeta = -1$, fit celeritas centri $v = e - 2\delta g t$, et celeritas radens

$$w = e - \varepsilon f \sin. \mathfrak{f} - 2\delta g (1 + \tfrac{ff}{aa}) t;$$

tum vero $\Phi = 0$; $\xi = 180°$ atque $\theta = 0$. Quare pro axe gyrationis praesente IO est DIO $= 90°$, et posito arcu ZO $= s$ et celeritate angulari $= \mathfrak{s}$ habemus $\mathfrak{s} \cos. s = \varepsilon \cos. \mathfrak{f}$ et ex (§. 22.)

$$\mathfrak{s} \sin. s = \varepsilon \sin. \mathfrak{f} + \tfrac{2\delta f g t}{aa},$$

vnde

vnde colligitur:

$$\text{tang.}\, s = \text{tang.}\, \mathfrak{f} + \frac{2\delta f g t}{\varepsilon a a \cos.\mathfrak{f}} \quad \text{et}$$

$$\vartheta = \sqrt{\left(\varepsilon\varepsilon + \frac{4\delta \varepsilon f g t \sin.\mathfrak{f}}{a a} + \frac{4\delta \delta f f g g t t}{a^4}\right)}.$$

Hoc autem tempore t percurrit centrum I lineam rectam $GX = X = t(e - \delta g t)$. Hic autem motus inaequabilis durabit per tempus $t = \frac{a a (e - \varepsilon f \sin.\mathfrak{f})}{2\delta g (a a + f f)}$, quo elapso erit spatium

$$X = \frac{a a (e - \varepsilon f \sin.\mathfrak{f})(e(a a + f f) + \varepsilon a a f \sin.\mathfrak{f})}{2\delta g (a a + f f)^2}$$

et celeritas $v = \frac{f(\varepsilon a^2 f \sin.\mathfrak{f} + e f)}{a a + f f}$. At pro motu gyratorio fit

$$\text{tang.}\, s = \text{tang.}\, \mathfrak{f} + \frac{f(e - \varepsilon f \sin.\mathfrak{f})}{\varepsilon(a a + f f)\cos.\mathfrak{f}} = \frac{e f + \varepsilon a a \sin.\mathfrak{f}}{\varepsilon(a a + f f)\cos.\mathfrak{f}},$$

(existente $DIO = 90^\circ$) et celeritas angularis:

$$\vartheta = \frac{\sqrt{(e e f f + 2\varepsilon e f a a \sin.\mathfrak{f} + \varepsilon\varepsilon a^4 \sin.\mathfrak{f}^2 - \varepsilon\varepsilon(a a + f f)^2 \cos.\mathfrak{f}^2)}}{a a + f f}.$$

Casus secundus

Sit $e < \varepsilon f \sin.\mathfrak{f}$, seu $k = \varepsilon f \sin.\mathfrak{f} - e$, quae est celeritas radens initio, eiusque directio IQ talis, vt sit $\sin.DQ = 0$, $\cos.DQ = 1$, ergo $DQ = \zeta = 0$, et Q in D cadat. Globus ergo a frictione δM secundum directionem IE constanter acceleratur, eiusque centrum I in eadem recta IE progreditur, atque elapso tempore t erit celeritas $v = e + 2\delta g t$ et celeritas radens

$$w = \varepsilon f \sin.\mathfrak{f} - e - 2\delta g\left(1 + \frac{f f}{a a}\right) t.$$

Tum vero fit $\Phi = 0$ et $\xi = 0$ atque $\theta = 90^\circ$. Quare pro axe gyrationis praesenti IO est $DIO = 90^\circ$ et posito arcu $ZO = s$ et celeritate angulari $= \vartheta$ habebimus $\vartheta \cos.s = \varepsilon \cos.\mathfrak{f}$ et $\vartheta \sin.s = \varepsilon \sin.\mathfrak{f} - \frac{2\delta f g t}{a a}$, vnde fit

tang.

$$\text{tang.}\, s = \text{tang.}\, f - \frac{2\delta f g t}{\varepsilon a a \cos. f} \quad \text{et}$$

$$\vartheta = \sqrt{\left(\varepsilon\varepsilon - \frac{4\delta\varepsilon f g t \sin. f}{a a} + \frac{4\delta\delta f f g g t t}{a^4}\right)},$$

hoc tempore t centrum globi percurrit lineam rectam $GX = X = t(e + \delta g t)$. Hic autem motus inaequabilis durabit tantum per tempus $t = \frac{a a (\varepsilon f \sin. f - e)}{2\delta g (a a + f f)}$, quo elapso erit celeritas $v = \frac{f(e f - \varepsilon a a \sin. f)}{a a + f f}$ et spatium

$$X = \frac{a a (\varepsilon f \sin. f - e)(e (a a + f f) + \varepsilon a a f \sin. f)}{2\delta g (a a + f f)^2}.$$

