XVI.

Tria Capita ex Opere quodam majori inedito de theoria lunae.

Caput ....

De loco lunae ex eclipsibus lunaribus determinando.

§ 1. Quo ex formulis hactenus inventis, quibus motus lunae continetur, ad quodvis tempus verus lunae locus in coelo definiri possit, primum aliquot ejus loca cognita esse oportet, ut deinceps tam longitudo media lunae quam ejus anomalia et locus nodi ad illa loca accommodari, ac praeterea vera excentricitas lunae exacte determinari possit. Si enim luna in motu suo eas sequatur regulas, quas ex theoria elicuimus, certum est, si formulae inventae ad aliquot loca observata accommodentur, eas perpetuo cum observationibus congruere debere. Ad hoc ergo institutum ejusmodi observationes eligi conveniet, ex quibus verus lunae locus geocentricus accurate concludi queat, ut nequidem ejus parallaxi sit opus, quippe quae postquam theoria penitus fuerit confirmata, demum exacte assignari poterit. Nullius ergo etiamnunc erunt usus neque observationes culminationis lunae, neque occultationes stellarum fixarum, quoniam ex iis sine cognita parallaxi verum lunae locum concludere non licet.

§ 2. Ad praesentem ergo scopum observationes eclipsium lunae sine dubio erunt aptissimae, quae in ubivis terrarum, ubi quidem lunam conspicere licet, eaedem apparent, neque a varietate parallaxeos inquinantur. Eveniunt autem hujusmodi eclipses circa oppositionem lunae et solis, in iisque datur temporis momentum, quo longitudo lunae e diametro opponitur longitudini solis: quod momentum si esset cognitum, quoniam pro eo locum solis definire liceret, hinc facillime vera longitudo lunae concludi posset. Verum hoc ipsum momentum verae oppositionis in observatione non ita distincte exprimitur, sed demum ex comparatione reliquarum circumstantiarum non obvio ratiocinio concludi potest. Quae enim in quavis eclipsi lunae attenta observatione distinguere licet, sunt ejus initium et finis; tum, si eclipsis fuerit totalis, momentum immersionis integrae et initium emersionis.

Quandoque etiam phases seu portiones obscuratas satis exacte dimetiri licet: verumtamen hujusmodi observationibus plerumque minor fides adhiberi potest, quia umbra non satis distincte terminatur.

§ 3. Repraesentet in superficie sphaerica (Fig. 205.) circulus ☊Ss eclipticam, et circulus ☊L orbitam lunae. Ponamus initio eclipsis centrum umbrae, seu punctum soli oppositum esse in S, centrum lunae vero in L; in fine autem eclipsis, centrum umbrae versari in s, centrum lunae vero in l. Ductis igitur arcubus SL et sl erit uterque aequalis summae semidiametrorum apparentium umbrae et lunae: ac propterea $SL = sl$. Dum enim eclipsis durat, tuto assumere licet, neque diametrum umbrae, neque diametrum lunae apparentem ullam mutationem pati: etiamsi enim revera in utroque quaepiam variatio contingere possit, tamen ea erit tam parva, ut ob reliquos leves errores, quae in observationibus evitari omnino nequeunt, attendi non mereatur. Deinde quoque consideramus locum nodi ☊ tanquam fixum, non quasi ejus motum negligeremus, sed quoniam promotiones tam solis quam lunae non veras sed relativas respectu nodi in calculum introducemus. Denique etiam durante eclipsi tam motum lunae quam solis uniformem statuemus, quae enim inaequalitas in motu lunae spatio aliquot horarum inesse potest, ea uti non ultra aliquot minuta assurgit, in hoc negotio erit imperceptibilis.

§ 4. Sit igitur tempore eclipsis semidiameter umbrae $= \alpha$, qui aequatur, uti constat, summae parallaxium lunae et solis, demto semidiametro apparente solis. Sit semidiameter lunae apparens $= \beta$, eritque tam pro initio quam pro fine eclipsis arcus $SL = sl = \alpha + \beta$. Sin autem in SL fuerit initium immersionis totius lunae in umbram, et in sl initium emersionis, erit quoque $SL = sl$ verum tum habebitur $SL = sl = \alpha - \beta$. Sive ergo cujuspiam eclipsis lunaris observetur initium et finis, sive immersio et emersio, siquidem fuerit totalis, utroque casu erit $SL = sl$, haecque aequalitas sufficit ad locum quemdam lunae verum eliciendum, etiamsi ipsi arcus SL et sl non sint cogniti.

§ 5. Ponamus ab initio eclipsis ad finem effluxisse h horas, seu ab immersione usque ad emersionem, siquidem hujusmodi observationibus uti velimus. Sit vero tempore eclipsis motus horarius solis $= m''$, et motus horarius lunae $= n''$, motus autem horarius nodi in antecedentia $= k''$, qui motus ex theoria lunae jam satis prope cognita sunt colligendi, etiamsi enim theoria aliquantillum a veritate discrepet, tamen discrimen, quod inde in motum horarium redundare potest, nullius prorsus erit momenti. Cum igitur hi motus sint uniformes saltem durante eclipsi, erit a nodo ☊ computando spatium $Ss = h(m'' + k'')$ et spatium $Ll = h(n'' + k'')$. Sin autem elapsis ab initio, cum centra umbrae et lunae erant in S et L, t horis, centrum umbrae sit in σ, et centrum lunae in λ, erit arcus $S\sigma = (m + k)\, t''$ et $L\lambda = (n + k)\, t''$.

§ 6. Si jam elapsis ab initio t horis vera luminarium oppositio contingat, erit arcus $\lambda\sigma$ perpendicularis in eclipticam ☊Ss, eoque momento erit longitudo lunae e diametro opposita longitudini solis. Verum ne reductione loci lunae ad eclipticam opus sit, expediet id temporis momentum investigasse, quo arcus $\sigma\lambda$ tam ab ecliptica, quam ab orbita lunae aequales arcus abscindat, ita ut sit ☊σ = ☊λ. Hoc enim momentum si fuerit cognitum, longitudo lunae in propria orbita aequalis esse debebit longitudini puncti soli oppositi. Pro qualibet scilicet eclipsi lunae id temporis momentum determinabimus, quo longitudo lunae in orbita exacte fit aequalis longitudini umbrae; hoc enim cognito, quia ex theoria solis locus centri umbrae constat, statim eum habebimus locum lunae orbita, quem tabulae lunares indicare debent, neque ad hoc reductione loci lunae ad eclipticam opus habebimus, uti vera oppositio postulat.

§ 7. Repraesentet ergo arcus $\sigma\lambda$ non veram oppositionem, sed eum luminarium situm, in quo arcus $\text{☊}\lambda$ aequalis sit arcui $\text{☊}\sigma$, hocque eveniat elapsis post initium SL horis t. Sit ergo hoc momento tam longitudo umbrae, quam longitudo lunae in orbita a nodo computata $\text{☊}\sigma = \text{☊}\lambda = x$, porro ob $S\sigma = (m+k)\,t''$, $L\lambda = (n+k)\,t''$, arcus $\text{☊}S = x - (m+k)\,t''$ et arcus $\text{☊}L = x - (n+k)\,t''$. Deinde vero erit arcus $\text{☊}s = x + (m+k)\,(h-t)''$ et $\text{☊}l = x + (n+k)\,(h-t)''$. Ponatur angulus $\lambda\text{☊}\sigma = \varrho$, erit ex natura triangulorum sphaericorum

$$\cos SL = \cos\varrho \sin\text{☊}L \sin\text{☊}S + \cos\text{☊}L \cos\text{☊}S,\quad \cos sl = \cos\varrho \sin\text{☊}l \sin\text{☊}s + \cos\text{☊}l \cos\text{☊}s.$$

Quare cum sit $SL = sl$, erit

$$\cos\text{☊}L \cos\text{☊}S - \cos\text{☊}l \cos\text{☊}s = \cos\varrho\,(\sin\text{☊}l \sin\text{☊}s - \sin\text{☊}L \sin\text{☊}S).$$

§ 8. Est vero per compositionem angulorum ut sequitur

$$\begin{aligned}
\cos\text{☊}L &= \cos x \cos(n+k)\,t'' + \sin x \sin(n+k)\,t''\\
\cos\text{☊}S &= \cos x \cos(m+k)\,t'' + \sin x \sin(m+k)\,t''\\
\cos\text{☊}l &= \cos x \cos(n+k)\,(h-t)'' - \sin x \sin(n+k)\,(h-t)''\\
\cos\text{☊}s &= \cos x \cos(m+k)\,(h-t)'' - \sin x \sin(m+k)\,(h-t)''\\
\sin\text{☊}l &= \sin x \cos(n+k)\,(h-t)'' + \cos x \sin(n+k)\,(h-t)''\\
\sin\text{☊}s &= \sin x \cos(m+k)\,(h-t)'' + \cos x \sin(m+k)\,(h-t)''\\
\sin\text{☊}L &= \sin x \cos(n+k)\,t'' - \cos x \sin(n+k)\,t''\\
\sin\text{☊}S &= \sin x \cos(m+k)\,t'' - \cos x \sin(m+k)\,t''
\end{aligned}$$

seu per alias sinuum proprietates

$$\begin{aligned}
\cos\text{☊}L \cos\text{☊}S &= \tfrac{1}{2}\cos(\text{☊}S - \text{☊}L) + \tfrac{1}{2}\cos(\text{☊}S + \text{☊}L)\\
\cos\text{☊}l \cos\text{☊}s &= \tfrac{1}{2}\cos(\text{☊}l - \text{☊}s) + \tfrac{1}{2}\cos(\text{☊}s + \text{☊}l)\\
\sin\text{☊}l \sin\text{☊}s &= \tfrac{1}{2}\cos(\text{☊}l - \text{☊}s) - \tfrac{1}{2}\cos(\text{☊}s + \text{☊}l)\\
\sin\text{☊}L \sin\text{☊}S &= \tfrac{1}{2}\cos(\text{☊}S - \text{☊}L) - \tfrac{1}{2}\cos(\text{☊}S + \text{☊}L).
\end{aligned}$$

§ 9. Cum igitur sit

$$\text{☊}S - \text{☊}L = (n-m)\,t'' \qquad \text{☊}S + \text{☊}L = 2x - (m+n+2k)\,t''$$

$$\text{☊}l - \text{☊}s = (n-m)\,(h-t)'' \qquad \text{☊}s + \text{☊}l = 2x + (m+n+2k)\,(h-t)''.$$

His ergo valoribus substitutis reperiemus

$$\left.\begin{array}{l}+\cos(n-m)t'' + \cos(2x-(m+n+2k)\,t'')\\ -\cos(n-m)(h-t)'' - \cos(2x+(m+n+2k)(h-t)'')\end{array}\right\} = \cos\varrho\left\{\begin{array}{l}\cos(n-m)(h-t)'' - \cos(2x+(m+n+2k)(h-t)'')\\ -\cos(n-m)t'' + \cos(2x-(m+n+2k)\,t'')\end{array}\right.$$

quae reducta abit in

$$(1+\cos\varrho)\big(\cos(n-m)t'' - \cos(n-m)(h-t)''\big) = (1-\cos\varrho)\big(\cos(2x+(m+n+2k)(h-t)'') - \cos(2x-(m+n+2k)\,t'')\big).$$

At cum sit

$$\cos(2x+(m+n+2k)(h-t)'') = \cos 2x \cos(m+n+2k)(h-t)'' - \sin 2x \sin(m+n+2k \ldots$$

$$\cos(2x-(m+n+2k)t'') = \cos 2x \cos(m+n+2k)t'' + \sin 2x \sin(m+n+2k)t''$$

quo calculus ad sinus et cosinus angulorum satis parvorum reducitur: qui anguli cum dentur in minutis secundis, ii multiplicentur per numerum $g = 0{,}0000048481$, seu $lg = 4{,}6855749$, ut reducantur ad partes radii, qui ponitur $= 1$; eritque sinibus et cosinibus horum angulorum per series convergentes expressis

$$(1+\cos\varrho)\left(1-\tfrac{1}{2}ggtt(n-m)^2 - 1 + \tfrac{1}{2}gg(h-t)^2(n-m)^2\right) =$$

$$(1-\cos\varrho)\begin{cases} +\cos 2x\left(1-\tfrac{1}{2}gg(h-t)^2(m+n+2k)^2\right) - \sin 2x\left(g(h-t)(m+n+2k)\right) \\ -\cos 2x\left(1-\tfrac{1}{2}ggtt(m+n+2k)^2\right) \qquad - \sin 2x \cdot gt(m+n+2k) \end{cases}$$

$$= (1-\cos\varrho)\left(\tfrac{1}{2}gg(m+n+2k)^2(2ht-hh)\cos 2x - gh(m+n+2k)\sin 2x\right)$$

§ 10. Hac ergo aequatione debite tractata reperiemus

$$(1+\cos\varrho)\cdot\tfrac{1}{2}gg(n-m)^2(hh-2ht) = (1-\cos\varrho)\cdot gh(m+n+2k)\left(\tfrac{1}{2}g(m+n+2k)(2t-h)\cos 2x - \sin 2x\right)$$

seu $g(h-2t)(n-m)^2 = \mathrm{tang}^2\tfrac{1}{2}\varrho\,(m+n+2k)\left(g(m+n+2k)(2t-h)\cos 2x - 2\sin 2x\right)$

vel hoc modo

$$\left.\begin{matrix} gh(n-m)^2 + gh(m+n+2k)^2\cos 2x\,\mathrm{tang}^2\tfrac{1}{2}\varrho \\ +2(m+n+2k)\sin 2x\,\mathrm{tang}^2\tfrac{1}{2}\varrho \end{matrix}\right\} = 2gt(n-m)^2 + 2gt(m+n+2k)^2\cos 2x\,\mathrm{tang}^2\tfrac{1}{2}\varrho$$

Hinc ergo eruetur

$$2t - h = \frac{2(m+n+2k)\sin 2x\,\mathrm{tang}^2\frac{1}{2}\varrho}{g(m+n+2k)^2\cos 2x\,\mathrm{tang}^2\frac{1}{2}\varrho + g(n-m)^2}$$

et ob $\mathrm{tang}^2\frac{1}{2}\varrho$ tantopere parvum, erit

$$t = \tfrac{1}{2}h + \frac{(m+n+2k)\sin 2x\,\mathrm{tang}^2\frac{1}{2}\varrho}{g(n-m)^2} - \frac{(m+n+2k)^3\sin 2x\cos 2x\,\mathrm{tang}^4\frac{1}{2}\varrho}{g(n-m)^4}$$

ubi ultimus terminus ob summam parvitatem facile negligitur.

