$\frac{1+n\cos s}{n+\cos s}=\cos 2\omega$, hincque $\cos s=\frac{n\cos 2\omega-1}{n-\cos 2\omega}$ et $\sin s=\frac{\sin 2\omega\sqrt{(nn-1)}}{n-\cos 2\omega}$; tum enim erit
$\sqrt{(nn-1)}=\text{tang}\,(45^0+\omega)$, ideoque

$$m\tau=\frac{n\,\text{tang}\,2\omega-\log.\text{tang}\,(45^0+\omega)}{(nn-1)\sqrt{(nn-1)}}.$$

Quodsi ergo ad singulos gradus anomaliae verae s valores quantitatis $m\tau$ computentur, inde vicissim pro dato $m\tau$ ipsa anomalia vera s simulque distantia $AB=\varrho=\frac{f}{1+n\cos s}$ facile colligitur.

107. **Coroll. 1.** Crescente ergo tempore t seu quantitate ipsi proportionali $m\tau$, crescit etiam anomalia vera $CAB=s$, atque elapso tempore infinito fit $\cos s=-\frac{1}{n}$ et $\sin s=\frac{\sqrt{(nn-1)}}{n}$, eodemque casu evadit distantia $AB=\varrho$ infinita.

108. **Coroll. 2.** Elapso tempore infinito locus corporis in asymtotam incidet, et asymtotae utrinque ad axem hyperbolae inclinantur angulo, cujus cosinus est $=\frac{1}{n}$ et tangens $=\sqrt{(nn-1)}$. Est vero $\frac{f}{nn-1}$ semiaxis transversus hyperbolae et $\frac{f}{\sqrt{(nn-1)}}$ semiaxis conjugatus.

109. **Scholion.** Evolvimus ergo omnes species motuum, quibus duo corpora se mutuo attrahentia, siquidem fuerint sphaerica circumferri possunt; vidimus orbitam, quam alterum circa alterum describere spectatur, esse sectionem conicam. Huc quidem proxime accedunt orbitae, quas planetae primarii et cometae circa solem describere videntur, dum illi in ellipsibus circumferuntur, hi vero quasi in parabolis, etsi adhuc incertum est, utrum hyperbola penitus sit excludenda. Verumtamen planetas non exacte in orbitis ellipticis circa solem circumferri vel exinde patet, quod lineae absidum in coelo non quietae deprehenduntur. Duplex scilicet perturbatio eorum motum afficit, quarum altera a figura planetarum non sphaerica, altera ab attractione reliquorum corporum coelestium proficiscitur, quam investigationem deinceps sumus suscepturi. Ante autem juvabit hoc idem argumentum de motu duorum corporum sphaericorum per calculos variatos pertractasse. Cum enim totum negotium resolutione aequationum differentio-differentialium innitatur, plurimum intererit hujusmodi aequationes variis methodis tentari, quandoquidem hoc casu de successu certi sumus, quacunque methodo utamur, etiamsi forte, nisi solutio jam ante esset cognita, calculi evolutio nimis ardua videretur. His autem difficultatibus superatis, aditus ad sublimiores investigationes, quando plura duobus corpora proponuntur, facilior forsitan redderetur. In sequente ergo capite aliis quibusdam methodis determinationem motus duorum corporum sphaericorum aggrediamur.

Caput III.

Aliae investigationes motus duorum corporum sphaericorum.

110. **Problema.** (Fig. 179.) Dum corpora sphaerica A et B se mutuo attrahunt, hujus motum, qualis ex illo spectatur, referre ad planum quodcunque per corpus A ductum.

Solutio. Repraesentet tabula planum, ad quod motum corporis B referri oportet, quod jam tempore elapso $=t$ versetur in B, unde demisso ad planum propositum perpendiculo BY, et ex Y

ad rectam fixam AE normali YX, sint ternae coordinatae $AX = x$, $XY = y$, $YB = z$; ipsa distantia $AB = \varrho = \sqrt{(xx + yy + zz)}$. Cum jam vis, qua B ad A urgetur, sit $= \frac{B(A+B)}{\varrho\varrho}$ massas corporum per litteras A et B indicando; ea secundum directiones ternarum coordinatarum resoluta dabit vim in directione

$$AX = \frac{-B(A+B)x}{\varrho^3},$$

$$XY = \frac{-B(A+B)y}{\varrho^3},$$

$$YB = \frac{-B(A+B)z}{\varrho^3},$$

unde sequentes aequationes elicimus sumendo elementum dt constans

$$ddx = \frac{-2g(A+B)x}{\varrho^3} \cdot dt^2, \quad ddy = \frac{-2g(A+B)y}{\varrho^3} \cdot dt^2, \quad ddz = \frac{-2g(A+B)z}{\varrho^3} \cdot dt^2.$$

