

DE INNVMERIS
GENERIBVS SERIERVM
MAXIME MEMORABILIVM, QVIBVS OMNIVM
AEQVATIONVM ALGEBRAICARVM
NON SOLVM RADICES IPSAE SED ETIAM
QVAECVNQVE EARVM POTESTATES
EXPRIMI POSSVNT.

Auctore

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Conuent. exhib. d. 11 April. 1776.

§. I.

Proposita aequatione quacunque algebraica ad hanc formam
reducta :

$$A x^a + B x^b + C x^c + D x^d + E x^e + \text{etc.} = 0,$$

ubi exponentes $a, b, c, d, e, \text{etc.}$ sint numeri quicunque, siue integri, siue fracti, siue positiui, siue etiam negatiui, in Tomo XV. Nouorum Commentariorum ostendi, non solum huius aequationis radicem x , sed etiam quamuis potestatem x^n , per seriem infinitam satis concinnam, cuius omnes termini secundum certam legem formentur, pluribus modis exprimi posse, quae eo magis omni attentione, atque adeo admiratione dignae videntur, quod earum ratio propemodum omnes vires Analyseos superare videtur,

tur, propterea quod istae series non solum per longas ambages sint inuentae, sed etiam in earum formatione plurimum inductioni sit tributum. Quamobrem plurimum operae pretium fore videtur, vt omnes operationes, quibus hae series formantur, dilucide exponantur et omnia momenta, quibus earum formatio innititur, summo studio perpendantur. Tum enim deum sperare licebit in veram originem istarum serierum penetrare atque omnia mysteria, quibus earum natura adhuc inuoluta deprehenditur, ad principia cognita reuocare.

§. 2. Prima operatio, qua aequationem quamcunque propositam ad hunc scopum praeparare oportet, in hoc consistit, vt tota aequatio per quempiam suorum terminorum diuidatur, hocque modo ipsa aequatio ad huiusmodi formam reducatur :

$$1 = \frac{A}{x^\alpha} + \frac{B}{x^\beta} + \frac{C}{x^\gamma} + \frac{D}{x^\delta} + \frac{E}{x^\epsilon} + \text{etc.}$$

quod ergo totidem diuersis modis fieri poterit, quot terminis ipsa aequatio fuerit composita; vbi iterum est monendum, exponentes $\alpha, \beta, \gamma, \delta, \epsilon$, etc. quoscunque numeros denotare posse, siue positiuos, siue negatiuos, siue integros, siue fractos. Quin etiam perinde est quonam ordine isti termini disponantur. Tantum hic in limine notasse iuuabit, quoniam ipsi radici x vna dimensio tribuitur, propter homogeneitatem omnium terminorum quantitibus A, B, C respectiue tot dimensiones assignari oportere, quot exponentes α, β, γ indicant. Ita littera A censenda est habere α dimensiones; numerus vero dimensionum litterae B est β , at litterae $C = \gamma$, etc., id quod ideo probe est obseruandum, quod in serie, quae valorem radicis x referat, omnes eius termini vnicam dimensionem constituere debent. In seriebus autem, quae radicis x potestatem

tatem quamcunque x^n exprimunt, evidens est in singulis terminis litteras A, B, C, D, etc. n dimensiones complere debere.

§. 3. His expositis pro quacunque radice potestate x^n tot diuersae series infinitae exhiberi possunt, quot litterae A, B, C, D, etc. in aequatione reperiuntur, propterea quod primus seriei cuiusque terminus ex quouis membro illius aequationis formari potest, quasi reliqui non adessent. Ita ex membro $x^\alpha = 1$ sequitur $x = A^{\frac{1}{\alpha}}$ et $x^n = A^{\frac{n}{\alpha}}$; ex membro autem secundo $\frac{B}{x^\beta} = 1$ prodiret $x^n = B^{\frac{n}{\beta}}$; similique modo ex tertio membro prodiret $x^n = C^\gamma$; et ita porro. Vocemus igitur illud aequationis membrum, ex quo primum seriei terminum constituere libet, aequationis membrum principale, pro quo in posterum assumamus perpetuo membrum primo loco positum $\frac{A}{x^\alpha}$, ita ut initium seriei inuestigandae sit $x^n = A^{\frac{n}{\alpha}}$.

§. 4. Constituto igitur primo termino seriei, quae valorem ipsius x^n exprimat, qui ergo est $x^n = A^{\frac{n}{\alpha}}$, manifestum est omnes terminos sequentes ex reliquis litteris B, C, D, E, etc. tam singulis quam utcunque inuicem combinatis, formari debere, unde pro reliquis terminis infinitos ordines constitui conueniet, quorum primus contineat singulas litteras B, C, D, etc. solitarias, secundus ordo omnia producta ex binis harum litterarum, quae ergo sunt primo earum quadrata B^2, C^2, D^2 tum vero producta ex binis diuersis BC, BD, CD, etc. tertius vero ordo complectetur omnia producta ex tribus harum littera-

litterarum, siue iisdem, siue diuersis, quae ergo erunt B^3 , C^3 , D^3 ; $BB C$, $BB D$, $B C C$, $B D D$, $B C D$, etc. quartus vero ordo continebit, praeter potestates quartas harum litterarum, omnia producta ex quaternis earum inter se coniunctis. Ex quo intelligitur ordinem quintum inuoluere omnia producta ex quinis, ordinem sextum ex senis, et ita porro in infinitum. Hinc totum negotium huc redit, quomodo omnes terminos cuiusque ordinis formari oporteat, vt omnibus in vnam summam collectis et ad primum $A^{\frac{n}{\alpha}}$ additis, prodeat series infinita verum valorem potestatis x^n exprimens, quamobrem pro quolibet ordine regulas singulos terminos formandi exponamus, quemadmodum in dissertatione initio commemorata sunt traditae.

PRIMA REGVLA.