§ 11. In hac aequatione terminus $\frac{1}{2}h$ designat medium totius eclipsis momentum, quod ex observationibus vel initii et finis eclipsis, vel immersionis et emersionis facile concluditur. Momentum ergo, quo longitudo lunae in orbita et longitudo umbrae in ecliptica inter se fiunt aequales, post medium eclipsis incidit elapsis horis $\frac{(m+n+2k)\sin 2x\,\mathrm{tang}^2\frac{1}{2}\varrho}{g(n-m)^2}$. Quamobrem si pro eclipsis momento medio computemus et longitudinem lunae in orbita, quae sit $= L$, et longitudinem umbrae seu oppositionem solis, quae sit $= U$, ex motu horario erit illo altero momento, quo utraque longitudo fit aequalis

$$\text{longitudo umbrae} = U + \frac{m(m+n+2k)\sin 2x\,\mathrm{tang}^2\frac{1}{2}\varrho}{g(n-m)^2}$$

$$\text{longitudo lunae} = L + \frac{n(m+n+2k)\sin 2x\,\mathrm{tang}^2\frac{1}{2}\varrho}{g(n-m)^2}$$

ob aequalitatem concluditur

longitudo lunae $L = U - \frac{(m+n+2k)\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho}{g\,(n-m)}$ sec.

§ 12. Hoc igitur jam sumus consecuti, ut pro eclipsis momento medio veram lunae longitudinem in orbita assignare valeamus. Pro isto scilicet momento quaeri debet longitudo solis vera, quae sex signis vel aucta vel minuta dabit longitudinem umbrae U: ab hac porro subtrahantur $\frac{(m+n+2k)\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho}{g\,(n-m)}$ minuta secunda *), atque remanebit longitudo lunae in orbita tempore medio eclipsis. Ad hoc ergo nosse oportet arcum x, qui habetur si longitudo nodi ascendentis a longitudine centri umbrae in media eclipsi subtrahatur: quamquam autem ob theoriam nondum satis perfectam, longitudo nodi nondum exactissime est cognita; tamen hinc ista determinatio non turbabitur, quoniam sufficit locum nodi proxime saltem nosse, dum error aliquot minutorum in arcu x commissus nullum sensibilem errorem in valore $\frac{(m+n+2k)\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho}{g\,(n-m)}$ gignit.

§ 13. Simili modo ex tabulis lunaribus adhuc constructis satis exacte habetur angulus inclinationis $☊ = \varrho$, ut minimus error in eo commissus nihili sit aestimandus. Quamobrem pro quovis angulo x valor $\frac{1}{g}\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho$ satis exacte assignari et in minutis secundis exprimi poterit, quem deinceps per $\frac{m+n+2k}{n-m} = 1 + \frac{2\,(m+k)}{n-m}$ multiplicari oportet, ut particula a longitudine umbrae subtrahenda obtineatur. Neglecto autem isto multiplicatore $\frac{m+n+2k}{n-m}$, valor alterius $\frac{1}{g}\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho$ ita se habebit, ut sequens tabella exhibet:

Tabula aequationum

pro vera lunae longitudine in orbita, tempore medio eclipsis lunaris invenienda.

Arg. Subtrahatur longitudo nodi a longitudine umbrae, et aequatio, quam sequens tabella exhibet, applicetur ad locum soli oppositum.

Grad.	○ Sign. VI Sign. Subtrahe.		Inclinatio orbitae ☽ ad eclipticam.
0	0′ 0″	30	5° 17′ 1″
1	0 15,3	29	5 17 0
2	0 30,6	28	5 16 59
3	0 45,9	27	5 16 58
4	1 1,1	26	5 16 56
5	1 16,2	25	5 16 53
6	1 31,2	24	5 16 50
7	1 46,1	23	5 16 45
8	2 0,8	22	5 16 40
9	2 15,3	21	5 16 35
10	2 29,7	20	5 16 29
Adde	V sign. XI sign.	Grad.	

Scripturae in margine. Hae autem aequationes insuper multiplicari debent per factorem $1 + \frac{2\,(m+k)}{n-m}$ ex motu horario inveniendum.

Multiplicatae per $2\frac{1}{6}$ dant correctionem pro longitudine lunae in ecliptica; multiplicatae autem per $\frac{2n+2k}{n-m}$ dabunt correctionem pro longitudine vera in ecliptica.

$n = 2023'' - 258''\cos\varphi$, $m = 148''$, $k = 8$

si $\cos\varphi = 0$, $\frac{4062}{1875}$; si $\cos\varphi = 1$, $\frac{3546}{1875}$;

si $\cos\varrho = -1$, $\frac{4578}{1875}$.

Scriptura ad marg. et si a loco lunae in orbita subtrahatur part. $\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho$ habebitur longitudo vera in ecliptica.

§ 14. Cum autem tempore medio eclipsis luna saepe nimis distet a vera oppositione, quum aequationes ab angulo η pendentes negligi possint, quo tamen calculus non mediocriter contra- expediet tam locum solis quam lunae non pro momento eclipsis medio, sed pro eo temporis momento quo longitudo lunae in orbita aequalis fit longitudini umbrae in ecliptica, supputare. Hoc momentum obtinebitur, si ad tempus eclipsis medium addantur $\frac{m+n+2k}{g(n-m)^2}\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho$ seu $\frac{3600(m+n+2k)}{g(n-m)^2}\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho$ minuta secunda temporis. Seorsim ergo evolvamus factorem $\frac{m+n+2k}{(n-m)^2}$, quippe qui ex motibus horariis determinatur, et alter factor $\frac{3600}{g}\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho$ in minutis secundis ex sequenti tabella cognoscetur.

Tabula ostendens temporis momentum,

quo longitudo lunae in orbita aequatur longitudini umbrae in ecliptica.

Arg. Subtrahatur locus nodi a longitudine solis, et aequatio hujus tabulae applicetur momento eclipsis medio.

0, VI Sign.

Grad.	Adde.	Logarithm.	
0	00000″	$-\infty$	30
1	55166	4,7416734	29
2	110254	5,0423929	28
3	165195	5,2179972	27
4	219900	5,3422263	26
5	274294	5,4382170	25
6	328311	5,5162854	24
7	381810	5,5818479	23
8	434787	5,6382771	22
9	487183	5,6876924	21
10	538930	5,7315327	20
	Subtrahe.	Logarithm.	Grad.

V, XI Sign.

Aequatio autem ex hac tabula in minutis secundis inventa multiplicari debet per $\frac{n+m+2k}{(n-m)^2}$ expressis motibus horariis n, m et k in minutis secundis; sicque obtinebitur tempus, quo tabulae eandem lunae longitudinem in orbita indicare debent, quae reperitur pro longitudine umbrae in ecliptica.

§ 15. Interim tamen tuto uti poterimus methodo priori, qua ad eclipsis momentum medium vera longitudo lunae in sua orbita colligitur. Hoc enim tempore angulus η seu ejus complementum ad 180° vix unquam ad 4′ exsurget, atque inaequalitas in motu lunae inde oriunda non ultra ascendet, qui error omnino erit contemnendus si perpendamus in determinatione momenti medii cujus vis eclipsis errorem unius minuti primi vix ac ne vix quidem evitari posse, unde in locum lunae error semissis minuti secundi redundat. Quamobrem superfluum foret nulliusque plane usus, si tantum in aliquot minutis secundis consistunt, nimis anxii esse vellemus. Praeterea lunae determinatione commode evenit, ut tempus durationis eclipsis, quo luna arcum nempe littera h ex calculo excesserit; sicque patet eandem operationem locum habere, momentum medium collectum sit ex initio et fine eclipsis, sive ex immersione et emersione, ex aequalibus lunae phasibus, quibus arcus LS et ls inter se sint aequales. Omnes hujusmodi observationes, si exacte instituantur, idem eclipsis momentum medium praebere debent.

§ 16. Tempus autem, quo arcus Ll et Ss percurruntur, una cum arcuum LS et ls magnitudine, inserviet loco nodi propius cognoscendo, sicque tabula nodorum lunae inde emendari poterit, correctione indigeat. Cum enim arcus SL et sl sint aequales, et ex diametris apparentibus et lunae satis exacte dentur, sit uterque arcus $SL = sl = f$. Tum vero sit in medio eclipsis centrum umbrae in σ, et centrum lunae in λ, ac ponantur arcus $☊\sigma = x$, et $☊\lambda = y$, erit $x - y = U - L = \frac{(m + n + 2k) \sin 2x \operatorname{tang}^2 \frac{1}{2} \varrho}{g(n - m)}$, hocque ergo discrimen, ob $\sin 2x$ et $\operatorname{tang} \frac{1}{2} \varrho$ proxime cognitos, cum sit minimum, pro dato haberi poterit. Sit intervallum temporis ab L ad $l = h$ horarum, eruntque arcus $L\lambda = l\lambda = \frac{1}{2} h (n + k)''$ et $S\sigma = \sigma s = \frac{1}{2} h (m + k)''$. Ideoque habebitur

$$☊L = y - \tfrac{1}{2} h (n + k)'' = x - \tfrac{1}{2} h (n + k)'' - \frac{(m + n + 2k) \sin 2x \operatorname{tang}^2 \frac{1}{2} \varrho}{g(n - m)} \text{ et } ☊S = x - \tfrac{1}{2} h (m + k)''.$$

§ 17. Ponamus brevitatis gratia

$$\tfrac{1}{2} h (n + k) = a, \quad \frac{(m + n + 2k) \sin 2x \operatorname{tang}^2 \frac{1}{2} \varrho}{g(n - m)} = c \quad \text{et} \quad \tfrac{1}{2} h (m + k) = b$$

ut sint in triangulo sphaerico $S☊L$ latera $☊L = x - a - c$, $☊S = x - b$, $SL = f$ et ang. $☊ = \varrho$; unde reperietur

$$\cos f = \cos \varrho \sin (x - a - c) \sin (x - b) + \cos (x - a - c) \cos (x - b)$$

atque eruetur

$$\cos (2x - a - b - c) = \frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos (a - b + c)}{\sin^2 \frac{1}{2} \varrho}.$$

Ex hac autem aequatione, quoniam levis error in angulo ϱ commissus fit admodum notabilis, anguli $2x - a - b - c$ non satis exacte inveniri potest. Cum igitur, si triangulum $l☊s$ consideretur, ad hanc perveniatur aequationem

$$\cos (2x + a + b - c) = \frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos (a - b - c)}{\sin^2 \frac{1}{2} \varrho}$$

illam aequationem per hanc dividendo

$$\frac{\cos (a + b) + \sin (a + b) \operatorname{tang} (2x - c)}{\cos (a + b) - \sin (a + b) \operatorname{tang} (2x - c)} = \frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos (a - b + c)}{\cos f - \cos^2 \frac{1}{2} \varrho \cos (a - b - c)}$$

$$\text{seu } \operatorname{tang} (2x - c) = \frac{\sin (a - b) \cos (a + b) \sin c \cos^2 \frac{1}{2} \varrho}{\sin (a + b) \cos f - \sin (a + b) \cos (a - b) \cos c \cos^2 \frac{1}{2} \varrho}$$