At pro motu gyratorio reperitur

$$\text{tang.}\, s = \text{tang.}\, ZO = \frac{e f + \varepsilon a a \sin. f}{\varepsilon (a a + f f) \cos. f},$$

(existente perpetuo $DIO = 90°$), et celeritas angularis

$$\vartheta = \frac{\sqrt{(e e f f + 2\varepsilon e a a f \sin. f + \varepsilon\varepsilon a^4 \sin. f^2 + \varepsilon\varepsilon (a a + f f)^2 \cos. f^2)}}{a a + f f}.$$

Corollarium 1.

§. 29. Si fuerit $e = \varepsilon f \sin. f$, globus statim ab initio motum prosequetur vniformem, tam progressiuum quam gyratorium, qui casus limitem constituit inter binos tractatus.

Corollarium 2.

§. 30. Ad priorem casum, quo $e > \varepsilon f \sin. f$, referendi sunt ii, quibus ε habet negatiuum valorem, seu globo impressus fuerit initio motus gyratorius in sensum $ZDTE$. Posito autem $-\varepsilon$ loco ε, fieri potest vt globus reuertatur, antequam ad vniformitatem peruenerit.

Corollarium 3.

§. 31. Casu hoc quo ε negatiue capitur, ad tempus t habebimus, $\Phi = 0$, $\theta = 0$, $\xi = 180°$, $v = e - 2\delta g t$,

$$w = e + \varepsilon f \sin. \mathfrak{f} - 2\delta g \left(1 + \frac{ff}{aa}\right) t;$$
$$\text{tang.}\, s = \text{tang.}\, \mathfrak{f} - \frac{2\delta f g t}{\varepsilon a a \cos. \mathfrak{f}} \text{ et}$$
$$\mathfrak{v} = \sqrt{\left(\varepsilon\varepsilon - \frac{4\delta \varepsilon f g t \sin. \mathfrak{f}}{aa} + \frac{4\delta\delta ff g g t t}{a^4}\right)};$$

at poſt tempus $t = \frac{aa(e + \varepsilon f \sin. \mathfrak{f})}{2\delta g (aa + ff)}$, percurſo ſpatio

$$X = \frac{aa(e + \varepsilon f \sin. \mathfrak{f})(e(aa + 2ff) - \varepsilon a^2 f \sin. \mathfrak{f})}{2\delta g (aa + ff)^2},$$

globi motus vniformitatem attinget, eritque tum

$$v = \frac{f(ef - \varepsilon aa \sin. \mathfrak{f})}{aa + ff};$$
$$\text{tang.}\, s = \frac{\varepsilon aa \sin. \mathfrak{f} - ef}{\varepsilon(aa + ff)\cos. \mathfrak{f}} \text{ et}$$
$$\mathfrak{v} = \frac{\sqrt{(eeff - 2\varepsilon e aa f \sin. \mathfrak{f} + \varepsilon\varepsilon a^4 \sin. \mathfrak{f}^2 + \varepsilon\varepsilon(aa + ff)^2 \cos. \mathfrak{f}^2)}}{aa + ff}.$$

Scholion.

§. 32. Caſus hic praecipue eſt memorabilis, quo globo eiusmodi motus imprimi poteſt, vt primo recedat, mox autem iterum reuertatur, quod experimento oſtendi ſolet, dum digito ad globum circa D applicato et deorſum preſſo duplex motus globo imprimitur, alter progreſſiuus in directione DIE, alter gyratorius in ſenſum ZDTE. Sed vt phaenomenon ſuccedat, neceſſe eſt vt celeritas angularis prae progreſſiua certum quendam limitem excedat, quem quo facilius agnoſcamus, calculum ad iſtum caſum accommodemus quo motus gyratorius globo circa axem horizontalem et ad directionem motus progreſſiui normalem imprimitur. Quod ſi ergo e denotet celeritatem progreſſiuam ſecundum directionem DIE, et ε celeritatem angularem retrogyrantem in ſenſum ZDTE, exiſtente f radio globi et Maa eius momento inertiae, frictioneque $= \delta M$; primo globus in directione DIE procedet, et elapſo tempore t eius celeritas ſecundum eandem directionem erit $v = e - 2\delta g t$, confecto ſpatio