Hinc dt^2 eliminando colligimus

$$yddx - xddy = 0, \quad zddy - yddz = 0, \quad xddz - zddx = 0,$$

quarum quidem quaelibet in binis reliquis jam continetur, ita ut duas tractasse sufficiat. Inde integrando obtinemus

$$ydx - xdy = Edt \quad \text{et} \quad zdy - ydz = Fdt, \quad \text{hincque}$$

$$F(ydx - xdy) + E(ydz - zdy) = 0,$$

quae per yy divisa et integrata dat

$$\frac{Fx}{y} + \frac{Ez}{y} + G = 0, \quad \text{seu} \quad Ez + Fx + Gy = 0,$$

ex qua liquet motum corporis B fieri in plano per punctum A transeunte. Cum igitur habeamus $Ez + Fx + Gy = 0$, ac praeterea has tres aequationes differentiales

$$Edt = ydx - xdy, \quad Fdt = zdy - ydz, \quad Gdt = xdz - zdx,$$

ob $xx + yy + zz = \varrho\varrho$ adipiscemur quadratis addendis

$$(EE + FF + GG)dt^2 = dx^2(\varrho\varrho - xx) + dy^2(\varrho\varrho - yy) + dz^2(\varrho\varrho - zz) - 2xydxdy - 2yzdydz - 2xzdxdz,$$

et quia $xdx + ydy + zdz = \varrho d\varrho$, obtinebimus

$$(EE + FF + GG)\,dt^2 = \varrho\varrho\,(dx^2 + dy^2 + dz^2) - \varrho\varrho d\varrho^2.$$

Verum si aequationum differentio-differentialium prima per $2dx$, secunda per $2dy$ et tertia per $2dz$ multiplicetur, summa erit

$$2dx\,ddx + 2dy\,ddy + 2dz\,ddz = \frac{-4g(A+B)\varrho d\varrho}{\varrho^3} \cdot dt^2,$$

cujus integrale, ob dt constans, est

$$dx^2 + dy^2 + dz^2 = Ddt^2 + \frac{4g(A+B)}{\varrho}dt^2,$$

qui valor ob superiorem aequationem est

$$= d\varrho^2 + \frac{(EE+FF+GG)\,dt^2}{\varrho\varrho},$$

ita ut per $\varrho\varrho$ multiplicando habeamus

$$\varrho\varrho\, d\varrho^2 = dt^2\,(D\varrho\varrho + 4g\,(A+B)\,\varrho - EE - FF - GG),$$

ideoque

$$dt = \frac{\varrho\, d\varrho}{\sqrt{(D\varrho\varrho + 4g\,(A+B)\,\varrho - EE - FF - GG)}}.$$

Superest ut reliquas variabiles x, y, z etiam per ϱ determinemus. Cum igitur sit $z = \frac{-Fx - Gy}{E}$, habebimus

$$EE\varrho\varrho = EExx + EEyy + FFxx + 2FGxy + GGyy,$$ hincque

$$y = \frac{-FGx + E\sqrt{((EE+GG)\,\varrho\varrho - (EE+FF+GG)\,xx)}}{EE+GG}.$$

Statuamus brevitatis gratia

$EE + FF + GG = HH$, sitque $\frac{Hx}{\varrho\sqrt{(EE+GG)}} = \cos\omega$, seu $x = \frac{\varrho\sqrt{(EE+GG)}}{H}\cos\omega$,

erit $y = \frac{-FG\cos\omega + EH\sin\omega}{H\sqrt{(EE+GG)}}\cdot\varrho$ et $z = \frac{-EF\cos\omega - GH\sin\omega}{H\sqrt{(EE+GG)}}\cdot\varrho$.