Pro formandis terminis primi ordinis.

§. 5. Hic ante omnia patet cum qualibet litterarum B , C , D , E , etc. eiusmodi potestatem ipsius A coniungi debere, vt dimensionum numerus prodeat $= n$. Quoniam igitur littera B censetur habere β dimensiones, litterae vero A numerus dimensionum est α , si pro B statuatur ista potestas A^λ , tum producti $A^\lambda B$ numerus dimensionum erit $\lambda\alpha + \beta = n$, vnde colligitur: $\lambda = \frac{n-\beta}{\alpha}$, sicque ex littera B nascitur $A^{\frac{n-\beta}{\alpha}} B$. Similique modo ex littera C oriatur $A^{\frac{n-\gamma}{\alpha}} C$. Tum vero ex littera D oriatur $A^{\frac{n-\delta}{\alpha}} D$, et ita porro. Praeterea vero his singulis productis praefigi oportet communem multiplicatorem $\frac{n}{\alpha}$; quamobrem termini seriei quaesitae ex singulis litteris B , C , D , E oriundi ita se habebunt:

K 3

$\frac{n}{\alpha} A$

$$\frac{n}{\alpha} A^{\frac{n-\beta}{\alpha}} B + \frac{n}{\alpha} A^{\frac{n-\gamma}{\alpha}} C + \frac{n}{\alpha} A^{\frac{n-\delta}{\alpha}} D + \frac{n}{\alpha} A^{\frac{n-\epsilon}{\alpha}} E + \text{etc.}$$

sicque habentur omnes termini primi ordinis pro serie quam inuestigamus.

SECUNDA REGULA.

Pro formandis terminis secundi ordinis.

§. 6. Quoniam hic occurrunt termini duplicis formae, scilicet vel ipsa quadrata B^2, C^2, D^2 , vel producta ex binis diversis BC, BD, BE, CD , etc. multiplicatores cuique formae iungendos seorsim euoluamus. Ac primo quidem pro forma B^2 , quae habet 2β dimensiones, iungi debet haec potestas $A^{\frac{n-2\beta}{\alpha}}$; deinde vero ex iis, quae in dissertatione iam allegata demonstrari, patet insuper adiungi debere multiplicatorem ex duobus factoribus constantem $\frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta}{2\alpha}$, unde termini seriei quaesitae, qui ex quadratis B^2, C^2, D^2 oriuntur, erunt sequentes:

Ex forma	nascitur terminus	
BB	$\frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta}{2\alpha} A^{\frac{n-2\beta}{\alpha}}$	BB
CC	$\frac{n}{\alpha} \cdot \frac{n+\alpha-2\gamma}{2\alpha} A^{\frac{n-2\gamma}{\alpha}}$	CC
DD	$\frac{n}{\alpha} \cdot \frac{n+\alpha-2\delta}{2\alpha} A^{\frac{n-2\delta}{\alpha}}$	DD
EE	$\frac{n}{\alpha} \cdot \frac{n+\alpha-2\epsilon}{2\alpha} A^{\frac{n-2\epsilon}{\alpha}}$	EE
etc.	etc.	

§. 7. Pro formis autem binas litteras diuersas inuolventibus sufficet considerasse formam BC , quae primo recipit coëfficientem numericum numero permutationum harum litterarum respondentem, qui est 2, vt hinc habeatur $2BC$. Deinde quia numerus dimensionum est $\beta + \gamma$, potestas ipsius A iun-

gen-

genda erit $A \frac{n-\beta-\gamma}{\alpha}$; ac denique multiplicator insuper adii-
ciendus erit $\frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-\gamma}{2\alpha}$. His igitur coniunctis ex singulis
nostris formis huius ordinis orientur sequentes termini pro
ferie quaesita:

Ex forma	nascitur terminus
B C	$2 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-\gamma}{2\alpha} A \frac{n-\beta-\gamma}{\alpha} B C$
B D	$2 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-\delta}{2\alpha} A \frac{n-\beta-\delta}{\alpha} B D$
B E	$2 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-\varepsilon}{2\alpha} A \frac{n-\beta-\varepsilon}{\alpha} B E$
C D	$2 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\gamma-\delta}{2\alpha} A \frac{n-\gamma-\delta}{\alpha} C D$
C E	$2 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\gamma-\varepsilon}{2\alpha} A \frac{n-\gamma-\varepsilon}{\alpha} C E$
D E	$2 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\delta-\varepsilon}{2\alpha} A \frac{n-\delta-\varepsilon}{\alpha} D E$
etc.	etc.

TERTIA REGVLA.

Pro formandis terminis tertii ordinis.

§. 8. In hoc ordine primo occurrit B^3 , cuius index na-
turalis ex permutatione ortus est = 1; deinde potestas ipsius
A, quia dimensionum numerus ipsius B^3 est 3^3 , erit $A \frac{n-3\beta}{\alpha}$.
At vero multiplicator insuper adiungendus nunc constabit ex
tribus factoribus, eritque $\frac{n}{\alpha} \cdot \frac{n+\alpha-3\beta}{2\alpha} \cdot \frac{n+2\alpha-3\beta}{3\alpha}$, quibus con-
iunctis termini ex singulis cubis oriundi se habebunt sequenti
modo:

Ex

Ex forma	nascitur terminus	
B^3	$\frac{n}{\alpha} \cdot \frac{n+\alpha-3\beta}{2\alpha} \cdot \frac{n+2\alpha-3\beta}{3\alpha} A$	$\frac{n-3\beta}{\alpha} B^3$
C^3	$\frac{n}{\alpha} \cdot \frac{n+\alpha-3\gamma}{2\alpha} \cdot \frac{n+2\alpha-3\gamma}{3\alpha} A$	$\frac{n-3\gamma}{\alpha} C^3$
D^3	$\frac{n}{\alpha} \cdot \frac{n+\alpha-3\delta}{2\alpha} \cdot \frac{n+2\alpha-3\delta}{3\alpha} A$	$\frac{n-3\delta}{\alpha} D^3$
etc.	etc.	

