E743

DE SERIE MAXIME MEMORABILI, QUA POTESTAS BINOMIALIS QUAECUNQUE EXPRIMI POTEST.

AUCTORE

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§. 1. Memini me olim vidisse seriem prorsus singularem pro potestate binomiali: $(1+x)^n$, quae abrumpebatur pro casibus quibus exponens n est tam numerus integer positivus, quam negativus. Quia autem ejus formae non amplius recordabar, eam sequenti modo sum perscrutatus. Quia ista series abrumpi debet, sive n fuerit numerus integer positivus, sive negativus, eam sub hac forma repraesento:

$$(1+x)^n = A + nB + n(n-1)C + (n+1)(n)(n-1)D$$
$$+ (n+1)(n)(n-1)(n-2)E + (n+2)\ldots\ldots(n-2)F$$
$$+ (n+2)\ \ldots\ldots\ (n-3)G + \text{etc.}$$

§. 2. Hac forma generali constituta, litteras A, B, C, D, etc. ita determinemus, ut casibus, quibus pro n numerus integer sive positivus sive negativus assumitur, satisfiat; unde casus simpliciores sequentes dabunt aequationes:

Si $n = 0$, erit $1 = A$,
- $n = 1$ - $1 + x = A + B$,
- $n = -1$ - $\frac{1}{1+x} = A - B + 2C$,
- $n = 2$ - $(1+x)^2 = A + 2B + 2C + 6D$,
- $n = -2$ - $\frac{1}{(1+x)^2} = A - 2B + 6C - 6D + 24E$,
- $n = 3$ - $(1+x)^3 = A + 3B + 6C + 24D + 24E + 120F$,
- $n = -3$ - $\frac{1}{(1+x)^3} = A - 3B + 12C - 24D + 120E - 120F + 720G$,
- $n = 4$ - $(1+x)^4 = A + 4B + 12C + 60D + 120E + 720F + 720G + 5040H$,
- $n = -4$ - $\frac{1}{(1+x)^4} = A - 4B + 20C - 60D + 360E - 720F + 5040G - 5040H + 40320I$,

etc.

§. 3. Jam resolutio harum aequationum pro litteris A, B, C, D, etc. sequentes nobis praebet valores:

1). $A = 1$, 5). $24E = \frac{x^4}{(1+x)^2}$,
2). $B = x$, 6). $120F = \frac{x^5}{(1+x)^2}$,
3). $2C = \frac{xx}{1+x}$, 7). $720G = \frac{x^6}{(1+x)^3}$,
4). $6D = \frac{x^3}{1+x}$, etc.

§. 4. Lex, qua hi valores ordine progrediuntur, satis est manifesta, dum quilibet terminus prodit, si praecedens vel per x vel per $\frac{x}{1+x}$ multiplicetur. Quo observato series quaesita sequenti forma expressa reperietur:

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{1.2}\cdot\frac{xx}{1+x} + \frac{(n+1)n(n-1)}{1.2.3}\cdot\frac{x^3}{1+x}$$
$$+ \frac{(n+1)\ldots(n-2)}{1\ldots4}\cdot\frac{x^4}{(1+x)^2} + \frac{(n+2)\ldots(n-2)}{1\ldots5}\cdot\frac{x^5}{(1+x)^2}$$
$$+ \frac{(n+2)\ldots(n-3)}{1\ldots6}\cdot\frac{x^6}{(1+x)^3} + \text{etc.}$$

cujus ordo quo clarius in oculos incurrat, statuamus $\frac{xx}{1+x} = zz$ et distinguamus terminos ordine pares ab imparibus, ut seriem geminatam obtineamus, eritque:

$$(1+x)^n = \begin{cases} 1 + \frac{n(n-1)}{1.2}z^2 + \frac{(n+1)\ldots(n-2)}{1\ldots4}z^4 + \frac{(n+2)\ldots(n-3)}{1\ldots6}z^6 + \text{etc.} \\ + z\left(n + \frac{(n+1)n(n-1)}{1.2.3}z^2 + \frac{(n+2)\ldots(n-2)}{1\ldots5}z^4 + \text{etc.}\right) \end{cases}$$

atque ob insignem ordinem, quo termini utriusque seriei procedunt, jam satis tuto concludere licet, eas esse veritati consentaneas. Quoniam vero haec lex per solam inductionem est conclusa, utique rigidiore demonstratione indiget, quam jam sum investigaturus.