Hinc erit $\frac{y}{x} = \frac{-FG\cos\omega + EH\sin\omega}{(EE+GG)\cos\omega}$ et $\frac{xdy - ydx}{xx} = \frac{EH\,d\omega}{(EE+GG)\cos^2\omega}$, ideoque

$$xdy - ydx = \frac{E\varrho\varrho\, d\omega}{H} = -E\,dt,$$ ita ut sit $d\omega = \frac{-H\,dt}{\varrho\varrho}$.

Quaeratur ergo angulus ω, ut sit

$$\omega = -\int\frac{H\,d\varrho}{\varrho\sqrt{(D\varrho\varrho + 4g\,(A+B)\,\varrho - HH)}},$$

eritque $\sin\omega = \frac{HH - 2g\,(A+B)\,\varrho}{\varrho\sqrt{(DHH + 4gg\,(A+B)^2)}}$ et $\cos\omega = \frac{H\sqrt{(D\varrho\varrho + 4g\,(A+B)\,\varrho - HH)}}{\varrho\sqrt{(DHH + 4gg\,(A+B)^2)}}$.

Invento hoc angulo ω, ad eum constantem angulum quemcunque adjicere licet, unde obtinebimus

$$\frac{x}{\varrho} = \frac{(EE+GG)\cos(\omega+\delta)}{H\sqrt{(EE+GG)}},$$

$$\frac{y}{\varrho} = \frac{-FG\cos(\omega+\delta) + EH\sin(\omega+\delta)}{H\sqrt{(EE+GG)}},$$

$$\frac{z}{\varrho} = \frac{-EF\cos(\omega+\delta) - GH\sin(\omega+\delta)}{H\sqrt{(EE+GG)}},$$

ita ut jam omnes quantitates variabiles sint determinatae per eandem variabilem ϱ.

111. Coroll. 1. Si ponatur $\frac{EH}{FG} = \operatorname{tang}\alpha$ et $\frac{GH}{EF} = \operatorname{tang}\gamma$, formulae posteriores transmutantur in has:

$$\frac{x}{\varrho} = \frac{\sqrt{(EE+GG)}}{H}\cos(\omega+\delta),\quad \frac{y}{\varrho} = \frac{-\sqrt{(EE+FF)}}{H}\cos(\omega+\delta+\alpha),\quad \frac{z}{\varrho} = \frac{-\sqrt{(FF+GG)}}{H}\cos(\omega+\delta-\gamma),$$

quae formulae exprimunt cosinus angulorum, quibus recta AB ad ternas directiones principales inclinatur.

112. **Coroll. 2.** Si ducta AY, angulus XAY tanquam longitudo vocetur $= \varphi$, ob $\frac{y}{x} =$ tang habebimus hanc longitudinis determinationem

$$\operatorname{tang} \varphi = \frac{-FG}{EE + GG} + \frac{EH \operatorname{tang} (\omega + \delta)}{EE + GG},$$

tum vero angulo YAB tanquam latitudine posito $= \psi$, erit

$$\sin \psi = \frac{-EF \cos (\omega + \delta) - GH \sin (\omega + \delta)}{H \sqrt{(EE + GG)}} = \frac{-\sqrt{(FF + GG)}}{H} \cos (\omega + \delta - \gamma).$$

113. **Coroll. 3.** Si recta $A☊$ fuerit intersectio plani, in quo corpus B movetur cum plano tabulae, motusque fiat in sensum EB, ita ut in ☊ sit nodus ascendens, erit pro longitudine hujus nodi tang $EA☊ = \frac{-F}{G}$, et pro inclinatione planorum tang $YNB = \frac{-\sqrt{(FF + GG)}}{E}$, seu cos $YNB =$ ducta YN ad $A☊$ normali.