§. 9. Secunda forma in hoc ordine occurrens est BBC, cui conuenit index naturalis = 3, et quia numerus dimensionum est $2\beta + \gamma$, potestas ipsius A iungenda erit $A^{\frac{n-2\beta-\gamma}{\alpha}}$; tum vero multiplicator insuper adiciendus erit

$$\frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta-\gamma}{2\alpha} \cdot \frac{n+2\alpha-2\beta-\gamma}{3\alpha}$$

vnde pro singulis formis formabuntur sequentes termini:

Ex forma	nascitur terminus	
BBC	$3 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta-\gamma}{2\alpha} \cdot \frac{n+2\alpha-2\beta-\gamma}{3\alpha} A$	$\frac{n-2\beta-\gamma}{\alpha} BBC$
BCC	$3 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-2\gamma}{2\alpha} \cdot \frac{n+2\alpha-\beta-2\gamma}{3\alpha} A$	$\frac{n-\beta-2\gamma}{\alpha} BCC$
BBD	$3 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta-\delta}{2\alpha} \cdot \frac{n+2\alpha-2\beta-\delta}{3\alpha} A$	$\frac{n-2\beta-\delta}{\alpha} BBD$
BDD	$3 \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-2\delta}{2\alpha} \cdot \frac{n+2\alpha-\beta-2\delta}{3\alpha} A$	$\frac{n-\beta-2\delta}{\alpha} BDD$
etc.	etc.	

In hoc ordine superest terminus BCD ternas litteras dispares inuoluens, cuius index numericus ex permutatione natus est 6; et quia numerus dimensionum hic est $\beta + \gamma + \delta$, potestas ipsius A iungenda erit $A^{\frac{n-\beta-\gamma-\delta}{\alpha}}$; at vero multiplicator insuper iungendus erit

$$\frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-\gamma-\delta}{2\alpha} \cdot \frac{n+2\alpha-\beta-\gamma-\delta}{3\alpha}$$

vnde

vnde ex forma B C D nascitur iste terminus pro serie quaesita:

$$6. \frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-\gamma-\delta}{2\alpha} \cdot \frac{n+2\alpha-\beta-\gamma-\delta}{3\alpha} A^{\frac{n-\beta-\gamma-\delta}{\alpha}} B C D;$$

vnde facile patet quomodo termini ex reliquis formis huius naturae B C E, C D E, etc. sint formandi, quos superfluum foret hic seorsim apponere, quandoquidem omnes has litteras inter se permutare licet, quamobrem etiam in sequentibus univiam formam ex quolibet ordine evoluisse sufficiet.

QVARTA REGVLA.

Pro formandis terminis quarti ordinis.

§. 10. Ex hactenus allatis facile intelligitur, quomodo pro singulis formis, quae in hoc ordine occurrunt, termini inde resultantis formari debeant.

I. Ex forma B⁴ nascitur iste terminus:

$$1. \frac{n}{\alpha} \cdot \frac{n+\alpha-4\beta}{2\alpha} \cdot \frac{n+2\alpha-4\beta}{3\alpha} \cdot \frac{n+3\alpha-4\beta}{4\alpha} A^{\frac{n-4\beta}{\alpha}} B^4.$$

II. Ex forma B³ C nascitur terminus

$$4. \frac{n}{\alpha} \cdot \frac{n+\alpha-3\beta-\gamma}{2\alpha} \cdot \frac{n+2\alpha-3\beta-\gamma}{3\alpha} \cdot \frac{n+3\alpha-3\beta-\gamma}{4\alpha} A^{\frac{n-3\beta-\gamma}{\alpha}} B^3 C.$$

III. Ex forma B² C² nascitur terminus

$$6. \frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta-2\gamma}{2\alpha} \cdot \frac{n+2\alpha-2\beta-2\gamma}{3\alpha} \cdot \frac{n+3\alpha-2\beta-2\gamma}{4\alpha} A^{\frac{n-2\beta-2\gamma}{\alpha}} B^2 C^2.$$

IV. Ex forma B² C D nascitur terminus

$$12. \frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta-\gamma-\delta}{2\alpha} \cdot \frac{n+2\alpha-2\beta-\gamma-\delta}{3\alpha} \cdot \frac{n+3\alpha-2\beta-\gamma-\delta}{4\alpha} A^{\frac{n-2\beta-\gamma-\delta}{\alpha}} B^2 C D.$$

V. Ex forma B C D E nascitur terminus

$$24. \frac{n}{\alpha} \cdot \frac{n+\alpha-\beta-\gamma-\delta-\epsilon}{2\alpha} \cdot \frac{n+2\alpha-\beta-\gamma-\delta-\epsilon}{3\alpha} \cdot \frac{n+3\alpha-\beta-\gamma-\delta-\epsilon}{4\alpha} A^{\frac{n-\beta-\gamma-\delta-\epsilon}{\alpha}} B C D E.$$

QVINTA REGVLA.

Pro formandis terminis quinti ordinis.