§. 5. Interim tamen statim casus memorabilis se offert, quo veritas hujus seriei egregie confirmatur, scilicet si exponens n statuatur infinite magnus, simul vero x infinite parvum, ita tamen ut productum nx sit quantitas finita, puta u; tum enim constat esse $(1+\frac{u}{n})^n = e^u$. Hoc autem casu series inventa sequentem induet formam:

$$e^u = 1 + \frac{u}{1} + \frac{u^2}{1.2} + \frac{u^3}{1.2.3} + \frac{u^4}{1.2.3.4} + \text{etc.}$$

quae series, ut cuique constat, veritati est consentanea.

§. 6. Ut autem in demonstrationem completam inquiramus, quia posuimus $\frac{xx}{1+x} = zz$, erit $x = \frac{zz + z\sqrt{(zz+4)}}{2}$.

Ad hanc fractionem tollendam statuamus $z = 2y$, ut fiat $x = 2yy + 2y\sqrt{yy+1}$, hincque fit:

$$1 + x = 1 + 2yy + 2y\sqrt{yy+1} = (y + \sqrt{1+yy})^2,$$

ita ut potestas nostra proposita evadat $(y + \sqrt{1+yy})^{2n}$. Quia igitur ista formula $y + \sqrt{1+yy}$ frequentissime occurret, brevitatis gratia ponamus $y + \sqrt{1+yy} = v$, ut potestas evolvenda sit v^{2n}.

§. 7. Cum igitur ista potestas v^{2n} aequetur binis seriebus supra exhibitis, pro priore statuamus:

$$s = 1 + \frac{n(n-1)}{1 \cdot 2} zz + \frac{(n+1)\ldots(n-2)}{1 \ldots 4} z^4 + \frac{(n+2)\ldots(n-3)}{1 \ldots 6} z^6 + \text{etc.}$$

pro altera vero statuamus:

$$\frac{tx}{z} = nx + \frac{(n+1)n(n-1)}{1 \cdot 2 \cdot 3} xz^2 + \frac{(n+2)\ldots(n-2)}{1 \ldots 4} xz^4 + \text{etc.}$$

ut fiat:

$$t = nz + \frac{(n+1)n(n-1)}{1 \cdot 2 \cdot 3} z^3 + \frac{(n+2)\ldots(n-2)}{1 \ldots 4} z^5 + \text{etc.}$$

quae series praecedenti magis est assimilata. Hinc igitur habebimus $v^{2n} = s + \frac{tx}{z}$.

§. 8. Cum nunc posuerimus $z = 2y$, atque hinc fiat $x = 2yy + 2y\sqrt{1+yy} = 2yv$, habebimus $\frac{x}{z} = v$, et aequatio nostra jam erit $v^{2n} = s + tv$. Ut nunc hanc aequationem per differentiationem tractemus, notetur esse $\partial v = \partial y + \frac{y\partial y}{\sqrt{1+yy}} = \frac{v\partial y}{\sqrt{1+yy}}$. Vicissim autem y per v ita exprimitur, ut sit $y = \frac{vv-1}{2v}$, hincque porro $\sqrt{1+yy} = \frac{vv+1}{2v}$.

Tum vero differentiando erit $\partial y = \frac{\partial v(vv+1)}{2vv}$, qui valor egregie convenit cum eo quem praecedens formula differentialis praeberet, unde fit:

$$\partial y = \frac{\partial v}{v}\sqrt{1+yy} = \frac{\partial v(vv+1)}{2vv}.$$

§. 9. Cum jam potestas nostra v^{2n} aequetur duabus seriebus, quarum alteram per s alteram per tv denotavimus, notasse hic juvabit priorem seriem s complecti terminos rationales, alteram vero terminos omnes continet irrationales. Hoc observato aequationis inventae primo sumamus logarithmos, ut habeamus $2nlv = l(s+tv)$ et sumtis differentialibus erit $\frac{2n\partial v}{v} = \frac{\partial s + v\partial t + t\partial v}{s+tv}$. Cum autem sit $v = y + \sqrt{1+yy}$ et $\partial v = \frac{v\partial y}{\sqrt{1+yy}}$, facta hac substitutione orietur haec aequatio:

$$\frac{2n\partial y}{\sqrt{1+yy}} = \frac{\partial s\sqrt{1+yy} + y\partial t\sqrt{1+yy} + \partial t(1+yy) + ty\partial y + t\partial y\sqrt{1+yy}}{(s+ty+t\sqrt{1+yy})\sqrt{1+yy}},$$

quae sublatis fractionibus hanc induet formam:

$$2ns\partial y + 2nty\partial y + 2nt\partial y\sqrt{1+yy}$$
$$= \partial s\sqrt{1+yy} + y\partial t\sqrt{1+yy} + \partial t(1+yy) + ty\partial y + t\partial y\sqrt{1+yy},$$

unde seorsim aequando partes rationales et irrationales nascuntur hae duae aequationes:

I. $2ns\partial y + (2n-1)ty\partial y = \partial t + yy\partial t,$

II. $(2n-1)t\partial y = \partial s + y\partial t.$

§. 10. Ut harum aequationum prior simplicior reddatur, ab ea subtrahatur posterior per y multiplicata, ejusque loco prodibit ista: $2ns\partial y = \partial t - y\partial s$. Altera aequatio, cum hac combinanda, erit $(2n-1)t\partial y = \partial s + y\partial t$. Jam videamus an ex his duabus aequationibus pro litteris s et t easdem series derivare queamus, quas supra per inductionem elicuimus. Quoniam autem illae series procedebant per potestates litterae $z = 2y$, hic loco y scribamus $\frac{1}{2}z$, sicque nostrae ambae aequationes erunt:

$$(2n-1)t\partial z = 2\partial s + z\partial t,$$

$$2ns\partial z = 2\partial t - z\partial s.$$

§. 11. Possemus nunc ex his duabus aequationibus eliminare quantitatem t, quo facto ptodiret aequatio differentialis secundi gradus inter s et z, unde haud difficulter series valorem ipsius s exprimens deduci posset. Deinde simili modo, eliminata littera s, talis aequatio prodiret inter t et z, ex qua eodem modo series pro t derivari posset; verum multo facilius ambae hae series immediate ex binis aequationibus inventis erui poterunt. Pro utraque scilicet littera statim fingamus sequentes series indefinitas:

$$s = 1 + Az^2 + Bz^4 + Cz^6 + Dz^8 + Ez^{10} + \text{etc.}$$

$$t = \alpha z + \beta z^3 + \gamma z^5 + \delta z^7 + \varepsilon z^9 + \text{etc.}$$

§. 12. Nunc istas series substituamus primo in aequatione priore:

$$(2n-1)t - \frac{z\partial t}{\partial z} - \frac{2\partial s}{\partial z} = 0,$$

sequenti modo:

$$\begin{array}{llllll}(2n-1)t = (2n-1)\alpha z & +(2n-1)\beta z^3 & +(2-n1)\gamma z^5 & +(2n-1)\delta z^7 & +\text{etc.}\\ -\frac{z\partial t}{\partial z} = -\alpha & -3\beta & -5\gamma & -7\delta & -\text{etc.}\\ -\frac{2\partial s}{\partial z} = -4A & -8B & -12C & -16D & -\text{etc.}\end{array}$$

Hic jam singulis partibus seorsim ad nihilum redactis obtinemus sequentes determinationes:

$$\begin{array}{ll} A = \frac{(n-1)}{2}\alpha, & D = \frac{(n-4)}{8}\delta,\\ B = \frac{(n-2)}{4}\beta, & E = \frac{(n-5)}{10}\varepsilon,\\ C = \frac{(n-3)}{6}\gamma, & F = \frac{(n-6)}{12}\zeta.\end{array}$$

etc.

§. 13. Eodem modo tractetur altera aequatio:

$$2ns + \frac{z\partial s}{\partial z} - \frac{2\partial t}{\partial z} = 0,$$

et facta substitutione serierum fictarum supra datarum fiet:

$$\begin{array}{llllll} 2ns = 2n + 2nAz^2 & + 2nBz^4 & + 2nCz^6 & + 2nDz^8 & +\text{etc.}\\ +\frac{z\partial s}{\partial z} = \ldots + 2A & + 4B & + 6C & + 8D & +\text{etc.}\\ -\frac{2\partial t}{\partial z} = -2\alpha - 6\beta & - 10\gamma & - 14\delta & - 18\varepsilon & -\text{etc.}\end{array}$$

Hincque ergo fluunt sequentes determinationes:

$$\begin{array}{ll} \alpha = n, & \delta = \frac{n+3}{7}C,\\ \beta = \frac{n+1}{3}A, & \varepsilon = \frac{n+4}{9}D,\\ \gamma = \frac{n+2}{5}B, & \zeta = \frac{n+5}{11}E,\end{array}$$

etc.