§ 18. Quamquam autem hic error in angulo ϱ in cosinu$^2 \frac{1}{2} \varrho$ fit plane imperceptibilis, tamen minimus error in angulo c commissus inventionem anguli $2x - c$ nimis incertam reddit, ita ut ex his observationibus solis locus nodi exacte definiri nequeat. Hancobrem in subsidium [illegible] debent aliae observationes, veluti si in eclipsi totali, praeter initium et finem ejus, quoque [illegible] et emersio observetur. Si enim tempus ab initio eclipsis ad finem elapsum sit $= h$ horarum [illegible] semidiameter apparens umbrae $= \alpha$, ac semidiameter lunae apparens $= \beta$, erit $f = $ [illegible] immersione autem et emersione erit arcus $SL = sl = \alpha - \beta$, qui ponatur $= F$; tempus [illegible] immersione ad emersionem elapsum sit $= H$ horarum. Quodsi ergo ponatur $\frac{1}{2} H(n$ [illegible] $\frac{1}{2} H(m + k) = B$, manente $\frac{(m + n + 2k) \sin 2x \tang^2 \frac{1}{2} \varrho}{g(n - m)} = c$, reperietur simili modo

$$\cos(2x - A - B - c) = \frac{\cos F - \cos^2 \frac{1}{2} \varrho \cos(A - B + c)}{\sin^2 \frac{1}{2} \varrho}.$$

§ 19. Si jam aequatio prius inventa

$$\cos(2x - a - b - c) = \frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos(a - b + c)}{\sin^2 \frac{1}{2} \varrho}$$

per istam dividatur, reperietur

$$\frac{\cos(2x - a - b - c)}{\cos(2x - A - B - c)} = \frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos(a - b + c)}{\cos F - \cos^2 \frac{1}{2} \varrho \cos(A - B + c)},$$

in quam valor $\sin^2 \frac{1}{2} \varrho$ non amplius ingreditur. Etsi autem in ea etiamnunc terminus $\cos^2 \frac{1}{2} \varrho$ [illegible] tamen levis incertitudo in angulo ϱ, cum ipse angulus sit valde parvus, valorem termini $\cos^2$ [illegible] non sensibiliter afficit, atque errores hinc oriundi in numeratore et denominatore fere se [illegible] compensabunt. Aequatio autem inventa resolvetur in sequentem

$$\frac{\cos(a + b + c) + \sin(a + b + c) \tang 2x}{\cos(A + B + c) + \sin(A + B + c) \tang 2x} = \frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos(a - b + c)}{\cos F - \cos^2 \frac{1}{2} \varrho \cos(A - B + c)}$$

ex qua valor $\tang 2x$, ac proinde ipse arcus ☊σ, quo nodus a loco centri umbrae medio eclipsis momento distat, assignari poterit. Sit enim brevitatis gratia

$$\frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos(a - b + c)}{\cos F - \cos^2 \frac{1}{2} \varrho \cos(A - B + c)} = d$$

$$\text{erit } \tang 2x = \frac{-\cos(a + b + c) + d \cos(A + B + c)}{-d \sin(A + B + c) + \sin(a + b + c)}.$$

§ 20. Invento loco nodi seu distantia ☊$\sigma = x$, existente σ loco centri umbrae momento medio eclipsis, inde porro inclinatio orbitae lunae ad eclipticam, seu angulus ☊ $= \varrho$, accuratius [illegible] poterit. Cum enim iste angulus ϱ jam tam prope sit cognitus, ut terminus $\cos^2 \frac{1}{2} \varrho$ minime [illegible] tate discrepet, erit

$$\sin^2 \frac{1}{2} \varrho = \frac{\cos f - \cos^2 \frac{1}{2} \varrho \cos(a - b + c)}{\cos(2x - a - b - c)}, \quad \text{seu} \quad \cos \varrho = \frac{\cos f - \cos(x - a - c) \cos(x - b)}{\sin(x - a - c) \sin(x - b)}$$

sive etiam $\sin^2 \frac{1}{2} \varrho = \frac{\cos(a - b + c) - \cos f}{2 \sin(x - a - c) \sin(x - b)} = \frac{\cos(a - b + c) - \cos f}{\cos(a - b + c) - \cos(2x - a - b - c)} = \frac{\sin\left(\frac{f + a - b + c}{2}\right) \sin\left(\frac{f - a + b - c}{2}\right)}{\sin(x - a - c) \sin(x - b)}$

…autem formula vicissim ipsa distantia x ita definiri poterit, ut angulus ϱ plane non ingre…quatenus particula minima c ab eo pendet. Cum enim sit simili modo

$$\sin^2 \tfrac{1}{2}\varrho = \frac{\cos(A-B+c)-\cos F}{\cos(A-B+c)-\cos(2x-A-B-c)}$$

erit $\dfrac{\cos(A-B+c)-\cos(2x-A-B-c)}{\cos(A-B+c)-\cos F} = \dfrac{\cos(a-b+c)-\cos(2x-a-b-c)}{\cos(a-b+c)-\cos f}$.

…hac autem aequatione difficilius valor ipsius x eruitur, unde methodo ante tradita potius uti conveniet.

§. 21. Qui autem laborem suscipere velit, atque ex aequationibus successive angulum ϱ et particulam c eliminare, is tandem sequentem reperiet aequationem

$$\tan 2x = \frac{(\cos F-\cos(A-B))(\cos f-\cos(a+b))\sin(A+B)\sin(a-b)-(\cos f-\cos(a-b))(\cos F-\cos(A+B))\sin(a+b)\sin(A-B)}{(\cos f-\cos(a-b))(\cos F+\cos(A+B))\sin(a+b)\sin(A-B)-(\cos F-\cos(A-B))(\cos f+\cos(a+b))\sin(A+B)\sin(a-b)}.$$

Inventa autem distantia x erit porro

$$c = \frac{(\cos f-\cos(a-b))\sin(a+b)\sin 2x}{\cos f\sin(a-b)+\sin 2b\cos 2x-\cos f\sin(a+b)\cos 2x}$$

$$\text{et } \tan^2 \tfrac{1}{2}\varrho = \frac{\cos(a-b)-\cos f-c\sin(a-b)}{\cos f-\cos(a+b)\cos 2x+c\sin(a+b)\cos 2x-\sin(a+b)\sin 2x-c\cos(a+b)\sin 2x}$$

$$\text{seu } \tan^2 \tfrac{1}{2}\varrho = \frac{\cos(a-b)-c\sin(a-b)-\cos f}{\cos f-\cos(2x-a-b)-c\sin(2x-a-b)}.$$

Sicque ex observationibus initii et finis eclipsis, itemque immersionis et emersionis in eclipsi totali, tam locus nodi quam inclinatio orbitae lunae ad eclipticam definiri poterunt. Interim tamen verendum est, ne istae formulae complicatae, ob minimos etiam errores in observationibus commissos, nimium a veritate seducant, ac fortasse saepe tutius erit formulis ante inventis uti.

§. 22. Non difficile hinc erit momentum quoque verae oppositionis solis ac lunae assignare, quo scilicet longitudo centri lunae congruat cum longitudine umbrae. Accidat enim vera oppositio z horis post medium eclipsis momentum; et cum in medio eclipsis esset longitudo centri umbrae a nodo

$$☊\sigma = x \quad\text{et}\quad ☊\lambda = x - \frac{(m+n+2k)\sin 2x\tan^2\frac{1}{2}\varrho}{g(n-m)},$$

erit momento verae oppositionis longitudo centri umbrae a nodo $= x+z(m+k)''$ et distantia lunae a nodo

$$= x+z(n+k)'' - \frac{(m+n+2k)\sin 2x\tan^2\frac{1}{2}\varrho}{g(n-m)}.$$

Sit brevitatis gratia

$$c = \frac{(m+n+2k)\sin 2x\tan^2\frac{1}{2}\varrho}{g(n-m)},$$

erit ex conditione verae oppositionis

$$\cos\varrho = \frac{\tan(x+z(m+k)'')}{\tan(x-c+z(n+k)'')} = \frac{(\tan x+\tan z(m+k)'') : (1-\tan x\tan z(m+k)'')}{(\tan x+\tan(z(n+k)''-c)) : (1-\tan x\tan(z(n+k)''-c))}.$$

Cum autem anguli c, $z(m+k)$, et $z(n+k)$ sint minimi, eorum tangentes ipsis angulis radii $=1$ conversis aequantur, quae conversio fit angulos per $g = 0{,}0000048481$ multipli... unde fiet

$$\frac{\cos\varrho\,(\text{tang}\,x + gz(n+k) - cg)}{1 - g(z(n+k) - c)\,\text{tang}\,x} = \frac{\text{tang}\,x + gz(m+k)}{1 - gz(m+k)\,\text{tang}\,x}$$

seu
$$\cos\varrho\,\text{tang}\,x + gz(n+k)\cos\varrho - cg\cos\varrho - gz(m+k)\cos\varrho\,\text{tang}^2 x =$$
$$\text{tang}\,x + gz(m+k) - g(z(n+k) - c)\,\text{tang}^2 x,$$

hincque eruitur

$$gz = \frac{(1-\cos\rho)\,\text{tang}\,x + gc\,\text{tang}^2 x + gc\cos\rho}{(n+k)\cos\rho - (m+k)\cos\rho\,\text{tang}^2 x - (m+k) + (n+k)\,\text{tang}^2 x}$$

$$\text{seu}\quad gz = \frac{(1-\cos\rho)\sin x\cos x + gc\sin^2 x + gc\cos\rho\cos^2 x}{(n+k)\cos\rho\cos^2 x - (m+k)\cos\rho\sin^2 x - (m+k)\cos^2 x + (n+k)\sin^2 x}$$

§ 23. Quia vero est $\sin x\cos x = \frac{1}{2}\sin 2x$, $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ et $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

erit
$$gz = \frac{(1-\cos\rho)\sin 2x + gc - gc\cos 2x + gc\cos\rho + gc\cos\rho\cos 2x}{(n+k)(1+\cos\rho) - (n+k)\cos 2x(1-\cos\rho) - (m+k)(1+\cos\rho) - (m+k)\cos 2x(1-\cos\rho)}$$

seu ob $1-\cos\varrho = 2\sin^2\frac{1}{2}\varrho$ et $1+\cos\varrho = 2\cos^2\frac{1}{2}\varrho$ habebitur

$$gz = \frac{\sin 2x\,\text{tang}^2\frac{1}{2}\rho + gc - gc\cos 2x\,\text{tang}^2\frac{1}{2}\rho}{n - m - (m+n+2k)\cos 2x\,\text{tang}^2\frac{1}{2}\rho}.$$

Cum autem sit
$$gc = \frac{(m+n+2k)\sin 2x\,\text{tang}^2\frac{1}{2}\rho}{n-m},\ \text{erit}$$

$$gz = \frac{2(n+k)\sin 2x\,\text{tang}^2\frac{1}{2}\rho - (m+n+2k)\sin 2x\cos 2x\,\text{tang}^4\frac{1}{2}\rho}{(n-m)^2 - (n-m)(m+n+2k)\cos 2x\,\text{tang}^2\frac{1}{2}\rho}$$

ideoque proxime $gz =$

$$\frac{2(n+k)}{(n-m)^2}\sin 2x\,\text{tang}^2\tfrac{1}{2}\varrho - \frac{(m+n+2k)}{(n-m)^2}\sin 2x\cos 2x\,\text{tang}^4\tfrac{1}{2}\varrho + \frac{2(n+k)(m+n+2k)}{(n-m)^3}\sin 2x\cos 2x\,\text{tang}\ldots$$

ob terminos autem posteriores minimos erit

$$z = \frac{2(n+k)}{g(n-m)^2}\sin 2x\,\text{tang}^2\tfrac{1}{2}\varrho$$

qui numerus indicabit partem horae ad momentum eclipsis medium addendam, ut prodeat momentum verae oppositionis. Vel ad hoc obtinendum, ad momentum eclipsis medium addantur tot minuta secunda, quot haec expressio $\frac{7200(n+k)}{g(n-m)^2}\sin 2x\,\text{tang}^2\frac{1}{2}\varrho$ continet unitates. In vera autem oppositione longitudo tam lunae quam centri umbrae a nodo computata erit

$$x + \frac{2(m+k)(n+k)}{g(n-m)^2}\sin 2x\,\text{tang}^2\tfrac{1}{2}\varrho$$

seu si medio eclipsis fuerit longitudo umbrae $= U$, erit in oppositione utraque longitudo

$$= U + \frac{2(m+k)(n+k)}{g(n-m)^2}\sin 2x\,\text{tang}^2\tfrac{1}{2}\varrho\ \text{min. sec.}$$