114. **Coroll. 4.** Si ponamus $\frac{F}{H} = \cos \alpha$, $\frac{G}{H} = \cos \beta$, $\frac{E}{H} = \cos \gamma$, ut sit

$$x \cos \alpha + y \cos \beta + z \cos \gamma = 0 \quad \text{et} \quad \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1;$$

tum vero statuamus

$$\frac{x}{v} = \sin \alpha \cos (\omega + \zeta), \quad \frac{y}{v} = \sin \beta \cos (\omega + \eta), \quad \frac{z}{v} = \sin \gamma \cos (\omega + \vartheta),$$

anguli ζ, η, ϑ ita sunt comparati ut sit

$$\operatorname{tang} (\vartheta - \eta) = \frac{\cos \alpha}{\cos \beta \cos \gamma}, \quad \operatorname{tang} (\zeta - \vartheta) = \frac{\cos \beta}{\cos \alpha \cos \gamma}, \quad \operatorname{tang} (\eta - \zeta) = \frac{\cos \gamma}{\cos \alpha \cos \beta},$$

unde ordo in his formulis facilius perspicitur.

115. **Scholion.** Evidens est hanc solutionem ad superiorem perducere. Si enim anguli ☊AB ponatur $= \varphi$, qui est angulus, quem corpus B in sua orbita plana tempore t absolvit, erit $\frac{BN}{v} = \sin \varphi$; at est $\frac{z}{BN} = \sin YNB = \frac{\sqrt{(FF + GG)}}{H}$, ideoque

$$BN = \frac{Hz}{\sqrt{(FF + GG)}} \quad \text{et} \quad \sin \varphi = \frac{H}{\sqrt{(FF + GG)}} \cdot \frac{z}{v} = -\cos (\omega + \delta - \gamma) \quad \text{per § 111.}$$

Ergo $\sin \varphi = \sin (\omega + \delta - \gamma - 90^0)$, ita ut sit

$$\varphi = \omega + \text{Const. ac } d\varphi = d\omega, \quad \text{seu} \quad d\varphi = \frac{-H dv}{v \sqrt{(Dvv + 4G(A + B) v - HH)}},$$

quae aequatio cum supra inventa plane congruit. Signum enim $-$, quod supra erat $+$, ob signum radicalis ambiguum nihil turbat. Quin etiam poteramus ponere

$$\sin \varphi = + \cos (180^0 + \omega + \delta - \gamma) = \sin (-90^0 - \omega - \delta + \gamma),$$

unde deducitur $\varphi = \text{Const.} - \omega$ et $d\varphi = -d\omega$, eademque prorsus aequatio obtinetur. Ceterum cum hic motum corporis B etiam, qualis ex A spectatur, definiverimus, nunc in motus absolutos utriusque corporis inquiramus, quales scilicet ambo ex puncto quodam fixo visi apparerent; ac primo quidem ambos motus in eodem plano absolvi assumamus.

116. **Problema.** (Fig. 180.) Si duo corpora sphaerica se mutuo attrahentia A et B moveantur in eodem plano, definire eorum motum absolutum.

Solutio. Moveantur ambo corpora A et B, quorum massae iisdem litteris indicentur, in plano tabulae, in quo assumta recta fixa OV, in eaque puncto fixo O, ad quodvis tempus elapsum pro utroque corpore coordinatas orthogonales OX, XA et OP, PB assignari oportet. Ponamus ergo pro corpore A coordinatas $OX = X$, $XA = Y$, et ducta AL rectae fixae OV parallela, vocatisque distantia $AB = \rho$ et angulo $LAB = \varphi$, pro corpore B erunt coordinatae

$$OP = X + \rho\cos\varphi \quad \text{et} \quad PB = Y + \rho\sin\varphi.$$

Jam quia vis, qua corpora se mutuo attrahunt, est $= \frac{AB}{\rho\rho}$, corpus A sollicitabitur secundum directiones fixas

$$\text{sec. } OX \text{ vi} = \frac{AB}{\rho\rho}\cos\varphi; \qquad \text{sec. } XA \text{ vi} = \frac{AB}{\rho\rho}\sin\varphi;$$

corpus vero B sollicitabitur

$$\text{sec. } OP \text{ vi} = \frac{-AB}{\rho\rho}\cos\varphi; \qquad \text{sec. } PB \text{ vi} = \frac{-AB}{\rho\rho}\sin\varphi.$$