§. 14. Cum nunc litterarum graecarum prima $\alpha = n$ sit cognita, alternatim binas superiores determinationes consulendo sequentes valores reperientur:

$$\alpha = n, \qquad A = \frac{n(n-1)}{2},$$

$$\beta = \frac{(n+1)n(n-1)}{1 \cdot 2 \cdot 3}, \qquad B = \frac{(n+1)n(n-1)(n-2)}{1 \cdot 2 \cdot 3 \cdot 4},$$

$$\gamma = \frac{(n+2)\ldots\ldots(n-2)}{1 \ldots\ldots 5}, \qquad C = \frac{(n+2)\ldots\ldots(n-3)}{1 \ldots\ldots 6},$$

$$\delta = \frac{(n+3)\ldots\ldots(n-3)}{1 \ldots\ldots 7}, \qquad D = \frac{(n+3)\ldots\ldots(n-4)}{1 \ldots\ldots 8},$$

etc. etc.

Demonstratum igitur nunc est legem progressionis, quam supra, quasi divinando, attulimus, cum veritate perfecte consentire.

§. 15. Cum igitur inventis istis seriebus sit:

$$(1+x)^n = v^{2n} = s + t\,\frac{x}{z},$$

quaestio hic omni attentione digna occurrit, quinam prodituri sint valores pro utraque littera s et t seorsim sumta, quam investigationem in sequente problemate sum suscepturus.

Problema.

Propositis his duabus seriebus:

$$s = 1 + \frac{n(n-1)}{1 \cdot 2} z^2 + \frac{(n+1)\ldots(n-2)}{1 \cdot 2 \cdot 3 \cdot 4} z^4 + \text{etc.}$$

$$t = \frac{n}{1} z + \frac{(n+1)n(n-1)}{1 \cdot 2 \cdot 3} z^3 + \frac{(n+2)\ldots\ldots(n-2)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} z^5 + \text{etc.}$$

investigare utriusque summam.

Solutio.

§. 16. Determinatio harum duarum summarum repetenda est ex binis aequationibus differentialibus supra inventis, dum loco z et ∂z scribitur $2y$ et $2\partial y$:

$$(2n-1)t\partial y = \partial s + y\partial t,$$
$$2ns\partial y = \partial t + y\partial s.$$

Hic quidem iterum posset alterutra litterarum s et t eliminari, quo pacto perveniretur ad aequationem differentialem secundi gradus: verum etiam isto labore supersedere poterimus. Utamur scilicet tantum aequatione priore hac forma relata: $\partial s = 2nt\partial y - \partial . ty$, cum qua combinemus aequationem principalem: $v^{2n} = s + tv$, unde fit $s = v^{2n} - tv$, ideoque:

$$\partial s = 2nv^{2n-1}\partial v - \partial . tv = 2nz\partial y - \partial . ty.$$

Est vero $\partial . tv - \partial . ty = \partial . t(v-y) = \partial . t\sqrt{1+yy}$, ob $v = y + \sqrt{1+yy}$. Sicque habebimus:

$$2nv^{2n-1}\partial v = 2nt\partial y + \partial t\sqrt{1+yy} + \frac{ty\partial y}{\sqrt{1+yy}},$$

quae aequatio per $\sqrt{1+yy}$ divisa, dat:

$$\partial t + \frac{ty\partial y}{1+yy} + 2nt\frac{\partial y}{\sqrt{1+yy}} = \frac{2nv^{2n-1}\partial v}{\sqrt{1+yy}}.$$