§ 24. Interest deinde etiam in contemplatione eclipsium nosse angulum apparentem, quem [illegible] lunae cum ecliptica constituit. Hic scilicet centrum umbrae in ecliptica tanquam immobile [illegible], et angulus quaeritur, quem luna in orbita sua secundum motum relativum cum ecliptica format. Ad hunc inveniendum sit (Fig. 206) σ centrum umbrae in vera oppositione, et λ centrum lunae, ita ut arcus $\lambda\sigma$ sit ad eclipticam perpendicularis. Sit ut hactenus, medio eclipsis momento longitudo umbrae a nodo $= x$, erit ut vidimus in vera oppositione arcus

$$☊\sigma = x + \frac{2(m+k)(n+k)}{g(n-m)^2} \sin 2x \operatorname{tang}^2 \tfrac{1}{2}\varrho = x + d'',$$

posito brevitatis gratia

$$d = \frac{2(m+k)(n+k)}{g(n-m)^2} \sin 2x \operatorname{tang}^2 \tfrac{1}{2}\varrho.$$

Jam tempusculo infinite parvo procedat centrum umbrae per spatiolum $\sigma s = u$, et luna per $\lambda l = vu$, existente $v = \frac{n+k}{m+k}$. Ponatur angulus $☊\lambda\sigma = \varphi$, erit $\cos\varphi = \cos(x+d)\sin\varrho = \cos x \sin\varrho - gd \sin x \sin\varrho$. Jam ex l in $\sigma\lambda$ productum demittatur perpendiculum lu, erit ob λl infinite parvum,

$$lu = vu\sin\varphi \quad \text{et} \quad \lambda u = vu\cos\varphi = vu(\cos x \sin\varrho - gd \sin x \sin\varrho);$$

erit

$$\sin^2\varphi = 1 - \cos^2 x \sin^2\varrho + 2gd \sin x \cos x \sin^2\varrho,$$

ideoque ob $\sin^2\varrho$ valde parvum

$$\sin\varphi = \cos\varrho + \tfrac{1}{2}\sin^2 x \sin\varrho \operatorname{tang}\varrho + gd \sin 2x \sin\varrho \operatorname{tang}\varrho.$$

§ 25. Jam ad motum relativum repraesentandum removeamus totum systema motu sibi parallelo, ut punctum s perveniat in σ, et l in t, eritque t locus centri lunae ex centro umbrae σ spectatus, ideoque interea centrum lunae ex λ in t pervenisse et arcum λt descripsisse censebitur; ac propterea angulus λtu erit ille angulus, quem orbita lunae apparens cum ecliptica facere videbitur; erit autem $lt = u$, ideoque $tu = vu\sin\varphi - u$. Quare ob $\lambda u = vu\cos\varphi$, reperietur tangens anguli quaesiti $ut\lambda$, nempe

$$\operatorname{tang} ut\lambda = \frac{v\cos\varphi}{v\sin\varphi - 1} = \frac{(n+k)\cos\varphi}{(n+k)\sin\varphi - m - k}, \text{ seu } = \frac{(n+k)\sin\varrho(\cos x - gd\sin x)}{(n+k)\cos\varrho + \frac{1}{2}(n+k)\sin^2 x \sin\varrho \operatorname{tang}\varrho - m - k}.$$

Facilius autem hic angulus reperietur ex angulo φ, quem ante investigari oportebit, ex formula $\cos\varphi = \cos(x+d)\sin\varrho$, existente

$$d = \frac{2(m+k)(n+k)}{g(n-m)^2} \sin 2x \operatorname{tang}^2 \tfrac{1}{2}\varrho;$$

posito $v = \frac{n+k}{m+k}$ erit tang: anguli quaesiti $= \frac{v\cos\varphi}{v\sin\varphi - 1}$. Potest quoque particula d prorsus negligi, cum perinde sit, sive ista inclinatio apparens ad medium eclipsis momentum, sive ad veram oppositionem supputetur.

§ 26. Quo igitur has formulas in usum vocare queamus, requiritur, ut tempore eclipsis veros motus horarios tam lunae quam solis et nodi cognoscamus, cui investigationi sequens caput est destinatum. Interim tamen juvabit hic valorem medium horum motuum perpendisse. Ex tabulis astronomicis invenimus motum medium horarium solis $m = 2'27''50''' = 147\tfrac{5}{6}'' = 147'',833$; motum medium horarium lunae $n = 32'56''28''' = 1976\tfrac{7}{15}'' = 1976'',4666$, et motum horarium medium nodi $= 7'' \, 56''' = 7\frac{14}{15} = 7,9333$. Hi valores, quia a veris nunquam adeo notabiliter discrepant, sufficiunt ad formulam, quam supra (§ 13) eruimus, ad longitudinem lunae in orbita, medio eclipsis inveniendam. Cum enim sit

$$\begin{aligned} m &= 147,833 \\ n &= 1976,466 \\ k &= 7,933 \end{aligned} \qquad \text{erit} \qquad \begin{aligned} m + n + 2k &= 2140,166 \\ n - m &= 1828,633 \end{aligned}$$

ideoque coëfficiens $\frac{m+n+2k}{n-m} = 1,17036 = 1 + \frac{1}{6} + \frac{1}{300} + \frac{1}{3000}$. Quamobrem haec habebitur

Tabula aequationum

pro vera lunae longitudine in orbita tempore eclipsis medio invenienda.

Arg. Subtrahatur longitudo nodi a longitudine solis, et aequatio, quam sequens tabula praebet, locum soli oppositum applicetur.

0, VI Sign. Grad.	Subtrah.	
0	0′ 0″	30
1	0 18	29
2	0 36	28
3	0 54	27
4	1 12	26
5	1 29	25
6	1 47	24
7	2 4	23
8	2 21	22
9	2 38	21
10	2 55	20
	Adde	Grad. V, XI Sign.

§ 27. Ex hac ergo tabula satis exacte locus lunae in sua orbita ad medium cujusque eclipsis momentum assignari poterit, etiamsi motus horarius verus diversus sit a medio. Quanquam enim verus motus horarius lunae a medio fere 5′, et verus motus horarius solis a medio 5″ discrepare potest, tamen inde in coëfficientem $\frac{n+m+2k}{n-m}$ non adeo magnum discrimen redundat. Augeamus namque, ut discrimen fiat maximum, valorem litterae n, 300″, at valorem litterae m minuamus 5″, ita ut numerator augmentum capiat 295″ et denominator 305″, eritque $\frac{n+m+2k}{n-m} = \frac{2435}{2134} = 1,14105$, cujus valoris defectus a praecedente medio est $= 0,02931 = \frac{1}{34}$. Hinc ergo aequatio maxima distantiae 10° solis a nodo respondens tantum 5″ diminuetur, quod discrimen praecipue in evolutione penitus est contemnendum, propterea quod ex observationibus medium eclipsis momentum accurate definire non licet, ut error $5''$ ullius censendus sit momenti. Interim tamen non difficile etiam hunc errorem, postquam veros motus horarios determinaverimus, penitus evitare.

§ 28. Aliter vero est comparata ratio reliquarum formularum, quas in hoc capite elicuimus, quae ex valoribus mediocribus litterarum n, m, k sine notabili errore ad usum vocari nequeunt. In his scilicet formulis littera n, quae maximis variationibus est obnoxia, non eundem obtinet dimensionum numerum, tam in numeratore quam in denominatore, quemadmodum in casu tractato evenit, ubi ob mutabilitatem litterae n numerator et denominator coëfficientis $\frac{n+m+2k}{n-m}$ fere in eadem ratione mutabantur. Hinc sine exacta motuum horariorum cognitione neque temporis momentum, quo longitudo in orbita aequatur longitudini umbrae in ecliptica, definiri poterit, neque momentum verae oppositionis luminarium, neque etiam angulus, quem apparens lunae semita cum ecliptica constituit. Multo minus licebit ex observationibus eclipsium verum locum nodorum et inclinationem orbitae lunaris ad eclipticam assignare. Quamobrem in sequenti capite tam veros motus horarios solis, lunae et lineae nodorum investigabimus, quam diametros apparentes et lunae et umbrae terrestris, quae a parallaxi lunae pendet.

Caput ....

De vero loco nodi atque vera inclinatione orbitae lunaris ad eclipticam.

§ 1. Antequam valores litterarum α, δ, ε, quae in expressione loci lunae φ adhuc insunt, per observationes lunae extra syzygias factas definiamus, conveniet verum nodi ascendentis locum cum ejus vera ad eclipticam inclinatione determinari, quoniam hoc commodissime ex observationibus eclipsium totalium effici potest. Ad hoc ergo primo eligamus eclipsin sextam, cujus cum duplex habeatur observatio, sumamus inter utramque media momenta, quae erunt

	A. 1722 Jun. 28^d	initium:	$12^h 16' 28''$	immersio:	$13^h 24' \ 4''$
		finis:	15 36 13	emersio:	14 27 29
medium erat	A. 1722 Jun. 28^d		13 56 0	tempore vero	
seu	A. 1722 Jun. 28^d		13 58 41	tempore medio.	

Pro hoc tempore medio reperitur ex meis tabulis

longitudo nodi ☊ media $3^s 2^\circ 20' 43''$

cujus aequationes cum inclinatione ad eclipticam erunt

	Aequat. ☊	Inclinatio
Anomalia media lunae	$-\ 0' 54''$	
Anom. media solis	$-\ 0\ 12$	
long. ☊ a longit. solis	$+ 13\ 46$	$5\ \ 16\ \ 59\frac{1}{2}$
long. ☉ a longit. ☽	$0\ \ 0$	-42
long. ☊ a longit. ☽	$+\ 1\ \ 1$	$+37$

Hinc erit ex tabulis longitudo nodi vera $= 3^s 2^\circ 34' 24''$ et inclinatio orbitae lunaris ad eclipticam $= \ 5\ 16\ 54$.

§ 2. Ut igitur ex observatione quoque has res eruamus, primo diametrum solis appar... ejus parallaxin quaeramus, ex ejus anomalia excentrica

$$V = 11^s 28^0 25' 22'', \text{ unde fit } \sin V = -\sin 1^0 34' 38'' \text{ et } \cos V = +\cos 1^0 34' 38''$$

Erat ergo diameter solis apparens $= 1933'' - 32'',2 = 1901'' = 31'41''$, ejus ergo semidia... apparens $= 15'50\frac{1}{2}''$. Porro autem ejus parallaxis horizontalis erit $= 12''$. Tum vero motus hora... rius solis erit $= 143'' = 2'23''$.

§ 3. Pro luna vero, cum sit ejus anomalia excentrica

$$\varphi = 4^s 26^0 26' 6'', \ \sin\varphi = +\sin 33^0 33' 54'', \ \cos\varphi = -\cos 33^0 33' 54'',$$

reperietur secundum praecepta supra data

diameter lunae apparens	$= 33'15''$
semidiameter apparens	$= 16\ 37\frac{1}{2}$
parallaxis lunae horizontalis. . . .	$= 60\ 18$

Tum vero quoque invenietur

motus lunae horarius verus. . . .	$= 37\ 20$
motus autem horarius nodi erit. .	$= 0\ 8.$

Jam ad semidiametrum umbrae inveniendum, secundum regulam cognitam

ad parallaxin lunae horizontalem .	$= 60\ 18$
addatur parallaxis solis	$= 12$
summa	$= 60\ 30$
subtrahatur semidiameter solis . .	$= 15\ 50\frac{1}{2}$
eritque semidiameter umbrae . . .	$= 44\ 39\frac{1}{2}$

§ 4. Ut igitur formulas supra Cap.... inventas ad hunc casum accomodemus, erit

semidiameter umbrae	$\alpha = 44'\,39\frac{1}{2}'' = 2679'',5$
semidiameter lunae apparens	$\beta = 16\ 37\frac{1}{2} = 997,5$
ergo pro initio ac fine eclipsis	$\alpha + \beta = 3677'' = f$
et pro immersione et emersione	$\alpha - \beta = 1682 = F$
tempus ab initio eclipsis ad finem . .	$h = 3^h 19' 45'' = 3^h, 32916$
tempus ab immersione ad emersionem	$H = 1\ \ 3\ 25 = 1, 05694$
motus horarius solis	$m = 143$
motus horarius lunae	$n = 2240$
motus horarius nodi	$k = 8$

ergo erit $m + k = 151$, $n + k = 2248$, $m + n + 2k = 2399$ et $n - m = 2097$; unde inveni...

$$lh = 0{,}5223355, \quad la = 3{,}5731018, \quad lA = 3{,}0748184$$
$$lH = 0{,}0240521, \quad lb = 2{,}4002825, \quad lB = 1{,}9019991$$

ergo $a = 3742''$, $b = 251''$, $A = 1188''$ et $B = 80''$. Cum jam porro sit

longitudo umbrae.	$= 9^s\, 6^0\, 51'\ \ 7''$
et longitudo nodi vera tabularis	$= 3\ \ 2\ \ 34\ \ 24$

erit valor vero proximus . . . . . . $x = 6^s\,4^0\,16'\,43''$

seu a nodo descendente computando $x = \quad 4\;\;16\;\;43$

quem valorem autem nunc accuratius definiri oportet.