Sumto ergo elemento temporis dt constante, habebimus has quatuor aequationes:

$$\text{I. } ddX = \frac{2gB\,dt^2}{\rho\rho}\cos\varphi, \qquad \text{II. } ddY = \frac{2gB\,dt^2}{\rho\rho}\sin\varphi,$$

$$\text{III. } ddX + dd.\rho\cos\varphi = \frac{-2gA\,dt^2}{\rho\rho}\cos\varphi, \qquad \text{IV. } ddY + dd.\rho\sin\varphi = \frac{-2gA\,dt^2}{\rho\rho}\sin\varphi,$$

unde sublatis ddX et ddY supererunt hae duae aequationes

$$(1)\ dd.\rho\cos\varphi = \frac{-2g(A+B)\,dt^2}{\rho\rho}\cos\varphi, \qquad (2)\ dd.\rho\sin\varphi = \frac{-2g(A+B)\,dt^2}{\rho\rho}\sin\varphi,$$

quibus definitur motus respectivus corporis B, qualis spectatori in A posito esset appariturus, quippe qui motus per distantiam $AB = \rho$ et angulum $LAB = \varphi$ determinatur. Conveniuntque hae formulae perfecte cum iis, quas in superiori capite invenimus. Definito autem hoc motu respectivo, pro absoluto deinceps colligimus

$$(A+B)\,ddX + B\,dd.\rho\cos\varphi = 0 \quad \text{et} \quad (A+B)\,ddY + B\,dd.\rho\sin\varphi = 0,$$

ac proinde bis integrando

$$(A+B)\,X + B\rho\cos\varphi = Et + \mathfrak{E} \quad \text{et} \quad (A+B)\,Y + B\rho\sin\varphi = Ft + \mathfrak{F},$$

quibus motus uniformis communis centri inertiae corporum in directum declaratur. Ad motum ergo absolutum utriusque corporis cognoscendum primo motum respectivum investigari convenit, quod etsi jam in superiori capite est praestitum, solutionem tamen ex binis aequationibus hic expositis petamus. Ac primo quidem haec combinatio $(1).\rho\sin\varphi - (2).\rho\cos\varphi$ praebet

$$\rho\sin\varphi\,dd.\rho\cos\varphi - \rho\cos\varphi\,dd.\rho\sin\varphi = 0,$$

quae integrata dat $v\sin\varphi\, d.v\cos\varphi - v\cos\varphi\, d.v\sin\varphi = 2Cdt$,

$$\text{seu } vv\,d\varphi = Cdt.$$

Deinde ista combinatio (1) $.d.v\cos\varphi +$ (2) $.d.v\sin\varphi$ praebet

$$d.v\cos\varphi.\,dd.v\cos\varphi + d.v\sin\varphi.\,dd.v\sin\varphi = \frac{-2g(A+B)\,dt^2}{v^3}(\cos\varphi\, d.v\cos\varphi + \sin\varphi\, d.v\sin\varphi)$$

$$= \frac{-2g(A+B)\,dt^2}{vv}\, dv,$$

unde integrando impetramus

$$(d.v\cos\varphi)^2 + (d.v\sin\varphi)^2 = D + \frac{4g(A+B)\,dt^2}{v}, \quad \text{sive} \quad dv^2 + vv\,d\varphi^2 = Ddt^2 + \frac{4g(A+B)\,dt^2}{v}.$$

Quare cum ex illa sit $dt = \frac{vv\,d\varphi}{C}$, fiet

$$CCdv^2 + CCvv\,d\varphi^2 = Dv^4\,d\varphi^2 + 4g(A+B)v^3 d\varphi^2,$$

$$\text{hincque} \qquad d\varphi = \frac{C\,dv}{v\sqrt{(Dv^2 + 4g(A+B)v - CC)}}$$

$$\text{atque} \qquad dt = \frac{v\,dv}{\sqrt{(Dv^2 + 4g(A+B)v - CC)}}.$$

Definitis autem ad tempus t quantitatibus v et φ, ex superioribus formulis colligentur coordinatae X et Y pro corpore A, ex quibus hujus corporis motus absolutus innotescit, indeque etiam corporis alterius B.

117. **Coroll. 1.** Si J sit commune centrum inertiae amborum corporum, erit

$$(A+B)\,OK = A.OX + B.OP = (A+B)\,X + Bv\cos\varphi$$

$$\text{et} \quad (A+B)\,KJ = A.XA + B.PB = (A+B)\,Y + Bv\sin\varphi,$$

unde in superioribus formulis E est celeritas ejus in directione OV, et F in directione KJ.