§. 11. In hoc ordine sequentes occurrunt casus feor-
sim evoluendi:

I. Ex forma B^5 nascitur terminus

$$\frac{n}{\alpha} \cdot \frac{n+\alpha-5\beta}{2\alpha} \cdot \frac{n+2\alpha-5\beta}{3\alpha} \cdot \frac{n+3\alpha-5\beta}{4\alpha} \cdot \frac{n+4\alpha-5\beta}{5\alpha} A^{\frac{n-5\beta}{\alpha}} B^5.$$

II. Ex forma $B^4 C$, posito brev. gr. $4\beta + \gamma = \theta$, na-
scitur terminus

$$5. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \cdot \frac{n+4\alpha-\theta}{5\alpha} A^{\frac{n-\theta}{\alpha}} B^4 C.$$

III. Ex forma $B^3 C^2$, posito brev. gr. $3\beta + 2\gamma = \theta$,
nascitur terminus

$$10. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \cdot \frac{n+4\alpha-\theta}{5\alpha} A^{\frac{n-\theta}{\alpha}} B^3 C^2.$$

IV. Ex forma $B^3 C D$, posito brev. gr. $3\beta + \gamma + \delta = \theta$,
nascitur terminus

$$20. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \cdot \frac{n+4\alpha-\theta}{5\alpha} A^{\frac{n-\theta}{\alpha}} B^3 C D.$$

V. Ex forma $B^2 C^2 D$, posito brev. gr. $2\beta + 2\gamma + \delta = \theta$,
nascitur terminus

$$30. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \cdot \frac{n+4\alpha-\theta}{5\alpha} A^{\frac{n-\theta}{\alpha}} B^2 C^2 D.$$

VI. Ex forma $B^2 C D E$, posito brev. gr. $2\beta + \gamma + \delta + \varepsilon = \theta$,
nascitur

$$60. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \cdot \frac{n+4\alpha-\theta}{5\alpha} A^{\frac{n-\theta}{\alpha}} B^2 C D E.$$

VII. Ex forma $B C D E F$, posito brev. gr. $\beta + \gamma + \delta$
 $+ \varepsilon + \zeta = \theta$, nascitur

$$120. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \cdot \frac{n+4\alpha-\theta}{5\alpha} A^{\frac{n-\theta}{\alpha}} B C D E F.$$

Hinc

Hinc iam lex progressionis ita manifesto perspicitur, vt superfluum foret vltiores ordines euoluere.

REGVLA GENERALIS.

Pro terminis formae B^i inueniendis.

§. 12. Quoniam igitur hic terminus ad ordinem i pertinet, per se patet eius exponentem i necessario esse debere numerum integrum posituum, dum reliqui exponentes α , β , γ , δ omnes plane numeros tam positivos quam negativos, siue integros, siue fractos admittant. Deinde quia hic terminus est simplex potestas, eius index numericus erit $= 1$. Porro quia litterae A tribuuntur β dimensiones, hic numerus dimensionum erit $i\beta = \theta$, hincque potestas ipsius A iungenda erit $A^{\frac{n-\theta}{\alpha}}$. Denique multiplicator ab exponente n pendens continebit i factores, qui sunt

$$\frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha}.$$

His ergo coniunctis terminus ex forma B^i oriundus erit

$$1. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha} A^{\frac{n-\theta}{\alpha}} B^i$$

quae formula facile ad terminos formae vel C^i , vel D^i , vel E^i , vel etc. transfertur.

REGVLA GENERALIS.

Pro terminis formae $B^b C^c$ inueniendis.

§. 13. Hic vt ante sponte intelligitur exponentes b et c non nisi numeros integros positivos designare posse, vnde ille terminus pertinebit ad ordinem $b+c$. Ponamus autem $b+c = i$. Primo igitur huius termini index numericus defini debet, quem indicemus littera N , et ex Theoria Permuta-

tionum. constat fore $N = \frac{1 \cdot 2 \cdot 3 \dots i}{1 \cdot 2 \cdot 3 \dots b \times 1 \cdot 2 \cdot 3 \dots c}$. Deinde quia litterae B dimensiones β , litterae vero C dimensiones γ tribuuntur, numerus dimensionum erit $b\beta + c\gamma = \theta$, hincque potestas ipsius A adiungenda erit $A^{\frac{n-\theta}{a}}$. Tertio vero multiplicator ab exponente n pendens erit vt ante

$$\frac{n}{a} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha},$$

nisi quod hic ipse θ debitus valor est assignandus, quamobrem his coniunctis reperietur terminus ex forma proposita $B^b C^c$ oriundus iste:

$$N \cdot \frac{n}{a} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha} A^{\frac{n-\theta}{a}} B^b C^c.$$

REGVLA GENERALIS.

Pro terminis formae $B^b \cdot C^c \cdot D^d$ formandis.

§. 14. Ponatur hic index ordinis, ad quem haec forma est referenda, $b + c + d = i$; numerus vero dimensionum in hoc termino contentarum statuatur vt haecenus $b\beta + c\gamma + d\delta = \theta$, vnde tam potestas ipsius A quam multiplicator definientur vt ante; quare cum index numericus sit

$$N = \frac{1 \cdot 2 \cdot 3 \cdot 4 \dots i}{1 \cdot 2 \dots b \times 1 \cdot 2 \dots c \times 1 \cdot 2 \dots d}$$

his coniungendis terminus formae propositae respondens erit

$$N \cdot \frac{n}{a} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha} A^{\frac{n-\theta}{a}} B^b \cdot C^c \cdot D^d.$$

FORMATIO SERIEI

Valorem ipsius x^n exprimentis.

§. 15. Postquam secundum praecepta exposita omnes termini cuiusque ordinis in infinitum fuerint inuenti, nil aliud opus

opus est, nisi ut omnes isti termini in unam summam colligantur, quae ad primum terminum addita praebebit valorem potestatis x^n . Hinc ergo si summa omnium terminorum primi ordinis ponatur = \mathfrak{A} , secundi ordinis = \mathfrak{B} , tertii ordinis = \mathfrak{C} , etc. erit

$$x^n = A \frac{n}{\alpha} + \mathfrak{A} + \mathfrak{B} + \mathfrak{C} + \mathfrak{D} + \text{etc.},$$

hicque valor propterea satisfaciet aequationi propositae

$$1 = \frac{A}{x^\alpha} + \frac{B}{x^\beta} + \frac{C}{x^\gamma} + \frac{D}{x^\delta} + \text{etc.},$$

quae cum habere queat plures radices, seu valores ipsius x , hic in genere vix definire licet, quemnam valorem expressio inuenta sit exhibitura: plerumque quidem omnium radicum maxima quandoque etiam minima hic locum habere deprehenditur.