§ 5. Deinde cum inclinatio orbitae lunaris ad eclipticam jam prope sit

$$\varrho = 5^0\,16'\,54''$$

erit ejus semissis $\tfrac{1}{2}\varrho = 2\;\;38\;\;27$

et distantiae x duplum $2x = 8\;\;33\;\;26$

unde quaeratur angulus ille parvus $c = \dfrac{(m+n+2k)\sin 2x \operatorname{tang}^2 \frac{1}{2}\varrho}{g(n-m)}$ sec. existente $lg = 4{,}6855749$, qui invenitur $c = 75'' = 1'\,15''$.

Tum quaerantur porro anguli:

$$\begin{array}{llll} a+b = 3993'' = 1^0\;\;6'\;33''\;; & a+b+c = 1^0\;\;7'\;48'' \\ A+B = 1268\;\; = 0\;\;21\;\;\;8\;\;; & A+B+c = 0\;\;22\;\;23 \\ a-b = 3491\;\; = 0\;\;58\;\;11\;\;; & a-b+c = 0\;\;59\;\;26 \\ A-B = 1108\;\; = 0\;\;18\;\;28\;\;; & A-B+c = 0\;\;19\;\;43 \end{array}$$

et anguli $f = 1\;\;1\;\;17 \qquad F = 0\;\;28\;\;\;2$

ex quibus quaeratur $d = \dfrac{\cos f - \cos^2 \frac{1}{2}\varrho \cos(a-b+c)}{\cos F - \cos^2 \frac{1}{2}\varrho \cos(A-B+c)}$ fietque $ld = 0{,}001647$, et cum sit

$$\operatorname{tang} 2x = \frac{d\cos(A+B+c) - \cos(a+b+c)}{\sin(a+b+c) - d\sin(A+B+c)}$$

hinc autem reperitur $x = 8^0 22' 45''$, qui valor fere duplo est major, quam ex tabulis invenitur. Sin autem utamur formula

$$\cos(2x-a-b-c) = \frac{\cos f - \cos^2 \frac{1}{2}\varrho \cos(a-b+c)}{\sin^2 \frac{1}{2}\varrho}$$

invenitur $x = 3^0\,19'\,59''$; unde patet ex his formulis nimis esse lubricum locum nodi assignare.

§ 6. Certior videtur formula alia supra inventa

$$\frac{\cos(a+b) + \sin(a+b)\operatorname{tang}(2x-c)}{\cos(a+b) - \sin(a+b)\operatorname{tang}(2x-c)} = \frac{\cos f - \cos^2 \frac{1}{2}\varrho \cos(a-b+c)}{\cos f - \cos^2 \frac{1}{2}\varrho \cos(a-b-c)}$$

ex qua calculo subducto reperitur $2x = 8^0\,21'\,52''$ et $x = 4^0\,10'\,56''$, qui valor ab eo, quem tabulae exhibent $x = 4^0\,16'\,43''$ deficit $5'\,47''$, ita ut locus nodi medius hac particula $5'\,47''$ promovendus videatur. Hac autem correctione adhibita, cum sit

$$\sin^2 \tfrac{1}{2}\varrho = \frac{\cos(a-b+c) - \cos f}{2\sin(x-a-c)\sin(x-b)} = \frac{\sin\dfrac{f+a-b+c}{2}\sin\dfrac{f-a+b-c}{2}}{\sin(x-a-c)\sin(x-b)}$$

hi autem anguli nimis sunt parvi, quam ut inde inclinatio vera recte concludi possit.

§ 7. Quoniam si hac ultima methodo utamur, immersionis et emersionis nulla ratio habetur, eclipses quoque partiales ad hunc scopum adhibere poterimus, quae etiam erunt aptiores ad inclinationem definiendam, cum anguli in denominatore $x-a-c$ et $x-b$ non fiant adeo parvi. Sumamus ergo eclipsin decimam

A. 1731 Jun. 19^d init. $13^h\,14'\,21''$ fin. $14\;\;40\;\;41$ medium $13^h\,57'\,31''$ t. v.

at medium A. 1731 Jun. 19^d $13^h\,58'\,15''$ tempore medio.

*

Pro hoc tempore ex tabulis meis colligitur

longitudo nodi media . . . . $9^s 8^\circ 45' 38''$

cujus correctiones sunt

	Long. ☊	Inclin.
anom. media lunae	$-$ 1′ 12″	
anom. media solis	$-$ 1 19	
distantia ☊ a ☉.	$-$ 31 52	$5^\circ 16' 35''$
dist. ☉ a ☽.	0 0	$-$ 42
dist. ☊ a ☽.	$-$ 2 23	$+$ 36
	$-$ 36′ 46″	$\varrho = 5^\circ 16' 29''$

et longit. ☊ vera $= 9^s\ 8^\circ 8' 52''$

longit. umbrae $= 8\ 28\ 5\ 41$ $\qquad \frac{1}{2}\varrho = 2^\circ 38' 15''$

hinc erit $x = -10\ 3\ 11$

§ 8. Jam est porro

$$\begin{array}{rlrl} V = & 11^s 19^\circ 38' 30'' & \cos V = & +\cos 10^\circ 21' 30'' \\ v = & 4\ 12\ 54\ 33 & \cos v = & -\cos 47\ \ 5\ 27 \\ 2v = & 8\ 25\ 49\ \ 6 & \cos 2v = & -\cos 85\ 49\ \ 6 \\ v - V = & 4\ 23\ 16\ \ 3 & \cos(v - V) = & -\cos 36\ 43\ 57 \end{array}$$

Unde invenitur:

diameter solis apparens	$= 31' 41''$
parallaxis solis horizontalis	$=\ \ 12$
motus horarius solis	$= 2\ 23 = m = 143''$
diameter lunae apparens	$= 32\ 55$
parallaxis lunae horizontalis	$= 59\ 38$
motus lunae horarius	$= 36\ 35 = n = 2195$

Ex his fiet:

semidiameter umbrae $\qquad \alpha = 44\ \ 0 - \frac{1}{2}$

semidiam. lunae apparens $\qquad \beta = 16\ 27 + \frac{1}{2}$

ergo $\alpha + \beta = 60\ 27 = f$

duratio porro eclipsis est $\qquad h = 1{,}4389$ horas

atque ob $a = \frac{1}{2}h(n+k)$ fiet $\quad a = 1585'' = 26' 25'$

et $b = \frac{1}{2}h(m+k)$, $\qquad b = \ 109 \ = \ 1\ 49$.

§ 9. Cum jam sit $n - m = 2052$ et $m + n + 2k = 2354$ atque $2x = -20^\circ 6' 22$ $\frac{1}{2}\varrho = 2^\circ 38' 15''$, reperietur particula illa $c = -2' 53''$. Deinde, quia habemus

$$a + b = 28' 14'',\ a - b = 24' 36'',\ a - b + c = 21' 43'' \text{ et } a - b - c = 27' 29''$$

reperietur
$$-\operatorname{tang}(2x - c) = \frac{120}{39776 \operatorname{tang}(a+b)}, \text{ unde colligitur}$$

$$-(2x - c) = 20^\circ 10' 14'' = -2x + c = -2x - 2' 53''$$

ideoque $\quad 2x = -20^\circ 13' 7''$ et $x = -10^\circ 6' 34''$. Erat autem per tabulas

$$\begin{array}{r} x = -10\ \ 3\ 11 \\ \hline \text{diff.} \qquad 3\ 23 \end{array}$$

longitudo nodi vera hoc tempore non $9^s 8^0 8' 52''$, ut tabulae praebent, sed $9^s 8^0 12' 15''$ scilicet $3' 23''$ promotior esse debebat. Hinc ergo longitudo media nodi tabularis $3' 23''$ augeri debere videtur, cum ante augmentum $5' 47''$ esset inventum; ita ut vix dubitari liceat, quin ad longitudines nodi in tabulis exhibitas nonnulla minuta prima adjici debeant. Hinc autem porro ob angulos $y - c = -10^0\, 30'\, 6''$ et $x - b = -10^0\, 8'\, 23''$ reperitur certius $l \sin^2 \frac{1}{2}\varrho = 7{,}3218378$, $l \sin \frac{1}{2}\varrho = 8{,}6609189$ et $\frac{1}{2}\varrho = 2^0\, 37' 21''$, et hoc tempore inclinatio $\varrho = 5^0\, 14' 42''$, quae a tabularis deficit $1' 47''$.

§ 10. Verumtamen ob hoc ipsum, quod hic tantum initium ac finis eclipsis in calculum inducuntur, huic determinationi non admodum confidere licet; propterea quod reipsa tres habemus quantitates incognitas x, ϱ et c, ad quas definiendas duae aequationes ex initio ac fine eclipsis deductae non sufficiunt. Etsi enim valorem ipsius c hic jam tanquam cognitum assumsimus, notandum tamen est, ex minimo errore in eo commisso errores satis grandes in determinationes arcuum x et ϱ irrepere posse. Vulgo quidem, si eclipsis partialis adhibetur, quantitas maximae obscurationis insuper in subsidium vocari solet, quae quoniam per observationem exactissime assignari nequit, expedire videtur eclipsibus totalibus, in quibus tam initium ac finis, quam immersio et emersio omni cura sunt observata, ad hoc institutum uti. Ne autem summa formularum supra inventarum complicatio calculum impediat, ternas aequationes, quas observationes initii, finis et immersionis suppeditant contemplemur, aliamque methodum aperiamus, ex iis immediate quantitates incognitas x, ϱ et c determinandi.

§ 11. Sit tempore eclipsis medio motus solis horarius $= m$

motus lunae horarius $= n$

motus nodi horarius $= k = 8$

semidiameter umbrae $= \alpha$, semidiameter lunae $= \beta$; $\alpha + \beta = f$, $\alpha - \beta = F$

tempus ab initio eclipsis ad finem elapsum $= h$ hor.

tempus ab immersione ad emersionem $= H$ hor.

ponaturque

$$a = \tfrac{1}{2} h (n + k) \qquad A = \tfrac{1}{2} H (n + k)$$

$$b = \tfrac{1}{2} h (m + k) \qquad B = \tfrac{1}{2} H (m + k)$$

Tum sint incognitae quantitates

distantia nodi ☊ a centro umbrae $= x$

distantia nodi ☊ a centro lunae $= y$

inclinatio orbitae lunaris ad eclipt. $= \varrho$

atque habebuntur hae aequalitates

$$\cos\varrho = \frac{\cos f - \cos(y - a)\cos(x - b)}{\sin(y - a)\sin(x - b)} = \frac{\cos f - \cos(y + a)\cos(x + b)}{\sin(y + a)\sin(x + b)}$$

itemque

$$\cos\varrho = \frac{\cos F - \cos(y - A)\cos(x - B)}{\sin(y - A)\sin(x - B)} = \frac{\cos F - \cos(y + A)\cos(x + B)}{\sin(y + A)\sin(x + B)}$$

§ 12. Quatuor harum formularum, quibus idem valor $\cos\varrho$ exprimitur, sufficiet tres assumsi cum quarta jam sponte in iis involvatur. Manifestum autem est, si binarum incognitarum alteram eliminare voluerimus, ut unica in aequatione supersit, expressionem esse prodituram pere complicatam, ut per calculum difficillime explicetur. Rejecta ergo praevia alterius incogni eliminatione ope regulae *falsi* dictae, utriusque valorem simul per fictas hypotheses definiamus, quo eo promtius fieri poterit, cum utriusque valor jam proxime constet. Quae operatio, quo clarius per spiciatur, eam statim ad eclipsin totalem A. 1722 accommodemus. Erit ergo $f = 1^0 1' 17''$, $F = 0^0 28' 2$ $a = 1^0 2' 22''$, $b = 4' 11''$, atque $A = 19' 48''$, $B = 1' 20''$. Proxime vero jam constat esse $x = 4^0 16' 43''$ et $y = x - c = 4^0 15' 28''$. Sit autem revera $x = 4^0 16' 43'' - p'$ et $y = 4^0 15' 28'' - q$ et ad correctiones p et q inveniendas constituantur tres hypotheses:

I.	II.	III.
$x = 4^0 16' 43''$	$x = 4^0 16' 43''$	$x = 4^0\ 6' 43''$
$y = 4\ 15\ 28$	$y = 4\ \ 5\ 28$	$y = 4\ 15\ 28$

Verum postea alias hypotheses fingi conveniet.