118. **Coroll. 2.** Positis ergo $E = 0$ et $F = 0$, commune centrum inertiae J quiescet. Ac si punctum O in eo ipso accipiamus, insuper constantes $\mathfrak{E}$ et $\mathfrak{F}$ evanescunt, eritque tum

$$OX = X = \frac{-Bv\cos\varphi}{A+B} \quad \text{et} \quad XA = Y = \frac{-Bv\sin\varphi}{A+B}.$$

119. **Coroll. 3.** Cum in superiori capite per anomaliam veram s has determinationes invenerimus:

$$v = \frac{f}{1+n\cos s}, \quad \varphi = s + \alpha \quad \text{et} \quad t = \frac{f\sqrt{f}}{\sqrt{2g(A+B)}}\int\frac{ds}{(1+n\cos s)^2},$$

habebimus in genere pro curva, quam corpus A motu absoluto describit, constantibus parum immutatis,

$$OX = X = \mathfrak{E} + E\int\frac{ds}{(1+n\cos s)^2} - \frac{Bf\cos(s+\alpha)}{(A+B)(1+n\cos s)},$$

$$XA = Y = \mathfrak{F} + F\int\frac{ds}{(1+n\cos s)^2} - \frac{Bf\sin(s+\alpha)}{(A+B)(1+n\cos s)}.$$

120. **Coroll. 4.** Pro curva vero, quam alterum corpus B motu absoluto describit, erunt coordinatae

$$OP = X + v\cos\varphi = \mathfrak{E} + E\int\frac{ds}{(1+n\cos s)^2} + \frac{Af\cos(s+a)}{(A+B)(1+n\cos s)},$$

$$PB = Y + v\sin\varphi = \mathfrak{F} + F\int\frac{ds}{(1+n\cos s)^2} + \frac{Af\sin(s+a)}{(A+B)(1+n\cos s)}.$$

Unde patet casu, quo $E = 0$ et $F = 0$, utramque curvam fore sectionem conicam.

121. **Scholion.** Hinc perspicitur egregius consensus inter ambas methodos, quibus sum usus ad motus binorum corporum determinandos, ac simul patet, utramque methodum ita inter se cohaerere, ut determinatio motus respectivi praecipuam partem in motus absoluti investigatione constituat. Haec methodus scilicet latius patet, quam illa, cum non solum motum respectivum perinde ac illa complectatur, sed etiam motum absolutum utriusque corporis declaret, atque hoc quidem ita, ut ratio motuum absolutorum facillime e calculo eliminetur, totumque negotium ad motus respectivi determinationem perducatur. Hoc enim cognito nihil aliud superest, nisi ut motus communis centri inertiae, qui semper est uniformis secundum lineam rectam, in computum introducatur. Quare etiam in investigatione motus plurium corporum se mutuo attrahentium semper sufficit motus respectivos, qui spectatori in uno eorum collocato sint apparituri, determinasse. Etsi enim hic unum corpus tanquam quiescens consideratur, tamen facile est deinceps toti systemati ejusmodi motum mente saltem induere, quo commune centrum inertiae vel ad quietem vel motum uniformem rectilineum redigatur, hocque modo ad motuum absolutorum cognitionem pervenietur. Istud etiam eo clarius patebit ex sequente problemate, ubi motus binorum corporum, quando non in eodem plano absolvuntur, sum evoluturus.

122. **Problema.** Si duo corpora sphaerica se mutuo attrahentia ita moveantur, ut motus eorum non in eodem plano absolvatur, definire utriusque corporis motum absolutum.