Quia vero est $\frac{\partial y}{\sqrt{1+yy}} = \frac{\partial v}{v}$, aequatio nostra erit:

$$\partial t + \frac{ty\partial y}{1+yy} + 2nt\frac{\partial v}{v} = \frac{2nv^{2n-1}\partial v}{\sqrt{1+yy}},$$

quae multiplicata per $v^{2n}\sqrt{1+yy}$ reddet membrum sinistrum integrabile, eritque:

$$\partial . tv^{2n}\sqrt{1+yy} = 2nv^{4n-1}\partial v,$$

cujus ergo integrale erit:

$$tv^{2n}\sqrt{1+yy} = \tfrac{1}{2}v^{4n} + \tfrac{C}{2},$$

consequenter habebimus:

$$t = \frac{v^{2n} + C v^{-2n}}{2\sqrt{1+yy}}.$$

§. 17. Jam pro constante C, quia casu $y = 0$ et $v = 1$, fieri debet $t = 0$, erit $C = -1$, ita ut sit:

$$t = \frac{v^{2n} - v^{-2n}}{2\sqrt{1+yy}}, \text{ unde deducitur:}$$

$$s = v^{2n} - tv = v^{2n} - \frac{v^{2n+1} + v^{1-2n}}{2\sqrt{1+yy}}.$$

Supra autem vidimus esse: $\sqrt{1+yy} = \frac{vv+1}{2v}$, quo substituto reperietur: $s = \frac{v^{2n} + v^{2-2n}}{vv+1}$.

Interim tamen etiam videamus, quomodo aequationem differentialem supra memoratam tractari oporteat.

Alia Solutio.

ex differentialibus secundi gradus petita.

§. 18. Cum nostrae binae aequationes differentiales sint: $\partial s = 2nt\partial y - \partial . ty$,

$$\partial t = 2ns\partial y + y\partial s,$$

erit ex priore $s = 2n\int t\partial y - ty$, quibus valoribus in altera substitutis fiet:

$$\partial t = 4nn\partial y\int t\partial y - y . \partial . ty,$$

quae evoluta dat:

$$\partial t = 4nn\partial y\int t\partial y - ty\partial y - yy\partial t.$$

§. 19. Ut hinc signum summatorium tollamus, statuamus $\int t\partial y = u$, ut sit $t = \frac{\partial u}{\partial y}$ et $\partial t = \frac{\partial\partial u}{\partial y}$, quibus valoribus substitutis prodit: $\frac{\partial\partial u}{\partial y}(1+yy)+y\partial u = 4nnu\partial y$, quam aequationem, ponendo $u = e^{\int p\partial y}$, ob $\partial u = p\partial y\, e^{\int p\partial y}$ et $\partial\partial u = (\partial p\partial y + pp\partial y)\, e^{\int p\partial y}$, ad differentialia prima reducere licet; erit enim:

$$(\partial p + pp\partial y)(1+yy) + py\partial y = 4nn\partial y, \text{ sive}$$

$$\partial p + pp\partial y + \frac{py\partial y}{1+yy} = 4nn\frac{\partial y}{1+yy}.$$

§. 20. Ut nunc primum terminum et tertium in unum contrahamus, ponamus $p = \frac{q}{\sqrt{1+yy}}$, eritque aequatio: $\frac{\partial q}{\sqrt{1+yy}} + \frac{qq\partial y}{1+yy} = 4nn\frac{\partial y}{1+yy}$, sive $\frac{\partial q}{\sqrt{1+yy}} = \frac{(4nn-qq)\partial y}{1+yy}$, quae commode separationem admittit; evidens enim est prodire:

$$\frac{\partial q}{4nn-qq} = \frac{\partial y}{\sqrt{1+yy}},$$

quae aequatio per $4n$ multiplicata et integrata dat:

$$l\frac{2n+q}{2n-q} = 4n\int\frac{\partial y}{\sqrt{1+yy}} = 4nlv,$$

consequenter erit $\frac{2n+q}{2n-q} = Cv^{4n}$, unde elicitur:

$$q = \frac{2n(Cv^{4n}-1)}{Cv^{4n}+1}.$$

§. 21. Hinc igitur, ob $p = \frac{q}{\sqrt{1+yy}}$, erit

$$p\partial y = \frac{q\partial y}{\sqrt{1+yy}} = \frac{q\partial v}{v}, \text{ ideoque } p\partial y = \frac{2n(Cv^{4n}-1)\partial v}{v(Cv^{4n}+1)},$$

quae expressio resolvitur in has partes:

$$p\partial y = -\frac{2n\partial v}{v} + \frac{4nCv^{4n-1}\partial v}{Cv^{4n}+1},$$

cujus ergo integrale erit:

$$\int p\partial y = -2nlv + l(Cv^{4n}+1) + lD,$$

consequenter erit:

$$e^{\int p\partial y} = \frac{D}{v^{2n}}(1+Cv^{4n}) = Dv^{-2n} + CDv^{+2n} = u.$$