§ 13. Pro his jam ternis hypothesibus evolvantur singuli valores pro $\cos\varrho$ inventi, ex iisque colligi poterunt ii valores, qui ex positis veris valoribus ipsarum y et x essent prodituri, qui deinde inter se aequales sunt ponendi. Commodius autem erit his formulis uti

$$\text{I. } \sin^2 \tfrac{1}{2}\varrho = \frac{-\sin\frac{1}{2}(x - y + a - b - f)\sin\frac{1}{2}(x - y + a - b + f)}{\sin(y - a)\sin(x - b)}$$

$$\text{II. } \sin^2 \tfrac{1}{2}\varrho = \frac{-\sin\frac{1}{2}(x - y - a + b - f)\sin\frac{1}{2}(x - y - a + b + f)}{\sin(y + a)\sin(x + b)}$$

$$\text{III. } \sin^2 \tfrac{1}{2}\varrho = \frac{-\sin\frac{1}{2}(x - y + A - B - F)\sin\frac{1}{2}(x - y + A - B + F)}{\sin(y - A)\sin(x - B)}$$

$$\text{IV. } \sin^2 \tfrac{1}{2}\varrho = \frac{-\sin\frac{1}{2}(x - y - A + B - F)\sin\frac{1}{2}(x - y - A + B + F)}{\sin(y + A)\sin(x + B)}$$

Cum jam sit

$$\begin{array}{rl} a = & 1^0\ \ 2' 22'' \\ b = & 0\ \ \ 4\ \ 11 \\ a - b = & 0\ \ 58\ \ 11 \\ f = & 1\ \ \ 1\ \ 17 \end{array} \quad \text{atque} \quad \begin{array}{rl} A = & 0^0 19' 48'' \\ B = & 0\ \ \ 1\ \ 20 \\ A - B = & 0\ \ 18\ \ 28 \\ F = & 0\ \ 28\ \ \ 2 \end{array}$$

erit

$$\begin{array}{rl} a - b + f = & 1^0 59' 28'', \\ a - b - f = & -0\ \ 3\ \ 6 \\ \frac{1}{2}(a - b + f) = & 0\ \ 59\ \ 44 \\ \frac{1}{2}(a - b - f) = & -0\ \ 1\ \ 33 \end{array} \qquad \begin{array}{rl} A - B + F = & 0^0 46' 30'' \\ A - B - F = & -0\ \ 9\ \ 34 \\ \frac{1}{2}(A - B + F) = & 0\ \ 23\ \ 15 \\ \frac{1}{2}(A - B - F) = & -0\ \ 4\ \ 47. \end{array}$$

§ 14. Jam secundum ternas hypotheses sit

	I.	II.	III.	revera
$\frac{1}{2}(x - y) =$	$0' 38''$	$0' 38''$	$0' 28''$	$0' 38'' - p''$
$\frac{1}{2}(x + y) =$	$4^0 16\ \ 0$	$4^0 11\ \ 0$	$4^0 16\ \ 0$	$4^0 16\ \ 0 - q'$
$x =$	$4\ 16\ 38$	$4\ 11\ 38$	$4\ 16\ 28$	
$y =$	$4\ 15\ 22$	$4\ 10\ 22$	$4\ 15\ 32$	

	I.	II.	III.
I.	$\frac{\sin 0'55''.\sin 1^0 0'22''}{\sin 3^0 13'0''.\sin 4^0 12'27''}$	$\frac{\sin 0'55''.\sin 1^0 0'22''}{\sin 3^0 8'0''.\sin 4^0 7'27''}$	$\frac{\sin 1'5''.\sin 1^0 0'12''}{\sin 3^0 13'10''.\sin 4^0 12'17''}$
II.	$\frac{\sin 59'6''.\sin 2'11''}{\sin 5^0 17'44''.\sin 4^0 20'49''}$	$\frac{\sin 59'6''.\sin 2'11''}{\sin 5^0 12'44''.\sin 4^0 15'49''}$	$\frac{\sin 59'16''.\sin 2'1''}{\sin 5^0 17'54''.\sin 4^0 20'39''}$
III.	$\frac{\sin 4'9''.\sin 23'53''}{\sin 3^0 55'34''.\sin 4^0 15'18''}$	$\frac{\sin 4'9''.\sin 23'53''}{\sin 3^0 50'34''.\sin 4^0 10'18''}$	$\frac{\sin 4'19''.\sin 23'43''}{\sin 3^0 55'44''.\sin 4^0 15'8''}$
IV.	$\frac{\sin 22'37''.\sin 5'25''}{\sin 4^0 35'10''.\sin 4^0 17'58''}$	$\frac{\sin 22'37''.\sin 5'25''}{\sin 4^0 30'10''.\sin 4^0 12'58''}$	$\frac{\sin 22'47''.\sin 5'15''}{\sin 4^0 35'20''.\sin 4^0 17'48''}$

Si autem calculus secundum has formulas evolvatur, reperitur $p = 102$, foretque ergo $y > x$, quod tamen admitti nequit.

§ 15. Ratio hujus incommodi, praeter incertitudinem momentorum, quibus eclipsis vel incipit vel finitur, vel tota luna in umbram terrae immergitur, vel ex ea emergere incipit, in hoc potissimum posita videtur, quod umbra terrae ob ejus atmosphaeram revera amplior est, quam in calculo admisimus. Etsi enim atmosphaera terrae, ob radiorum solis refractionem, conum terrae umbrosum imminuit, ut ejus vertex ne quidem ad lunam usque porrigatur, sicque luna nunquam in veram terrae umbram ingrediatur, tamen pelluciditas atmosphaerae in tanta distantia tantopere diminuitur, ut ipsa quoque atmosphaera perinde ac terra ipsa tanquam corpus opacum spectari debeat: quamobcausam semidiameter umbrae augeri debebit tanta particula, quanta altitudo atmosphaerae est ipsius radii terrae. Quare cum ex crepusculis altitudo atmosphaerae sit quasi 12 milliarium conclusa, radio telluris existente 860 mill., semidiameter umbrae augeri debebit parte sui $\frac{1}{71}$. Hinc in nostro exemplo semidiameter umbrae $44'40''$ augeri debebit $38''$, idemque erit incrementum angulorum f et F, unde anguli exigui illi in numeratoribus fractionum ipsi $\sin^2 \frac{1}{2}\varrho$ aequalium augeri debebunt $19''$.

§ 16. Calculo expedito minus utique incommodum oritur, si semidiameter umbrae $19''$ augeatur, neque tamen hoc modo veritas, quae jam proxime est cognita, satis salvatur: perspicuum fiet umbram adhuc magis augeri oportere. Videntur autem omnia incommoda optime tolli, si semidiameter umbrae $30''$ augeatur; ita ut atmosphaera plus quam semissi amplior sit statuenda, quam ex crepusculis conclusimus, sive aër etiamnunc in altitudinem fere 20 milliarium in regione lunae tanquam corpus opacum cernitur. Ob incognitam vero umbrae terrestris veram quantitatem, ex eclipsibus neque verus nodorum locus, neque vera orbitae lunaris inclinatio ad eclipticam accuratius definiri potest, quam in tabulis exhibetur. Unde his elementis tabularibus tantisper uti conveniet, donec ex observationibus exquisitissimis latitudinis lunae, vel maximae vel evanescentis, tam inclinationem tabularem quam locum nodi accurate definire liceat.

Caput ....

De diametris apparentibus motuque horario vero Solis ac Lunae, in eclipsibus lunaribus.

§ 1. Sit U anomalia media solis, V ejus anomalia excentrica, θ longitudo vera et e excentricitas orbitae, quam invenimus esse $e = 0{,}0167595$; erit ergo

$$U = V + e \sin V \text{ et } d\theta = \frac{dV\sqrt{(1-ee)}}{1 + e\cos V}.$$

Porro si ponatur y distantia solis a terra, et a distantia media, erit

$$y = a\,(1 + e\cos V),$$

pro a vero in tabulis usurpari solet numerus 100000. Quando autem sol in hac distantia media a terra versatur, ejus diameter apparens deprehenditur $32'\,13''$. In distantia ergo $y = a\,(1 + e\cos V)$ erit diameter solis apparens

$$= \frac{32'13''}{1 + e\cos V} = \frac{1933''}{1 + e\cos V}:$$

Evoluto autem hoc denominatore prodibit diameter apparens

$$= 1933''\left(1 - e\cos V + \tfrac{1}{2}ee + \tfrac{1}{2}ee\cos 2V - \tfrac{3}{4}e^3\cos V - \tfrac{1}{4}e^3\cos 3V\right);$$

et si parallaxis horizontalis in distantia media statuatur $= 12\tfrac{1}{2}''$, erit pro quavis distantia y, quae anomaliae excentricae V convenit, parallaxis horizontalis solis

$$= 12{,}5\left(1 + \tfrac{1}{2}ee - (e + \tfrac{3}{4}e^3)\cos V + \tfrac{1}{2}ee\cos 2V - \tfrac{1}{4}e^3\cos 3V\right).$$

§ 2. Posito autem pro e valore supra invento erit

$$1 + \tfrac{1}{2}ee = 1{,}0001405, \qquad e + \tfrac{3}{4}e^3 = 0{,}0167630$$

$$\tfrac{1}{2}ee = 0{,}0001405, \qquad \tfrac{1}{4}e^3 = 0{,}0000012.$$

Hinc anomaliae excentricae V respondebit diameter solis apparens in minutis secundis

$$\underset{}{1933''} - \underset{(1{,}5105718)}{32{,}4\cos V} + \underset{(9{,}4323278)}{0{,}27\cos 2V}$$

cum ergo terminus ultimus ne dimidium quidem minutum secundum praebeat, erit

$$\text{diameter solis apparens} = 32'13'' - \underset{(1{,}5105718)}{32{,}4\cos V}$$

$$\text{semidiam. solis apparens} = 16\ \ 6\tfrac{1}{2} - \underset{(1{,}2095418)}{16{,}2\cos V}$$

Parallaxis autem solis horizontalis anomaliae excentricae V respondens erit $= 12\tfrac{1}{2}'' - 0{,}2\cos V$. In apogeo ergo parallaxis solis fere erit $12''$, in perigeo vero $13''$, unde hoc calculo fere supersedere poterimus.

§ 3. Quod ad motum solis horarium attinet, eum ex aequatione differentiali $d\theta = \frac{dV\sqrt{(1-ee)}}{1 + e\cos V}$ definiri conveniet. Cum enim sit $dV = \frac{dU}{1 + e\cos V}$, erit $d\theta = \frac{\sqrt{(1-ee)}}{(1 + e\cos V)^2}\,dU$, unde si dU denotet motum horarium anomaliae solis mediae, qui est $= 2'\,27\tfrac{5}{6}''$, valor ipsius $d\theta$ erit motus horarius verus solis. Hinc ergo pro anomalia excentrica V erit motus horarius solis

$$= 147'',833\left(1 + ee - (2e + 2e^3)\cos V + \tfrac{3}{2}ee\cos 2V - e^3\cos 3V\right)$$

$$= 147'',833\left(1{,}0002809 - 0{,}0335284\cos V + 0{,}0004214\cos 2V\right).$$

His ergo factoribus evolutis erit

$$\text{motus horarius solis} = 147'',87 - \underset{(0{,}6951839)}{4{,}9566\cos V}.$$

In apogeo ergo est motus horarius solis $= 142'',92 = 2'23''$

in perigeo vero erit motus horarius solis $= 152'',86 = 2\ \ 33$.

Ad quodvis igitur tempus hinc tam diameter solis apparens cum ejus parallaxi horizontali, quam verus motus horarius assignari poterit.