§. 16. Si praeter terminum principalem $\frac{A}{x^\alpha}$ aequatio unicum contineat terminum $\frac{B}{x^\beta}$, in omnibus ordinibus unicus tantum habebitur terminus, vnde pro hoc casu series inuenta se ita habebit:

$$\begin{aligned} x^n &= A \frac{n}{\alpha} + \frac{n}{\alpha} A \frac{n-\beta}{\alpha} B + \frac{n}{\alpha} \cdot \frac{n+\alpha-2\beta}{2\alpha} A \frac{n-2\beta}{\alpha} B B \\ &+ \frac{n}{\alpha} \cdot \frac{n+\alpha-3\beta}{2\alpha} \cdot \frac{n+2\alpha-3\beta}{3\alpha} A \frac{n-3\beta}{\alpha} B^3 \\ &+ \frac{n}{\alpha} \cdot \frac{n+\alpha-4\beta}{2\alpha} \cdot \frac{n+2\alpha-4\beta}{3\alpha} \cdot \frac{n+3\alpha-4\beta}{4\alpha} A \frac{n-4\beta}{\alpha} B^4 \\ &+ \frac{n}{\alpha} \cdot \frac{n+\alpha-5\beta}{2\alpha} \cdot \frac{n+2\alpha-5\beta}{3\alpha} \cdot \frac{n+3\alpha-5\beta}{4\alpha} \cdot \frac{n+4\alpha-5\beta}{5\alpha} A \frac{n-5\beta}{\alpha} B^5 \\ &\text{etc.} \end{aligned}$$

quae ergo series conuenit aequationi $1 = \frac{A}{x^\alpha} + \frac{B}{x^\beta}$, ita ut radix potestatis n ex hac serie extracta exhibeat quampiam radicem istius aequationis; et quoniam pro n numerum quemcumque, siue positium, siue negatiuum, siue integrum, siue fractum accipere licet, hinc innumerabiles diuersae formae pro eadem aequationis radice x exhiberi possunt.

§. 17. Sin autem aequatio praeter terminum principalem $\frac{A}{x^n}$ duos pluresque complectatur terminos, tum etiam in serie inuenta numerus terminorum cuiusque ordinis continuo magis augebitur. Scilicet si numerus litterarum B, C, D, etc. fuerit $= k$, qui ergo numerum terminorum in primo ordine contentorum indicabit, tum numerus terminorum secundi ordinis erit $= \frac{k(k+1)}{1 \cdot 2}$, tertii vero ordinis $= \frac{k(k+1)(k+2)}{1 \cdot 2 \cdot 3}$, quarti ordinis $= \frac{k(k+1)(k+2)(k+3)}{1 \cdot 2 \cdot 3 \cdot 4}$, atque adeo in genere numerus terminorum ordinis i erit $= \frac{k(k+1)(k+2) \dots (k+i-1)}{1 \cdot 2 \cdot 3 \dots i}$.

§. 18. Quoniam igitur in enumeratione terminorum cuiusque ordinis probe cauendum est, ne vlla forma praetermittatur, hoc obtinebitur si generatim pro ordine i enoluetur ista potestas: $(B + C + D + E + \text{etc.})^i$, tum enim non solum omnes formae ad hunc ordinem pertinentes prodibunt, sed etiam quaelibet forma iam adiunctum habebit suum indicem numericum, qui ipsi iuxta regulam permutationum conuenit, et quem supra designauimus littera N, ita ut non opus sit eum seorsim inuestigare.

PRAEPARATIO.

ad demonstrationem serierum inuentarum.

§. 19. Quanquam istas series iam in Tomo XV. Nouorum Commentariorum exhibui, tamen iam obseruavi, eas methode maxime indirecta ac per plures ambages esse inuentas, praeterquam quod plerumque etiam soli inductioni innituntur, quamobrem ob summam dignitatem harum serierum maxime est optandum, vt earum veritas per demonstrationem solidam extra omne dubium collocetur; inprimis autem desideratur methodus directa, qua ad easdem series, per operationes iam vsu receptas, peruenire liceat. Quoniam autem posteriori requisito nondum satisfacere valui, saltem veritatem harum serierum, quam firma demonstratione corroborare mihi licuit, hic sum expositurus.

§. 20. Docebo igitur series, quarum constructionem hic tradidi, apprime satisfacere ipsi aequationi propositae:

$$1 = \frac{A}{x^\alpha} + \frac{B}{x^\beta} + \frac{C}{x^\gamma} + \frac{D}{x^\delta} + \text{etc.},$$

quae per x^n multiplicata induit hanc formam:

$$x^n = A x^{n-\alpha} + B x^{n-\beta} + C x^{n-\gamma} + D x^{n-\delta} + \text{etc.}$$

quamobrem ostendi oportet seriem, quam supra pro exprimenda potestate x^n inuenire docui, reuera aequalem esse summae serierum, quae secundum eandem legem pro potestatibus $x^{n-\alpha}$, $x^{n-\beta}$, $x^{n-\gamma}$, etc. formantur, si scilicet respectiue per litteras A, B, C, D, etc. multiplicentur. Euidens autem est has posteriores series ex nostra serie generali erui posse, si loco exponentis n successiue scribantur numeri $n-\alpha$, $n-\beta$, $n-\gamma$, $n-\delta$, etc. Hoc autem sequenti modo pro qualibet terminorum forma ostendere licebit, vbi quidem sufficiet

ficiet veritatem ternarum postremarum regularum generalium demonstrasse.

DEMONSTRATIO

primae regulae generalis pro terminis formae B^i .