X =

$X = t(e - \delta g t)$: tum vero etiamnunc circa eundem axem retrouoluetur celeritate angulari $\vartheta = \varepsilon - \frac{2\delta f g t}{aa}$. Motus autem aequabilis euadit elapſo tempore $t = \frac{aa(e + \varepsilon f)}{2\delta g(aa + ff)}$, eritque tum celeritas progreſſiua $v = \frac{f(ef - \varepsilon aa)}{aa + ff}$ et angularis $\vartheta = \frac{\varepsilon aa - ef}{aa + ff}$. Quare ſi fuerit $\varepsilon > \frac{ef}{aa}$, globus nunc retro mouetur, gyratorio motu adhuc retro vergente: ſin autem fuerit $\varepsilon < \frac{ef}{aa}$, globus adhuc procedit, et gyratio in ſenſum contrarium eſt verſa. Illo caſu globus regredi coepit elapſo tempore $t = \frac{e}{2\delta g}$ et percurſo ſpatio $X = \frac{ee}{4\delta g}$.

Si globus ſit homogeneus, erit $aa = \frac{2}{5}ff$ et ef exprimit celeritatem gyrationis in puncto contactus, quae ſi vocetur $= b$, erit poſt tempus t celeritas progreſſiua $v = e - 2\delta g t$, et gyratoria in puncto contactus, quae ſit $u = b - 5\delta g t$, et ſpatium percurſum $= t(e - \delta g t)$: motus vero aequabilis euadet elapſo tempore $t = \frac{e + b}{7\delta g}$, et confecto ſpatio $= \frac{(6e - b)(e + b)}{49\delta g}$, vbi erit celeritas progreſſiua $v = \frac{5e - 2b}{7}$ et gyratoria $u = \frac{2b - 5e}{7}$. Vt ergo phoenomenon memoratum ſuccedat, debet eſſe initio $b > \frac{5}{2}e$. Sin autem eſſet $b = \frac{5}{2}e$, vterque motus ſimul extingueretur, elapſo ſcilicet tempore $\frac{e}{2\delta g}$ min. ſec. et confecto ſpatio $\frac{ee}{4\delta g}$.

Concluſiones.

Pro determinatione motus, quo globus quomodocunque impulſus ſuper plano horizontali progreditur.

I. *Status quaeſtionis.* Globus hic ita comparatus ſupponitur, vt non ſolum eius centrum grauitatis in ipſum

ſum figurae centrum incidat, ſed etiam omnia momenta inertiae reſpectu cuiusque diametri inter ſe ſint aequalia. Talis globi radius hic ponitur $= f$, eiusque maſſa ſeu pondus $=$ M et momentum inertiae reſpectu axis cuiuscunque per centrum grauitatis tranſeuntis $= Maa$, ita vt, ſi globus ex materia homogenea conſtet, futurum ſit $aa = \frac{2}{5}ff$. Praeterea vero tam ipſum planum horizontale quam tota globi ſuperficies ita aequaliter laeuigata aſſumitur, vt dum globus ſuper plano radendo ingreditur, vbique eandem frictionem patiatur, quae, cum preſſioni ſeu ipſi ponderi globi ſit proportionalis, hic ſtatuitur $= \delta$ M.

Tab. IV. Fig. 1. II. *Status initialis.* Ponamus globum initio in puncto D plano inſiſtere eique motum progreſſiuum ſecundum directionem D O eſſe impreſſum cum ea celeritate, vt globus vno minuto ſecundo ſpatium $= e$ eſſet percurſurus, quae celeritas non tam puncto contactus D quam centro globi impreſſa eſt intelligenda. Tum vero referat circulus A B C D ſectionem verticalem globi ſecundum directionem D O factam, qui ſimul hemiſphaerium globi conuexum nobis obuerſum referat, in quo ſit E polus, circa quem globo motus gyratorius initio fuit impreſſus, cuius celeritas angularis in ſenſum A B C D vergens ſit $= \varepsilon$, ita vt ε deſignet angulum vno minuto ſecundo abſoluendum. Pro ſitu autem huius puncti E ſit B punctum globi ſummum, puncto contactus D diametraliter oppoſitum, vnde per polum E agatur circulus maximus B E et vocetur arcus $BE = \mathfrak{f}$ et angulus $ABE = \mathfrak{h}$; quibus ergo poſitis tota vis viua globo initio impreſſa erit $= M(ee + \varepsilon\varepsilon aa)$. Poſtquam igitur globo talis duplex motus fuerit impreſſus, quaeritur quomodo is deinceps ſit pro-