§ 4. Diameter lunae apparens ejusque parallaxis horizontalis pendet ab ejus distantia a terra. Scilicet si distantia media ponatur $= c$, et distantia vera $= z$, erit, uti supra vidimus, diameter lunae apparens $= \frac{c}{z} \cdot 31'13\frac{1}{2}''$,*) et parallaxis lunae horizontalis $= \frac{c}{z} \cdot 56'39''$. Commodissime ergo primo quaeritur distantia lunae a terra z, ex qua cum sit $c = 100000$, levi calculo tam diameter apparens quam parallaxis lunae horizontales definitur. Interim tamen quoque formula $\frac{c}{z}$ per divisionem evolvi poterit, sicque ex ea immediate tam diametrum apparentem, quam parallaxin horizontalem definire licebit. Ponamus loco coëfficientium numericorum litteras alphabeti, sitque

$$\frac{z}{c} = 1 + k\cos v - \mathfrak{a}\cos V + \mathfrak{b}\cos(v+V) - \mathfrak{c}\cos(v-V) + \mathfrak{d}\cos\eta - \mathfrak{e}\cos 2\eta - \mathfrak{f}\cos(2\eta-2v)$$
$$- \mathfrak{g}\cos(2\eta+v) + \mathfrak{h}\cos(2\eta-v) + \mathfrak{j}\cos(2\eta+V) + \mathfrak{k}\cos(2\eta-V) - \mathfrak{l}\cos(2\eta-v+V) - \mathfrak{m}\cos(2\eta-v-V).$$

§ 5. Coëfficientes ergo hi, quorum valores supra exhibuimus, sequenti modo per logarithmos determinabuntur, ut sit

$lk = 8{,}7359900$	$l\mathfrak{d} = 6{,}4639186$	$l\mathfrak{h} = 8{,}0092614$
$l\mathfrak{a} = 6{,}1580247$	$l\mathfrak{e} = 7{,}8437310$	$l\mathfrak{j} = 6{,}2771879$
$l\mathfrak{b} = 6{,}2430230$	$l\mathfrak{f} = 6{,}3091788$	$l\mathfrak{k} = 6{,}3908075$
$l\mathfrak{c} = 6{,}2803776$	$l\mathfrak{g} = 6{,}5773131$	$l\mathfrak{l} = 6{,}2581595$
		$l\mathfrak{m} = 6{,}0152034$

Quibus notatis habebimus convertendo fractionem $\frac{z}{c}$:

$$\begin{array}{lllllll}
\frac{c}{z} = 1 & -k\cos v & +\frac{1}{2}kk\cos 2v & -\frac{1}{4}k^3\cos 3v & +\mathfrak{a}\cos V & -\mathfrak{b}\cos(v+V) & +\mathfrak{c}\cos(v-V) \\
+\frac{1}{2}kk - \mathfrak{e}\mathfrak{h} & & & & +k\mathfrak{b} & -\mathfrak{a}k & -\mathfrak{a}k \\
+\frac{1}{2}\mathfrak{h}\mathfrak{h} - \mathfrak{f}\mathfrak{h} & & & & -k\mathfrak{c} & +\mathfrak{j}\mathfrak{h} & +\mathfrak{b}\mathfrak{h} \\
 & -\frac{3}{4}k^3 & & & -\mathfrak{h}\mathfrak{l} & & \\
 & & & & -\mathfrak{h}\mathfrak{m} & &
\end{array}$$

$$\begin{array}{lllll}
-\mathfrak{d}\cos\eta & +\mathfrak{e}\cos 2\eta & +\mathfrak{f}\cos(2\eta-2v) & +\mathfrak{g}\cos(2\eta+v) & -\mathfrak{h}\cos(2\eta-v) \\
 & -k\mathfrak{g} & +k\mathfrak{h} & -k\mathfrak{e} & -k\mathfrak{e} \\
 & +k\mathfrak{h} & & & -k\mathfrak{f}
\end{array}$$

$$\begin{array}{llll}
-\mathfrak{j}\cos(2\eta+V) & -\mathfrak{k}\cos(2\eta-V) & +\mathfrak{l}\cos(2\eta-v+V) & +\mathfrak{m}\cos(2\eta-v-V) \\
-k\mathfrak{l} & -k\mathfrak{m} & +k\mathfrak{j} & +k\mathfrak{k} \\
+\mathfrak{b}\mathfrak{h} & -\mathfrak{c}\mathfrak{h} & -\mathfrak{a}\mathfrak{h} & -\mathfrak{a}\mathfrak{h}
\end{array}$$

§ 6. Restitutis ergo loco harum litterarum valoribus erit

$$\begin{array}{llll}
\frac{c}{z} = 1{,}00160 & -\,0{,}05464\cos v & +\,0{,}00148\cos 2v & +\,0{,}00012\cos V \\
\quad 0{,}00069 & \quad 8{,}73751 & \quad 7{,}17095 & \quad 6{,}07918
\end{array}$$

*) *Ad marginem:* potius $31'14\frac{1}{2}''$.

$$-\underset{6,25527}{0,00018}\cos(\varphi+V)+\underset{6,25527}{0,00018}\cos(\varphi-V)-\underset{6,46392}{0,00029}\cos\eta+\underset{7,87564}{0,00752}\cos 2\eta$$

$$+\underset{6,88081}{0,00076}\cos(2\eta-2\varphi)-\underset{8,02571}{0,01061}\cos(2\eta-\varphi)-\underset{6,29226}{0,00020}\cos(2\eta+V)$$

$$-\underset{6,40654}{0,00025}\cos(2\eta-V)+\underset{6,27875}{0,00019}\cos(2\eta-\varphi+V)+\underset{6,06070}{0,00012}\cos(2\eta-\varphi-V)$$

Hinc ergo pro primo invenitur *Diameter Lunae apparens:*

$$=31'17''-\underset{2,01016}{102,''4}\cos\varphi+\underset{0,44360}{2,8}\cos 2\varphi+\underset{9,352}{0,2}\cos V-\underset{9,527}{0,3}\cos(\varphi+V)+\underset{9,527}{0,3}\cos(\varphi-V)$$

$$-\underset{9,736}{0,6}\cos\eta+\underset{1,1483}{14,1}\cos 2\eta+\underset{0,153}{1,4}\cos(2\eta-2\varphi)-\underset{1,29836}{19,9}\cos(2\eta-\varphi)-\underset{9,564}{0,4}\cos(2\eta+V)$$

$$-\underset{9,679}{0,4}\cos(2\eta-V)+\underset{9,551}{0,4}\cos(2\eta-\varphi+V)+\underset{9,333}{0,2}\cos(2\eta-\varphi-V)$$

Similique modo *Parallaxis Lunae horizontalis:*

$$=56'44''-\underset{2,26879}{185,7}\cos\varphi+\underset{0,702}{5,1}\cos 2\varphi+\underset{9,610}{0,4}\cos V-\underset{9,786}{0,6}\cos(\varphi+V)+\underset{9,786}{0,6}\cos(\varphi-V)$$

$$-\underset{9,995}{1,0}\cos\eta+\underset{1,4069}{25,5}\cos 2\eta+\underset{0,4121}{2,6}\cos(2\eta-2\varphi)-\underset{1,5570}{36,1}\cos(2\eta-\varphi)-\underset{9,823}{0,7}\cos(2\eta+V)$$

$$-\underset{9,937}{0,9}\cos(2\eta-V)+\underset{9,809}{0,6}\cos(2\eta-\varphi+V)+\underset{9,592}{0,4}\cos(2\eta-\varphi-V)$$

§ 7. Cum igitur sufficiat tam diametrum lunae apparentem quam parallaxin horizontalem ad unum duove minuta secunda nosse, hae formulae multo fient succinctiores eritque

$$\text{diameter lunae apparens}=31'17''-\underset{2,01016}{102}\cos\varphi+\underset{0,444}{3}\cos 2\varphi+\underset{1,1843}{14}\cos 2\eta$$
$$+\underset{0,15}{}\cos(2\eta-2\varphi)-\underset{1,298}{20}\cos(2\eta-\varphi)$$

$$\text{parallaxis horizontalis}=56'44''-\underset{2,2688}{186}\cos\varphi+\underset{0,702}{5}\cos 2\varphi-\underset{9,99}{}\cos\eta+\underset{1,407}{25}\cos 2\eta$$
$$+\underset{0,412}{2}\cos(2\eta-2\varphi)-\underset{1,557}{36}\cos(2\eta-\varphi)$$

Neque tamen opus est, ut hinc tabulae peculiares construantur, cum in tabulis distantiam lunae a terra ubique exhibeamus, eaque facili negotio colligi queat. Ea vero cognita multo facilius tam diametrum lunae apparentem, quam parallaxin ejus horizontalem definire licebit. Ceterum diameter apparens, quem sic invenimus, ad centrum terrae spectat, indeque pro quavis altitudine super horizonte diameter visa non difficulter assignatur.

§. 8. Ex his ergo concludimus fore in conjunctione

		in Apog.	in Perig.
diametrum lunae appar.	$= 31'\,30'' - 122 \cos v + 4 \cos 2v$	$29'\,32''$	$33'\,36''$
parallaxin horizontalem	$= 57\;\;8 - 222 \cos v + 8 \cos 2v$	53 34	60 58
in oppositione vero erit			
diameter lunae app.	$= 31\;\;32 - 122 \cos v + 4 \cos 2v$	29 34	33 38
parallaxis horiz.	$= 57\;\;10 - 222 \cos v + 8 \cos 2v$	53 36	61 0
atque in quadraturis			
diameter lunae app.	$= 31\;\;3 - 83 \cos v + \cos 2v + \cos V$	29 41	32 27
parallaxis horiz.	$= 56\;\;18 - 150 \cos v + 2 \cos 2v + 2 \cos V$	53 50	58 50

Ope priorum ergo formularum ad momenta eclipsium tam solarium quam lunarinm facile definitur diameter lunae apparens et ejus parallaxis horizontalis: quoniam haec determinatio a sola anomalia lunae excentrica pendet, dum reliquae partes ab anomalia solis insuper pendentes tam fuerint exiguae, ut sine errore rejici queant.

§. 9. His expeditis investigationem motus lunae horarii suscipio: quae pariter ex aequatione differentiali, qua variatio longitudinis lunae momentanea $d\varphi$ continetur, est petenda. Quodsi vero ponamus anomaliam lunae mediam $= u$, excentricam $= v$, ut sit $du = dv\,(1 + k \cos v)$, distantiamque lunae a terra $= z$, erit $d\varphi = \frac{\alpha - \frac{3}{2} Mnn}{zz : cc}\, du$, existente

$$\alpha = 1{,}0070234,\quad l\alpha = 0{,}0030396,\quad l\tfrac{3}{2}\,nn = 7{,}9312851$$

Unde si du denotet motum horarium anomaliae mediae, ut sit

$$du = 32'\,39''\,48''',$$

valor differentialis $d\varphi$ dabit motum horarium lunae verum. Supra autem invenimus valorem litterae M sequentem

$$\begin{aligned} M = &+ A \cos 2\eta + Bk \cos (2\eta + v) + Ekk \cos 2(\eta - v) - Fe \cos (2\eta + V)\\ &+ akk \cos 2\eta + Ck \cos (2\eta - v) \qquad\qquad - Ge \cos (2\eta - V)\\ &- Kke \cos (2\eta - v - V) + M\varepsilon \cos \eta + N\varepsilon \cos 3\eta + O\varepsilon k \cos (\eta - v)\\ &- Lke \cos (2\eta - v + V) \qquad - nnS \cos 4\eta + nnUk \cos v \end{aligned}$$

§. 10. Si jam pro his litteris valores supra inventos substituamus, reperiemus numeratorem fractionis $\alpha - \frac{3}{2}\, nnM =$

$$\begin{aligned} &1{,}0070234 + 0{,}004662 \cos 2\eta + 0{,}0003385 \cos (2\eta + v) + 0{,}0011281 \cos (2\eta - v)\\ &0{,}0030396 \qquad 7{,}66857 \qquad 6{,}529607 \qquad 7{,}052357\\ &- 0{,}0005506 \cos (2\eta - 2v) - 0{,}0001170 \cos (2\eta + V) - 0{,}0001269 \cos (2\eta - V)\\ &6{,}740834 \qquad 6{,}068320 \qquad 6{,}103616\\ &- 0{,}0000364 \cos (2\eta - v - V) - 0{,}0000293 \cos (2\eta - v + V) + 0{,}0000084 \cos \eta\\ &5{,}560398 \qquad 5{,}466904 \qquad 4{,}921866\\ &+ 0{,}0000139 \cos 3\eta - 0{,}0000124 \cos (\eta - v) - 0{,}0000403 \cos 4\eta + 0{,}0001987 \cos v\\ &5{,}143714 \qquad 5{,}091308 \qquad 5{,}6050 \qquad 6{,}2981 \end{aligned}$$

*

Pro hoc numeratore ponatur brevitatis gratia

$$\alpha + A\cos 2\eta + B\cos(2\eta+\varphi) + C\cos(2\eta-\varphi) - D\cos(2\eta-2\varphi) - E\cos(2\eta+V)$$
$$- F\cos(2\eta-V) - G\cos(2\eta-\varphi-V) - H\cos(2\eta-\varphi+V) + J\cos\eta + K\cos 3\eta$$
$$- L\cos(\eta-\varphi) - M\cos 4\eta + N\cos\varphi.$$

Pro denominatore vero $\frac{zz}{cc}$ jam ejus reciprocum ante inventum consideremus, sitque brevitatis gratia

$$\frac{c}{z} = \mathfrak{A} - \mathfrak{B}\cos\varphi + \mathfrak{C}\cos 2\varphi + \mathfrak{D}\cos V - \mathfrak{E}\cos(\varphi+V) + \mathfrak{F}\cos(\varphi-V) - \mathfrak{G}\cos\eta + \mathfrak{H}\cos 2\eta$$
$$+ \mathfrak{J}\cos(2\eta-2\varphi) - \mathfrak{K}\cos(2\eta-\varphi) - \mathfrak{L}\cos(2\eta+V) - \mathfrak{M}\cos(2\eta-V) + \mathfrak{N}\cos(2\eta-\varphi+V)$$
$$+ \mathfrak{O}\cos(2\eta-\varphi-V).$$