§. 21. Supra vidimus terminum ex hac forma oriundum pro potestate x^n esse

$$1. \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot \frac{n+3\alpha-\theta}{4\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha} A^{\frac{n-\theta}{\alpha}} \cdot B^i,$$

existente $\theta = i\beta$, qui ergo aequalis esse debet terminis eiusdem formae, qui ex membris aequationis primo et secundo $A x^{n-\alpha} + B x^{n-\beta}$ oriuntur, quandoquidem in reliquis membris nulli termini formae B^i locum inveniunt, propterea quod horum membrorum termini omnes vel litteram C, vel D, vel E necessario inuoluunt. Ex primi igitur membri potestate $x^{n-\alpha}$ sumi debet terminus formae B^i , quippe qui ductus in A naturam non mutat; at vero ex secundi membri potestate $x^{n-\beta}$, tantum capi debet terminus formae B^{i-1} , quippe qui per B multiplicatus praebet formam propositam B^i . Sicque hos binos terminos iunctos aequales fieri oportet ipsi termino proposito.

§. 22. Quaeramus igitur primo terminum formae B^i , in potestate $x^{n-\alpha}$ occurrentem, quem ergo ex ipsa formula proposita deducemus, loco n scribendo $n-\alpha$, qui idcirco erit

$$\frac{n-\alpha}{\alpha} \cdot \frac{n-\theta}{2\alpha} \cdot \frac{n+\alpha-\theta}{3\alpha} \cdot \frac{n+2\alpha-\theta}{4\alpha} \dots \frac{n+(i-2)\alpha-\theta}{i\alpha} A^{\frac{n-\alpha-\theta}{\alpha}} B^i,$$

existente $\theta = i\beta$. Deinde vero terminus formae B^{i-1} potestati $x^{n-\beta}$ ex ipsa formula proposita eruetur, si primo loco i scribatur $i-1$, porro loco θ nunc scribi debet $\theta - \beta$, tertio

tio vero loco n scribendum est $n - \beta$, quo facto terminus resultans erit

$$\frac{n-\beta}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-2)\alpha-\theta}{(i-1)\alpha} A^{\frac{n-\theta}{\alpha}} B^{i-1},$$

quamobrem nunc priorem terminum ducamus in A, posteriorem vero in B, et quia ambo hi termini communem habebunt factorem

$$\frac{n+\alpha-\theta \cdot n+2\alpha-\theta \cdot n+3\alpha-\theta \dots n+(i-2)\alpha-\theta}{\alpha \cdot 2\alpha \cdot 3\alpha \cdot 4\alpha \dots (i-1)\alpha} A^{\frac{n-\theta}{\alpha}} B^i = \Delta,$$

eorum summa erit

$$\Delta \left(\frac{(n-\alpha) \cdot (n-\theta)}{i\alpha} + n - \beta \right) = \frac{n [n + (i-1)\alpha - \theta]}{i\alpha} \cdot \Delta,$$

ob $i\beta = \theta$. Quamobrem restituto pro Δ valore assumpto et factoribus in debitum ordinem dispositis summa istorum terminorum erit:

$$\frac{n(n+\alpha-\theta)(n+2\alpha-\theta)(n+3\alpha-\theta) \dots [n+(i-2)\alpha-\theta][n+(i-1)\alpha-\theta]}{\alpha \cdot 2\alpha \cdot 3\alpha \cdot 4\alpha \dots (i-1)\alpha \cdot i\alpha} A^{\frac{n-\theta}{\alpha}} \cdot B^i,$$

qui est ipse terminus potestati x^n respondens, ideoque huius regulae veritas est demonstrata.

DEMONSTRATIO

secundae regulae generalis, pro terminis formae $B^b C^c$.

§. 23. Postquam hic posuimus $b+c=i$ et $b\beta+c\gamma=\theta$, tum vero $N = \frac{1 \cdot 2 \cdot 3 \dots i}{1 \cdot 2 \dots b \times 1 \cdot 2 \dots c}$, ipse terminus huius formae in potestatem x^n ingrediens inuentus est

$$N \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots + \frac{n+(i-1)\alpha-\theta}{i\alpha} \cdot A^{\frac{n-\theta}{\alpha}} B^b C^c.$$

Haec igitur expressio aggregatum esse demonstrandum est 1°. ex termino formae $B^b C^c$, potestati $x^{n-\alpha}$ respondente, ducto in A; 2°. ex termino formae $B^{b-1} C^c$, potestati $x^{n-\beta}$ respondente,

ducto in B; 3°. ex termino formae $B^b C^{c-1}$, potestati $x^{n-\gamma}$ respondente, ducto in C. Evidens enim est in reliquis membris, quae per D, E, F, etc. multiplicata sunt, formam propositam occurrere non posse, quandoquidem aequatio, cui satisfieri oportet, est haec:

$$x^n = A x^{n-\alpha} + B x^{n-\beta} + C x^{n-\gamma} + D x^{n-\delta} + \text{etc.}$$

§. 24. Pro inveniendis autem terminis formae $B^b C^c$, in potestate $x^{n-\alpha}$ occurrente, perspicuum est in ipsa formula proposita tantum loco n scribi debere $n-\alpha$, litteras autem i , θ et N eosdem valores retinere, unde iste terminus, potestati $x^{n-\alpha}$ conueniens, erit

$$\text{I. } N \cdot \frac{n-\alpha}{\alpha} \cdot \frac{n-\theta}{2\alpha} \cdot \frac{n+\alpha-\theta}{3\alpha} \cdot \frac{n+2\alpha-\theta}{4\alpha} \dots \frac{n+(i-2)\alpha-\theta}{i\alpha} A^{\frac{n-\alpha-\theta}{\alpha}} B^b C^c.$$