progreſſurus; ac primo quidem ſtatim duos caſus notaſſe iuuabit, quibus globus eundem motum impreſſum perpetuo eſſet conſeruaturus: Alter ſcilicet caſus tum locum habebit, quando celeritas progreſſiua e fuerit nulla, ſimulque globus circa axem verticalem BD gyretur, ita vt hoc caſu fuerit $BE = \mathfrak{f} = 0$; quia enim tum nulla adeſt frictio, globus perpetuo in eodem loco gyrari perget. Alter vero caſus tum locum habet, quando axis gyrationis fuerit horizontalis ideoque angulus $\mathfrak{h} = 90^\circ$, ſimul vero inſuper $e = \varepsilon f \sin.\mathfrak{f}$, quandoquidem hoc caſu frictio pariter ceſſat. Reliquis autem caſibus omnibus globus ab initio per aliquod tempus motu inaequabili feretur, dum tam motus progreſſiuus quam gyratorius continuo variabitur, hocque temporis interuallum repertum eſt

$$= \frac{aa\sqrt{(ee - 2\varepsilon ef \sin.\mathfrak{f} \sin.\mathfrak{h} + \varepsilon\varepsilon ff \sin.\mathfrak{f}^2)}}{2\delta g (aa + ff)},$$

in minutis ſecundis expreſſum, ſiquidem g denotet altitudinem, per quam grauia vno minuto ſecundo delabuntur. Vnde vniuerſus globi motus ſponte in duas partes diſtinguitur, quarum priore motus erit inaequabilis, poſteriore vero aequabilis.

III. *Determinatio partis prioris.* Elapſum nunc ſit ab initio tempus quodcunque indefinitum $= t$ in minutis ſecundis expreſſum, quod autem minus ſit quam limes modo aſſignatus-

$$\frac{aa\sqrt{(ee - 2\varepsilon ef \sin.\mathfrak{f} \sin.\mathfrak{h} + \varepsilon\varepsilon ff \sin.\mathfrak{f}^2)}}{2\delta g (aa + ff)},$$

hocque tempore tangat globus planum horizontale in puncto T, ex quo ad rectam fixam DO ducatur normalis TX vocenturque coordinatae $DX = X$, $XT = Y$, pro linea curua DT, per quam punctum contactus hucusque proceſſit,

fit, ita vt centrum grauitatis globi fimilem viam defcripfiffe fit cenfendum. Tum vero ponatur angulus quo elementum T t ad directionem D O inclinatur $= \Phi$, vt fit tang. $\Phi = \frac{dY}{dX}$, ipfa autem celeritas, qua centrum globi hoc momento fecundum T t ingreditur, vocetur tantisper $= v$, eritque $\frac{dX}{dt} = v$ cof. Φ et $\frac{dY}{dt} = v$ fin. Φ. Hos autem valores demum ex motu gyratorio, qui nunc globo conuenit, determinari oportet, quos mox exhibebimus, poftquam fcilicet motum gyratorium fuerimus contemplati. Hunc in finem fecetur iterum globus plano in T infiftens, plano verticali MZNT, illi quod in ftatu initiali confiderauimus parallelo, ita vt circulus MZNT hemifphaerium nunc
Tab IV. Fig. 5. nobis obuerfum repraefentet, in quo punctum O fit polus, circa quem globus nunc gyratur in fenfum MZNT celeritate angulari $= 8$. His pofitis ifta motus determinatio ita fuccincte proponi poterit: Ex elementis ad ftatum initialem pertinentibus colligatur angulus ζ, vt fit

$$\text{tang.}\,\zeta = \frac{\varepsilon f \,\text{fin.}\, \mathfrak{f} \,\text{cof.}\, \mathfrak{h}}{e - \varepsilon f \,\text{fin.}\, \mathfrak{f} \,\text{fin.}\, \mathfrak{h}},$$