§ 11. Evolvatur ergo calculus, ac reperietur

$$\begin{array}{llllll}
\frac{d\varphi}{du} = \alpha\mathfrak{A}^2 & -2\alpha\mathfrak{A}\mathfrak{B}\cos\varphi & +2\alpha\mathfrak{A}\mathfrak{C}\cos 2\varphi & +2\alpha\mathfrak{A}\mathfrak{D}\cos V & -2\alpha\mathfrak{A}\mathfrak{E}\cos(\varphi+V) & +2\alpha\mathfrak{A}\mathfrak{F}\cos(\varphi-V) \\
+\frac{1}{2}\alpha\mathfrak{B}^2 & -\alpha\mathfrak{B}\mathfrak{C} & +\frac{1}{2}\alpha\mathfrak{B}^2 & +\alpha\mathfrak{B}\mathfrak{E} & -\alpha\mathfrak{B}\mathfrak{D} & -\alpha\mathfrak{B}\mathfrak{D} \\
+\frac{1}{2}\alpha\mathfrak{H}^2 & -\alpha\mathfrak{D}\mathfrak{E} & & -\alpha\mathfrak{B}\mathfrak{F} & +\alpha\mathfrak{K}^2 & +\alpha\mathfrak{K}\mathfrak{M} \\
+\frac{1}{2}\alpha\mathfrak{K}^2 & +\alpha\mathfrak{D}\mathfrak{F} - A\mathfrak{A}\mathfrak{K} & & & & \\
+A\mathfrak{A}\mathfrak{H} & -\alpha\mathfrak{H}\mathfrak{K} & & & & \\
-C\mathfrak{A}\mathfrak{K} & +N\mathfrak{A}^2 & & & &
\end{array}$$

$$\begin{array}{lllll}
-2\alpha\mathfrak{A}\mathfrak{G}\cos\eta & +2\alpha\mathfrak{A}\mathfrak{H}\cos 2\eta & +2\alpha\mathfrak{A}\mathfrak{J}\cos(2\eta-2\varphi) & -2\alpha\mathfrak{A}\mathfrak{K}\cos(2\eta-\varphi) & +\alpha\mathfrak{B}\mathfrak{G}\cos(\eta-\varphi) \\
+J\mathfrak{A}^2 & +\alpha\mathfrak{B}\mathfrak{K} & +\alpha\mathfrak{B}\mathfrak{K} & -\alpha\mathfrak{B}\mathfrak{H} & -L\mathfrak{A}^2 \\
 & +A\mathfrak{A}^2 & -D\mathfrak{A}^2 & -\alpha\mathfrak{B}\mathfrak{J} & \\
 & -C\mathfrak{A}\mathfrak{B} & -C\mathfrak{A}\mathfrak{B} & +C\mathfrak{A}^2 & \\
 & & & -A\mathfrak{A}\mathfrak{B} &
\end{array}$$

$$\begin{array}{llll}
-2\alpha\mathfrak{A}\mathfrak{L}\cos(2\eta+V) & -2\alpha\mathfrak{A}\mathfrak{M}\cos(2\eta-V) & +2\alpha\mathfrak{A}\mathfrak{N}\cos(2\eta-\varphi+V) & +2\alpha\mathfrak{A}\mathfrak{O}\cos(2\eta-\varphi-V) \\
-\alpha\mathfrak{B}\mathfrak{N} - C\mathfrak{A}\mathfrak{E} & -\alpha\mathfrak{B}\mathfrak{O} - C\mathfrak{A}\mathfrak{F} & +\alpha\mathfrak{B}^2 - A\mathfrak{A}\mathfrak{F} & +\alpha\mathfrak{B}\mathfrak{M} \\
-E\mathfrak{A}^2 & -F\mathfrak{A}^2 & -\alpha\mathfrak{D}\mathfrak{K} + C\mathfrak{A}\mathfrak{D} & -\alpha\mathfrak{D}\mathfrak{K} + C\mathfrak{A}\mathfrak{D} \\
+A\mathfrak{A}\mathfrak{D} & +A\mathfrak{A}\mathfrak{D} & -H\mathfrak{A}^2 & -G\mathfrak{A}^2 \\
 & & & -A\mathfrak{A}\mathfrak{E}
\end{array}$$

$$\begin{array}{lll}
-\alpha\mathfrak{B}\mathfrak{H}\cos(2\eta+\varphi) & +K\mathfrak{A}^2\cos 3\eta & -M\mathfrak{A}^2\cos 4\eta \\
+\mathfrak{B}\mathfrak{A}^2 & & +A\mathfrak{A}\mathfrak{H} \\
-A\mathfrak{A}\mathfrak{B} & &
\end{array}$$

§ 12. Valoribus ergo in numeris restitutis prodibit

$$\begin{array}{llllll}
\frac{d\varphi}{du} = 1{,}01188 & -0{,}11022\cos\varphi & +0{,}004493\cos 2\varphi & +0{,}000242\cos V & -0{,}000368\cos(\varphi+V) \\
0{,}0051293 & 9{,}04226 & 7{,}65257 & 6{,}38394 & 6{,}5658
\end{array}$$

$$\begin{array}{llll}
+0{,}000358\cos(\varphi-V) & -0{,}000579\cos\eta & +0{,}020356\cos 2\eta & +0{,}000014\cos 3\eta \\
6{,}5539 & 6{,}7627 & 8{,}30869 & 5{,}14579
\end{array}$$

$$0{,}000006 \cos 4\eta \quad + 0{,}000004 \cos(\eta - v) \quad + 0{,}001502 \cos(2\eta - 2v) \quad - 0{,}000328 \cos(2\eta + v)$$
4,778 — 4,6020 — 7,17667 — 6,51587

$$- 0{,}020979 \cos(2\eta - v) \quad - 0{,}000524 \cos(2\eta + V) \quad - 0{,}000649 \cos(2\eta - V)$$
3,32178 — 6,71933 — 6,81224

$$+ 0{,}000365 \cos(2\eta - v + V) \quad + 0{,}000209 \cos(2\eta - v - V)$$
6,56229 — 6,32015

Cum nunc sit $du = 32'39\frac{4}{5}'' = 1959'',8$, erit $l.du = 3{,}292212$. Hinc fiet in minutis secundis motus horarius lunae verus =

$$33'3'' - 216'' \cos v \quad + 9 \cos 2v \quad - \cos \eta \quad + 40 \cos 2\eta \quad + 3 \cos(2\eta - 2v) \quad - 41 \cos(2\eta - v)$$
2,3344 — 0,9448 — 0,055 — 1,6009 — 0,4689 — 1,6140

$$- \cos(2\eta + V) - \cos(2\eta - V)$$
0,005 — 0,104

omissis scilicet terminis, quorum valores ne quidem ad unum minutum secundum exsurgunt. Hinc ergo non erit adeo difficile ad quodvis tempus motum lunae horarium supputare; hicque labor multo erit facilior, quam si, ut vulgo fieri solet, duo loca lunae ad tempora horae unius intervallo discrepantia computari debeant.

§ 13. Haec autem formula adhuc quapiam correctione indiget, cum valor litterae α, ob plures particulas ex denominatore ad eum accedentes, non satis sit certus. Ad hanc ergo correctionem inveniendam, calculo quaesivi duo loca lunae ad duo momenta horae intervallo differentia. Primo scilicet assumsi esse $v = 0$, $u = 0$, $\eta = 0$ et $V = 90^0$; deinde, post horae intervallum erat $v = 1858'',6$, $u = 1959'',8$, et ob motum lunae horarium jam proxime cognitum $\eta = 1627''$, sicque ex tabulis prodiit promotio lunae horaria $= 1774'',7$. Formula autem hic inventa praebet pro hoc casu motum horarium $= 1775'',7$, ita ut haec formula unico tantum minuto secundo sit minuenda; quare primus terminus $33'3''$ transmutandus erit in $33'2''$. Hincque concludimus fore in conjunctione motum horarium lunae

$$= 33'40\tfrac{4}{5}'' - 258{,}3 \cos v \quad + 11{,}7 \cos 2v \quad - 1{,}8 \cos V \quad + 1{,}4 \cos(v - V).$$
2,41212 — 1,0696 — 0,2607 — 0,1495

In oppositione vero erit motus horarius lunae

$$= 33'43\tfrac{1}{10}'' - 258{,}3 \cos v + 11{,}7 \cos 2v - 1{,}8 \cos V + 1{,}4 \cos(v - V),$$

ubi quidem terminum N omisi, quoniam supra jam est monitum hunc terminum in excentricitate comprehendi posse.

§ 14. Si ergo luna fuerit in apogeo, erit ejus motus horarius

in Conjunctione $= 29'34\frac{1}{5}'' - 0{,}4 \cos V$, in Oppositione $= 29'36\frac{1}{2}'' - 0{,}4 \cos V$

Sin vero luna sit in perigeo, erit ejus motus horarius

in Conjunctione $= 38'10\frac{4}{5}'' - 3{,}2 \cos V$, in Oppositione $= 38'13\frac{1}{10}'' - 3{,}2 \cos V$.

Si igitur dum luna in perigeo versatur, sol fuerit in apogeo, erit motus horarius

$$\text{in Conjunctione} = 38'7\tfrac{3}{5}'', \text{ at in Oppositione} = 38'9\tfrac{9}{10}''.$$

Sin autem sol fuerit in perigeo perinde ac luna, erit motus horarius lunae

$$\text{in Conjunctione} = 38'14'', \text{ in Oppositione} = 38'16\tfrac{3}{10}''.$$

Dum autem sol est in apogeo, variatio motus horarii lunae inde orta ne ad dimidium quidem minutum secundum ascendit.

§ 15. Formula autem generalis pro motu horario lunae sic correcta ita se habebit, ut, posita anomalia lunae excentrica $= \varphi$, anomalia solis $= V$ et distantia lunae a sole $= \eta$ sit motus lunae horarius in minutis secundis expressus =

$$\begin{array}{llllll} 1982''{,}0 - 216{,}4 \cos\varphi & + 8{,}8 \cos 2\varphi & + 0{,}5 \cos V & - 0{,}7 \cos(\varphi + V) & + 0{,}7 \cos(\varphi - V) \\ \quad 2{,}33526 & 0{,}9448 & 9{,}676 & 9{,}858 & 9{,}846 \end{array}$$

$$\begin{array}{lllll} - 1{,}1 \cos\eta & + 39{,}9 \cos 2\eta & + 3{,}0 \cos(2\eta - 2\varphi) & - 0{,}6 \cos(2\eta + \varphi) & - 41{,}1 \cos(2\eta - \varphi) \\ 0{,}0549 & 1{,}6009 & 0{,}4690 & 9{,}808 & 1{,}6140 \end{array}$$

$$\begin{array}{llll} - 1{,}0 \cos(2\eta + V) & - 1{,}3 \cos(2\eta - V) & + 0{,}7 \cos(2\eta - \varphi + V) & + 0{,}4 \cos(2\eta - \varphi - V) \\ 0{,}0115 & 0{,}1044 & 9{,}854 & 9{,}612 \end{array}$$

Commode ergo tabulis lunaribus adjungi poterit tabula peculiaris motui horario inveniendo inserviens, quoniam tam ad eclipses quam ad occultationes accurate determinandas plurimum interest nosse motum lunae horarium. Hanc tabulam autem conveniet ad partes decimas minuti secundi accommodari, ne collectione singularum aequationum error unum minutum secundum superet. Constabit autem haec tabula duodecim quoque partibus, pariter ac ipsa motus lunae tabula, commodeque cum ea conjungi poterit.

§ 16. Motum horarium lunae in conjunctione et oppositione, quo in eclipsibus opus habemus, jam ante exhibuimus; videamus ergo quomodo se motus lunae horarius sit habiturus in quadraturis, quando angulus η est vel 90° vel 270°; utroque autem casu reperietur motus lunae horarius

$$\begin{array}{lllll} = 1942''{,}1 & - 174{,}7 \cos\varphi & + 5{,}8 \cos 2\varphi & + 2{,}8 \cos V & - 1{,}1 \cos(\varphi + V) \\ & \quad 2{,}24229 & 0{,}763 & 0{,}447 & 0{,}041 \end{array}$$

	☉ in Apog.	☉ in Perig.
Si ergo luna fuerit in apogeo, seu $\varphi = 0$, erit motus horarius lunae $= 1773''{,}2 + 1{,}7 \cos V$:	1775″, seu 29′35″	1771″ seu 29′31″
Sin autem luna sit in perigeo, seu $\varphi = 180°$, erit motus horarius lunae $= 2122''{,}6 + 3{,}9 \cos V$:	2126″ seu 35′26″	2119″ seu 35′19″

Motus ergo lunae horarius est minimus in quadraturis, quando luna in apogeo, sol vero in perigeo versatur. Maximus vero est motus horarius lunae in oppositione, quando simul luna et sol fuerint in perigeo.