§. 22. Cum igitur sit $u = \int t \partial y$, ideoque $t = \frac{\partial u}{\partial y}$, per differentiationem, mutatis constantibus arbitrariis, reperimus:

$$t = \frac{E v^{-2n} + F v^{+2n}}{\sqrt{1 + yy}}.$$

Ad constantes autem definiendas primo notetur posito $y = 0$ et $v = 1$ fieri debere $t = 0$, unde fit $F = -E$, ita ut jam sit: $t = \frac{E}{\sqrt{1+yy}} (v^{-2n} - v^{+2n})$. Deinde vero si y fuerit infinite parvum, fieri debet $t = nz = 2ny$, tum vero evadit $v = 1 + y$ et $v^{-1} = 1 - y$, ideoque $v^{2n} = 1 + 2ny$ et $v^{-2n} = 1 - 2ny$, ex quibus valoribus fiet $2ny = -4nEy$, ergo $E = -\frac{1}{2}$, sicque nanciscimur pro t eundem valorem ac supra invenimus, scilicet $t = \frac{v^{2n} - v^{-2n}}{2\sqrt{1+yy}}$, ex quo porro ut ante derivatur $s = \frac{v^{2n-1} + v^{1-2n}}{2\sqrt{1+yy}}$.

Solutio facillima Problematis.

§. 23. Hanc solutionem derivabimus ex sola aequatione $x^{2n} = s + tv$, in qua, ob $v = y + \sqrt{1+yy}$, littera s complectitur potestates pares ipsius y, t vero impares. Sumto igitur y negative, littera s manet eadem, littera t vero abibit in $-t$; tum autem loco v habebimus:

$$-y + \sqrt{1+yy} = v^{-1}.$$

Hoc observato, si loco litterarum s, t, v scribamus s, $-t$, v^{-1}, aequatio nostra etiamnunc subsistere debebit, sicque habebimus $v^{-2n} = s - \frac{t}{v}$, qua aequatione cum principali $v^{2n} = s + tv$ conjuncta, fiet subtrahendo:

$$v^{2n} - v^{-2n} = tv + \frac{t}{v},$$

unde fit $t = \frac{v^{2n} - v^{-2n}}{2\sqrt{1+yy}}$, et hinc reperietur $s = \frac{v^{2n-1} + v^{1-2n}}{2\sqrt{1+yy}}$. Cum enim ex prima aequatione sit $t = v^{2n-1} - \frac{s}{v}$, ex altera vero $-t = v^{1-2n} - v^3$, hi valores invicem coaequati dabunt $\frac{s(vv+1)}{v} = v^{2n-1} + v^{1-2n}$, unde ob $\frac{vv+1}{v} = 2\sqrt{1+yy}$ erit $s = \frac{v^{2n-1} + v^{1-2n}}{2\sqrt{1+yy}}$.

§. 24. Transferamus denique haec omnia ad ipsam potestatem $1+x$, et cum sit $1+x = vv$ et $\sqrt{1+yy} = \frac{vv+1}{2v} = \frac{x+2}{2\sqrt{1+x}}$, erit $2\sqrt{1+yy} = \frac{x+2}{\sqrt{1+x}}$, quibus valoribus substitutis fiet:

$$s = \frac{\sqrt{1+x}}{x+2} \times \left((1+x)^{n-\frac{1}{2}} + (1+x)^{\frac{1}{2}-n}\right),$$

$$t = \frac{\sqrt{1+x}}{x+2} \times \left((1+x)^{n} - (1+x)^{-n}\right),$$

sive

$$s = \frac{(1+x)^n + (1+x)^{-n}}{x+2},$$

$$t = \frac{(1+x)^{n+\frac{1}{2}} - (1+x)^{\frac{1}{2}-n}}{x+2}.$$

Hinc pro altera serie deducitur $\frac{tx}{z} = t\sqrt{1+x}$, ideoque valor seriei erit:

$$\frac{(1+x)^{n+1} - (1+x)^{1-n}}{2+x},$$

quae est summa terminorum ordine parium in serie pro potestate $(1+x)^n$ inventa.