Tab. IV. Fig. 4. ex eoque ftatim pro motu progreffiou oritur

$$DX = X = ct + \delta g t t \,\text{cof.}\, \zeta,$$
$$TX = Y = \delta g t t \,\text{fin.}\, \zeta,$$

vnde colligitur celeritas fecundum DX $= \frac{dX}{dt} = e + 2\delta g t \,\text{cof.}\, \zeta$ et celeritas fecundum XT, fiue $\frac{dY}{dt} = 2\delta g t \,\text{fin.}\, \zeta$, hincque fit tang. $\Phi = \frac{2\delta g t \,\text{fin.}\, \zeta}{e + 2\delta g t \,\text{fin.}\, \zeta}$, et ipfa celeritas progreffiua

$$v = \sqrt{(ee + 4\delta e g t \,\text{cof.}\, \zeta + 4\delta\delta g g t t)}.$$

Pro motu autem gyratorio circa polum O quaeratur angulus η, vt fit

$$\text{tang.}\,\eta = \text{tang.}\,(\zeta - \mathfrak{h}) + \frac{2\delta f f g t}{e a a \,\text{fin.}\, \zeta},$$

hinc-

hincque porro quantitas $q = \frac{e \sin. \zeta}{f \cos. \eta \xi}$, eritque pro distantia huius poli O a puncto globi summo Z, tang. $ZO = \frac{q}{\varepsilon \cos. f}$, ipsa vero celeritas angularis $\vartheta = \sqrt{(qq + \varepsilon\varepsilon \cos. f^2)}$, denique erit angulus $MZO = \zeta - \eta$. Sicque omnia quae ad motus determinationem requiruntur sunt definita.

IV. *Determinatio partis posterioris.* Iam notauimus motum aequabilem incipere elapso tempore

$$t = \frac{aa\sqrt{(ee - 2e\varepsilon f \sin. f \sin. \mathfrak{h} + \varepsilon\varepsilon ff \sin. f^2)}}{2\delta g (aa + ff)}.$$

Quod si ergo hic valor loco t substituatur, coordinatae X et Y dabunt punctum in curua DT, quod sit K, vbi motus aequabilis incipiet. Introducto autem angulo ζ erit tempus illud $t = -\frac{aa\varepsilon f \sin. f \cos. \mathfrak{h}}{2\delta g (aa + ff) \sin. \zeta}$, vbi notetur $\sin. \zeta$ esse negatiuum. Cum igitur corpus vsque ad punctum K peruenerit, erit eius celeritas progressiua in directione DT Tab IV. Fig. 4.

$$= e - \frac{aa\varepsilon f \sin. f \cos. \mathfrak{h} \cos. \zeta}{(aa + ff) \sin. \zeta} = \frac{eff - aa\varepsilon f \sin. f \sin. \mathfrak{h}}{aa + ff},$$

et celeritas in directione LK

$$= -\frac{aa\varepsilon f \sin. f - e \cos. \mathfrak{h}}{aa + ff}.$$

Pro motu autem gyratorio deinceps sequente habebimus primo

$$\text{tang. } \eta = \text{tang. } (\zeta - \mathfrak{h}) - \frac{f^3 \varepsilon \sin. f \cos. \mathfrak{h}}{e (aa + ff) \sin. \zeta^2},$$

$$\text{tang. } \eta = -\frac{e (aa + ff) - aak}{e (aa + ff) \text{ tang. } \zeta},$$

vnde innotescit pro nostro tempore angulus $MZO = \zeta - \eta$. Porro vero pro eodem tempore, vbi contactus fit in puncto K, celeritas centri inuenta est

$$v = \sqrt{\left(ee + \frac{2aaek \cos. \zeta}{aa + ff} + \frac{a^4 kk}{(aa + ff)^2}\right)},$$

inclinatio autem directionis motus in K ad rectam fixam

DO, quae in genere erat Φ, nunc fiet

$$\text{tang.}\, \Phi = \frac{a a k \sin.\zeta}{e(aa + ff) + aak \cos.\zeta},$$

vnde cognoscitur motus progressiuus, quo globus post hoc tempus vniformiter progredietur. Pro polo autem gyrationis O iam vidimus esse angulum $MZO = \zeta - \eta$; praeterea vero inuenimus $\text{tang.}\, ZO = \frac{v}{\varepsilon f \cos.\mathfrak{f}}$, et ipsam celeritatem gyratoriam:

$$\text{ȣ} = \frac{\surd(eeff + 2e\varepsilon aaf \sin.\mathfrak{f} \sin.\mathfrak{h} + \varepsilon\varepsilon a^4 \sin.\mathfrak{f}^2 + \varepsilon\varepsilon(aa+ff)^2 \cos.\mathfrak{f}^2)}{aa + ff}$$