Deinde terminus formae $B^{b-1} C^c$, in potestate $x^{n-\beta}$ occurrens, deriuabitur ex ipsa formula proposita, si primo loco n scribatur $n-\beta$; tum vero loco i hic scribi debet $b-1+c=i-1$; deinde loco θ scribi debet $(b-1)\beta + c\gamma = \theta - \beta$; denique pro indice N scribendum erit $\frac{1 \cdot 2 \cdot 3 \dots (i-1)}{1 \cdot 2 \dots b \times 1 \cdot 2 \dots c}$; sicque loco N scribi oportebit $N \cdot \frac{b}{i}$, unde ipse iste terminus erit

$$\text{II. } \frac{b}{i} N \cdot \frac{n-\beta}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-2)\alpha-\theta}{(i-1)\alpha} A^{\frac{n-\theta}{\alpha}} B^{b-1} C^c.$$

Denique terminus $B^b C^{c-1}$, in potestate $x^{n-\gamma}$ occurrens, deriuabitur ex ipsa formula proposita, si primo loco n scribatur $n-\gamma$; tum vero loco i hic scribi debet $b+c-1=i-1$; tertio loco θ scribi debet $b\beta + (c-1)\gamma = \theta - \gamma$; denique pro indice N scribendum erit $\frac{1 \cdot 2 \cdot 3 \dots (i-1)}{1 \cdot 2 \cdot 3 \dots b \times 1 \cdot 2 \cdot 3 \dots c-1}$, sicque pro N scribi oportebit $N \cdot \frac{c}{i}$, unde ipse iste terminus erit

$$\text{III. } \frac{c}{i} N \cdot \frac{n-\gamma}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-2)\alpha-\theta}{(i-1)\alpha} A^{\frac{n-\theta}{\alpha}} B^b C^{c-1}.$$

Quodsi ergo harum formularum prima ducatur in A, secunda in

in B et tertia in C, ostendendum est earum summam ipsi formulae propositae fore aequalem.

§. 25. Tum autem evidens est postremos factores literales inter se prodituros esse aequales, scilicet $A^{\frac{n-\theta}{\alpha}} B^b C^c$; priores autem factores hunc communem habent factorem:

$$\frac{x \cdot (n+\alpha-\theta) (n+2\alpha-\theta) \dots [n+(i-2)\alpha-\theta]}{\alpha \cdot 2\alpha \cdot 3\alpha \cdot 4\alpha \dots (i-1)\alpha},$$

qui si designetur per Δ , summa harum trium formularum erit

$$\Delta \left(\frac{(n-\alpha)(n-\theta)}{i\alpha} + \frac{b(n-\beta)}{i} + \frac{c(n-\gamma)}{i} \right),$$

quae expressio, facta evolutione, abit in hanc:

$$\frac{\Delta}{i\alpha} (nn - \alpha n - \theta n + \alpha b n + \alpha c n + \alpha \theta - \alpha b \beta - \alpha c \gamma).$$

Cum autem fit $b\beta + c\gamma = \theta$ et $b + c = i$, ista forma reducetur ad hanc:

$$\frac{\Delta}{i\alpha} (nn - \alpha n - \theta n + \alpha i n) = \frac{\Delta}{i\alpha} n [n + (i-1)\alpha - \theta],$$

quamobrem istud aggregatum reperietur hoc modo expressum:

$$N \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha} A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c,$$

quae cum congruat cum ipsa forma proposita, declarat veritatem nostrae regulae.

DEMONSTRATIO

tertia regulae generalis, pro terminis formae
 $B^b \cdot C^c \cdot D^d$.

§. 26. Posuimus hic primo $b + c + d = i$, secundo $b\beta + c\gamma + d\delta = \theta$ et tertio $N = \frac{1 \cdot 2 \cdot 3 \cdot 4 \dots i}{1 \cdot 2 \dots b \times 1 \cdot 2 \dots c \times 1 \cdot 2 \dots d}$, hincque terminum istius formae pro potestate x^n inuenimus:

M 2

N.

$$N \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha} A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c \cdot D^d.$$

Nunc igitur demonstrandum est hanc formulam compositam esse ex quatuor sequentibus partibus: 1°. ex termino formae eiusdem $B^b \cdot C^c \cdot D^d$, potestati $x^{n-\alpha}$ respondente, ducto in A; 2°. ex termino formae $B^{b-1} \cdot C^c \cdot D^d$, potestati $x^{n-\beta}$ respondente, ducto in B; 3°. ex termino formae $B^b \cdot C^{c-1} \cdot D^d$, pro potestate $x^{n-\gamma}$, ducto in C; 4°. ex termino formae $B^b \cdot C^c \cdot D^{d-1}$, pro potestate $x^{n-\delta}$, ducto in D.

§. 27. Prima igitur pars formabitur ex nostra formula generali, si modo loco n scribatur $n - \alpha$, vnde facta multiplicatione per A ista pars erit

$$I. N \cdot \frac{n-\alpha}{\alpha} \cdot \frac{n-\theta}{2\alpha} \cdot \frac{n+\alpha-\theta}{3\alpha} \dots \frac{n+(i-2)\alpha-\theta}{i\alpha} \cdot A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c \cdot D^d.$$

Pro parte autem secunda eruenda primo loco n scribi debet $n - \beta$, secundo loco i scribi debet $i - 1$, tertio loco θ scribendum est $\theta - \beta$, denique loco indicis N habebimus

$$\frac{1 \cdot 2 \cdot 3 \dots i-1}{1 \cdot 2 \dots (b-1) \times 1 \cdot 2 \dots c \times 1 \cdot 2 \dots d}$$

ita vt nunc loco N scribi oporteat $\frac{b}{i} \cdot N$, quibus obseruatis tota pars secunda ita se habebit:

$$II. \frac{b}{i} \cdot N \cdot \frac{n-\beta}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-2)\alpha-\theta}{(i-1)\alpha} A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c \cdot D^d.$$