Solutio. (Fig. 181) Sint jam elapso tempore $= t$ ambo corpora in A et B, quorum massae iisdem litteris A et B indicentur. Referantur eorum loca ad ternas directiones fixas inter se normales OE, OF, OG, quibus constituantur pro utroque parallelae coordinatae, quas pro A vocemus $OX = X$, $XY = Y$ et $YA = Z$. Pro corpore autem B statuamus primo distantiam $AB = v$, tum vero ejus inclinatio ad ternas illas directiones fixas OE, OF, OG indicetur angulis ζ, η, ϑ, ut sit $\cos^2\zeta + \cos^2\eta + \cos^2\vartheta = 1$, hincque coordinatae pro B erunt $OP = X + v\cos\zeta$, $PQ = Y + v\cos\eta$, $QB = Z + v\cos\vartheta$, seu posito $v\cos\zeta = x$, $v\cos\eta = y$, $v\cos\vartheta = z$, ut sit $vv = xx + yy + zz$, habebimus $OP = X + x$, $PQ = Y + y$, $QB = Z + z$. Cum jam vis attractrix secundum AB sit $= \frac{AB}{vv}$, corpus A ab ea sollicitatur

$$\text{secundum } OE \text{ vi} = \frac{AB}{vv}\cos\zeta, \quad \text{sec. } OF \text{ vi} = \frac{AB}{vv}\cos\eta, \quad \text{sec. } OG \text{ vi} = \frac{AB}{vv}\cos\vartheta,$$

corpus vero B his viribus

$$\text{sec. } OE \text{ vi} = \frac{-AB}{vv}\cos\zeta, \quad \text{sec. } OF \text{ vi} = \frac{-AB}{vv}\cos\eta, \quad \text{sec. } OG \text{ vi} = \frac{-AB}{vv}\cos\vartheta,$$

unde principia accelerationis suppeditabunt has aequationes

$$ddX = \frac{2gB}{vv}\,dt^2\cos\zeta, \qquad ddY = \frac{2gB}{vv}\,dt^2\cos\eta, \qquad ddZ = \frac{2gB}{vv}\,dt^2 \ldots$$

$$ddX + ddx = \frac{-2gA}{vv}\,dt^2\cos\zeta, \quad ddY + ddy = \frac{-2gA}{vv}\,dt^2\cos\eta, \quad ddZ + ddz = \frac{-2gA}{vv}\,dt^2 \ldots$$

Hic quantitates X, Y, Z referuntur ad motum absolutum corporis A, at x, y, z ad motum spectivum, quo corpus B ex A spectatum moveri cernitur. Pro hoc ergo colligimus:

$$\text{I.}\quad ddx = \frac{-2g(A+B)\,dt^2\cos\xi}{vv} = \frac{-2g(A+B)\,x\,dt^2}{v^3},$$

$$\text{II.}\quad ddy = \frac{-2g(A+B)\,dt^2\cos\eta}{vv} = \frac{-2g(A+B)\,y\,dt^2}{v^3},$$

$$\text{III.}\quad ddz = \frac{-2g(A+B)\,dt^2\cos\vartheta}{vv} = \frac{-2g(A+B)\,z\,dt^2}{v^3},$$

ex quibus cum aequatione $xx+yy+zz=vv$ omnes quantitates x, y, z et v ad tempus minari oportet. Inde autem primo has aequationes integrabiles deducimus

$$yddx - xddy = 0, \quad zddy - yddz = 0, \quad xddz - zddx = 0,$$

quae integratae dant

$$ydx - xdy = Edt, \quad zdy - ydz = Fdt, \quad xdz - zdx = Gdt.$$

Quare cum sit $F(ydx - xdy) = E(zdy - ydz)$, per yy dividendo nanciscemur $\frac{Fx}{y} = \frac{-Ez}{y}$ ...
Similique modo ob $G(zdy - ydz) = F(xdz - zdx)$, per zz dividendo adipiscimur $\frac{Gy}{z} = \frac{-Fx}{z}$ ...
ex quibus conjunctim deducimus $Fx + Gy + Ez = 0$, qua aequatione motus corporis B ex tatus in eodem plano fieri indicatur.