hunc ergo motum gyratorium globus posthac perpetuo conseruabit. Cum autem globus ad hanc vniformitatem peruenerit, erit eius vis viua

$$= \frac{M(eeff + 2\varepsilon eaaf \sin.\mathfrak{f} \sin.\mathfrak{h} + \varepsilon\varepsilon aa(aa+ff)\cos.\mathfrak{f}^2)}{aa+ff},$$

quae deficit ab initiali quantitate

$$\frac{Maa(ee - 2\varepsilon ef \sin.\mathfrak{f} \sin.\mathfrak{h} + \varepsilon\varepsilon ff \sin.\mathfrak{f}^2)}{aa+ff} = \frac{Maakk}{aa+ff},$$

vbi breuitatis gratia posuimus

$$kk = ee - 2\varepsilon ef \sin.\mathfrak{f} \sin \mathfrak{h} + \varepsilon\varepsilon ff \sin.\mathfrak{f}^2.$$

Pro hoc motu vniformi notasse iuuabit fore angulum $MZO = \zeta - \eta = 90° + \Phi$, tum vero $v = f\,\text{ȣ} \sin.s$, quibus ergo formulis conditio motus aequabilis continetur.

Additamentum.

Praeterea casus hic imprimis notatu dignus videtur quo globo initio nullus plane motus progressiuus fuit impressus, ita vt sit $e = 0$; tum enim erit $\text{tang.}\, \zeta = -\cot.\mathfrak{h}$ ideoque $\zeta = 90° + \mathfrak{h}$ et $k = \varepsilon f \sin.\mathfrak{f}$: tum vero pro via descripta erit $X = \delta g t t \cos.\zeta$ et $Y = \delta g t t \sin.\zeta$; vnde patet, hanc viam esse lineam rectam ad axem DO sub angulo

gulo $=\zeta$ inclinatam. Praeterea vero erit

$$v \operatorname{cof.} \Phi = 2\delta g t \operatorname{cof.} \zeta \text{ et } v \operatorname{fin.} \Phi = 2\delta g t \operatorname{fin.} \zeta,$$

vnde colligitur $\operatorname{tang.} \Phi = \operatorname{tang.} \zeta$ ideoque $\Phi = \zeta = 90° + \mathfrak{h}$, tum vero ipfa celeritas $v = 2\delta g t$. Deinde pro motu gyratorio erit $\operatorname{tang.} \eta = \operatorname{tang} (\zeta - \mathfrak{h}) + \frac{2\delta f f g t}{\varepsilon a a \operatorname{fin.} \zeta}$, vbi quia $\zeta - \mathfrak{h} = 90°$, erit $\operatorname{tang.} \eta = \infty$, ideoque $\eta = 90°$, confequenter angulus $MZO = \zeta - \eta = \mathfrak{h}$; vnde patet, polum gyrationis O perpetuo in eodem circulo verticali manere. Cum igitur fit

$$\vartheta \operatorname{cof.} s = \varepsilon \operatorname{cof.} \mathfrak{f} \text{ et } \vartheta \operatorname{fin.} s = \frac{2\delta g t}{\varepsilon},$$

colligitur

$$\operatorname{tang.} s = \frac{2\delta g t}{\varepsilon f \operatorname{cof.} \mathfrak{f}} \text{ et } \vartheta = \sqrt{\left(\varepsilon \varepsilon \operatorname{cof.} \mathfrak{f} + \frac{4\delta\delta g g f f t t}{a^4}\right)};$$

hincque motus inaequabilis durabit per temporis fpatium $t = \frac{\varepsilon a a f \operatorname{fin.} \mathfrak{f}}{2\delta g (a a + f f)}$, quo elapfo erit $v = \frac{\varepsilon a a f \operatorname{fin.} \mathfrak{f}}{a a + f f}$, manente $\Phi = \zeta = 90° + \mathfrak{h}$. Porro etiamnunc erit angulus $MZO = \mathfrak{h}$, at $\operatorname{tang.} s = \frac{a a}{a a + f f} \operatorname{tang.} \mathfrak{f}$ et

$$\vartheta = \frac{\varepsilon \sqrt{(a^4 \operatorname{fin.} \mathfrak{f}^2 + (a a + f f)^2 \operatorname{cof.} \mathfrak{f}^2)}}{a a + f f}.$$