Tertia forma simili modo formabitur ex ipsa proposita, scribendo primo $n - \gamma$ loco n , secundo $i - 1$ loco i et tertio $\theta - \gamma$ loco θ , at vero quarto pro indice N habebimus $\frac{c}{i} N$, quibus obseruatis ista pars tertia tota erit

$$III. \frac{c}{i} \cdot \frac{n-\gamma}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-2)\alpha-\theta}{(i-1)\alpha} A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c \cdot D^d.$$

Quarta forma orietur ex principali, scribendo primo $n - \delta$ loco

co n , secundo $i-1$ loco i , tertio $\theta-\delta$ loco θ et quarto $\frac{d}{i} N$ loco N , vnde fiet quarta pars

$$IV. \frac{d}{i} \cdot N \cdot \frac{n-\delta}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-2)\alpha-\theta}{(i-1)\alpha} \cdot A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c \cdot D^d.$$

§. 28. Quoniam quatuor istae partes communem habent factorem hunc:

$$\Delta = \frac{n \cdot (n+\alpha-\theta) \cdot (n+2\alpha-\theta) \cdot (n+3\alpha-\theta) \dots n+(i-2)\alpha-\theta}{\alpha \cdot 2\alpha \cdot 3\alpha \cdot 4\alpha \dots (i-1)\alpha} A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c \cdot D^d,$$

summa omnium harum partium erit

$$\Delta \left(\frac{(n-\alpha)(n-\theta)}{i\alpha} + \frac{b(n-\beta)+c(n-\gamma)+d(n-\delta)}{i} \right),$$

quae expressio euoluta, ob $b+c+d=i$ et $b\beta+c\gamma+d\delta=\theta$, reducitur ad hanc formam: $\frac{\Delta}{i\alpha} [n(n+(i-1)\alpha-\theta)]$, consequenter tota summa quatuor harum partium erit

$$N \cdot \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \dots \frac{n+(i-1)\alpha-\theta}{i\alpha} \cdot A^{\frac{n-\theta}{\alpha}} \cdot B^b \cdot C^c \cdot D^d,$$

quae est ipsa forma demonstranda. Vnde iam manifestum est omnes series hac methodo resultantes veritati esse consentaneas.

REGVLA FACILIS ET SVCCINCTA

omnes series huius generis expedite formandi

§. 29. Proposita aequatione quacunque huius formae:

$$1 = \frac{A}{x^\alpha} + \frac{B}{x^\beta} + \frac{C}{x^\gamma} + \frac{D}{x^\delta} + \text{etc.}$$

ponatur breuitatis gratia

$$A^{-\frac{\beta}{\alpha}} B + A^{-\frac{\gamma}{\alpha}} C + A^{-\frac{\delta}{\alpha}} D + \text{etc.} = V,$$

hincque semper habebitur:

$$\frac{x^n}{A^\alpha} = 1 + \frac{n}{\alpha} V + \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot V^2 + \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot V^3 + \text{etc.}$$

fi modo notetur omnes potestates ipsius V actu evolui debere, litteram vero θ non habere valorem fixum, sed pro quovis termino evoluto ipsi peculiarem valorem tribui debere. Scilicet pro termino, qui continet productum $B^b \cdot C^c \cdot D^d \cdot E^e$ etc. isti litterae θ tribui oportet hunc valorem:

$$\theta = b\beta + c\gamma + d\delta + e\epsilon + \text{etc.},$$

hocque in singulis terminis observato orientur eadem series, quas per regulas supra datas formare docuimus.

§. 30. Quanquam nulla adhuc methodus directa patet ad istam expressionem perueniendi, tamen sequenti modo eius ratio quadantenus explicari potest. Cum sit

$$1 - \frac{B}{x^\beta} - \frac{C}{x^\gamma} - \frac{D}{x^\delta} - \text{etc.} = \frac{A}{x^\alpha},$$

in membro sinistro loco x vbique scribatur eius valor ex prima parte oriundus, scilicet $A^{\frac{1}{\alpha}}$, fietque

$$x^\alpha = \frac{A}{1 - A^{-\frac{\beta}{\alpha}} \cdot B - A^{-\frac{\gamma}{\alpha}} \cdot C - A^{-\frac{\delta}{\alpha}} \cdot D - \text{etc.}},$$

unde si br. gr. ponatur

$$V = A^{-\frac{\beta}{\alpha}} B + A^{-\frac{\gamma}{\alpha}} C + A^{-\frac{\delta}{\alpha}} D + \text{etc.}$$

erit $x^\alpha = \frac{A}{1-V}$, hincque sumtis potestatibus exponentis $\frac{n}{\alpha}$, erit

$$x^n = \frac{A^{\frac{n}{\alpha}}}{(1-V)^{\frac{n}{\alpha}}}, \text{ quae expressio si more solito in seriem conver-$$

ver-

vertatur, prodibit

$$\frac{x^n}{A^\alpha} = (1-V)^{-\frac{n}{\alpha}} = 1 + \frac{n}{\alpha} V + \frac{n(n+\alpha)}{\alpha \cdot 2\alpha} V^2 + \frac{n}{\alpha} \cdot \frac{n+\alpha}{2\alpha} \cdot \frac{n+2\alpha}{3\alpha} V^3 + \text{etc.}$$

haec ergo expressio introductione litterae θ , modo ante exposito definiendae, transformabitur in ipsam formam modo traditam:

$$\frac{x^n}{A^\alpha} = 1 + \frac{n}{\alpha} V + \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot V^2 + \frac{n}{\alpha} \cdot \frac{n+\alpha-\theta}{2\alpha} \cdot \frac{n+2\alpha-\theta}{3\alpha} \cdot V^3 + \text{etc.}$$

haecque speculatio aliis forte occasionem praebere poterit formationem huius seriei a priori inuestigandi.