Porro si primam per $2dx$, secundam per $2dy$ et tertiam per $2dz$ multiplicemus, ob $xdx + ydy + zdz = vdv$, summa erit

$$2dx\,ddx + 2dy\,ddy + 2dz\,ddz = \frac{-4g(A+B)\,dv}{vv}\,dt^2,$$

cujus integrale ob dt constans dat

$$dx^2 + dy^2 + dz^2 = Ddt^2 + \frac{4g(A+B)\,dt^2}{v},$$

ex qua ope aequationum

$$Fx + Gy + Ez = 0, \quad xx + yy + zz = vv, \quad ydx - xdy = Edt, \quad zdy - ydz = Fdt,$$
$$xdz - zdx = Gdt$$

eadem solutio deducitur, quam jam supra dedimus. Denique inventis variabilibus x, y, ... respectivum spectantibus, ex iis pro motu absoluto corporis A colliguntur coordinatae X, Y, ... has aequationes

$$X = \frac{-Bx + \mathfrak{E}t + \mathfrak{e}}{A+B}, \quad Y = \frac{-By + \mathfrak{F}t + \mathfrak{f}}{A+B}, \quad Z = \frac{-Bz + \mathfrak{G}t + \mathfrak{g}}{A+B}.$$

123. **Coroll. 1.** Ex aequationibus differentialibus

$$ydx - xdy = Edt, \quad zdy - ydz = Fdt, \quad xdz - zdx = Gdt,$$

integratione immediate colligitur, multiplicando primam per z, secundam per x, et tertiam per z, sumendo:

$$0 = Ezdt + Fxdt + Gydt, \quad \text{hincque} \quad Ez + Fx + Gy = 0.$$

124. **Coroll. 2.** Eaedem aequationes differentiales quadratae et additae, posito

$$EE + FF + GG = HH, \quad \text{praebent}$$

$$HHdt^2 = vv(dx^2 + dy^2 + dz^2) - vvdv^2, \quad \text{ideoque} \quad dx^2 + dy^2 + dz^2 = dv^2 + \frac{HHdt^2}{vv}.$$

125. **Coroll. 3.** Si illae aequationes differentiales combinentur cum hac

$$xdx + ydy + zdz = vdv,$$

differentialia dx, dy et dz inde ita definiuntur, ut sit

$$dx = \frac{xdv}{v} + \frac{(Ey - Gz)\,dt}{vv}, \quad dy = \frac{ydv}{v} + \frac{(Fz - Ex)\,dt}{vv}, \quad dz = \frac{zdv}{v} + \frac{(Gx - Fy)\,dt}{vv}.$$

126. **Coroll. 4.** Cum autem sit $Ez + Gy = -Fx$, si ponamus $Ey - Gz = p$, erit quadratis tis

$$(EE + GG)(yy + zz) = (EE + GG)(vv - xx) = FFxx + pp,$$

que, ob $FF + GG + EE = HH$, erit $p = Ey - Gz = \sqrt{((EE + GG)vv - HHxx)}$, hincque

$$\frac{vdx - xdv}{vv} = d \cdot \frac{x}{v} = \frac{dt}{vv}\sqrt{\left(EE + GG - \frac{HHxx}{vv}\right)}.$$

127. **Scholion.** Hae investigationes non solum inserviunt motui absoluto definiendo, etsi in astronomia parum interest eum nosse, sed imprimis eas ideo hic attulimus, ut intelligatur, quomodo hujusmodi solutiones, ubi plures aequationes differentio-differentiales occurrunt, tractari conveniat. Cum enim in sequentibus omnia a resolutione talium aequationum pendeant, in hujusmodi maxime juvabit vires analyseos exercuisse, unde haec tractatio utilitate non caritura videtur. ergo motu duorum corporum sphaericorum, quod quidem argumentum jam passim satis est pertractatum, antequam casum trium corporum aggrediamur, in motum duorum corporum non sphaericorum inquiramus, ut pateat, quantum discrimen a defectu sphaericitatis proficiscatur. Cum enim tam solis quam planetarum corpora a figura sphaerica recedant, jam ob hanc causam irregularitates quaedam se motui, qui per regulas consuetas in hypothesi corporum sphaericorum determinatur, admiscebunt, quarum cognitio eo magis est necessaria, ne phaenomena actioni aliorum corporum tribuantur. Hic vero alteri tantum corpori figuram a sphaerica diversam assignabimus, alterum perfecte sphaericum relinquentes; si enim ambo non fuerint sphaerica, primo alterum tanquam sphaericum spectetur, tum vero alterum, quo facto ex combinatione phaenomenorum solutio haud difficulter colligetur, praecipue cum viderimus a defectu figurae sphaericae motus parum perturbari.