

XVIII.

De motu Cometarum in orbitis parabolicis, solem in foco habentibus.

1. **Problema I.** Cometae in data orbita parabolica moti invenire locum heliocentricum ad datum tempus.

Solutio. (Fig. 214) Sit $MEAF$ orbita cometae parabolica, in cujus foco F versetur sol, quae cum ponatur data, dabitur locus perihelii A in coelo ex sole visus, atque cometa ex sole cernitur in circulo maximo moveri, cujus planum pariter erit datum. Datum porro etiam sit tempus, quo cometa in perihelio versabitur, atque vel ante vel post hoc tempus quaeratur locus cometae M ex sole visus. Elapsum sit scilicet jam tempus T , postquam cometa in perihelio A extiterit, sitque hoc tempore M locus cometae, ita ut cometa in coelo appareat a loco perihelii A distare angulo ASM , qui angulus erit cometae anomalia vera. Sit iste angulus, qui quaeritur, $ASM = \varphi$, ponatur distantia perihelii a sole $SA = a$, erit parameter parabolae $= 4a$. Ex loco cometae M in axem parabolae AS demittatur perpendicularum MP , et vocetur $AP = x$, $PM = y$, erit $yy = 4ax$ atque ob $PS = x - a$, erit radius $SM = x + a$. Hinc anguli $ASM = \varphi$ sinus erit $= \frac{y}{x+a}$ et cosinus $= \frac{a-x}{x+a}$ posito sinu toto $= 1$. Cum igitur sit $\cos \varphi = \frac{a-x}{x+a}$, erit

$$x = \frac{a(1 - \cos \varphi)}{1 + \cos \varphi}, \text{ et distantia cometae a sole } MS = a + x = \frac{2a}{1 + \cos \varphi}.$$

Inventa ergo anomalia vera φ innotescit distantia cometae a sole $MS = \frac{2a}{1 + \cos \varphi}$. Quoniam vero tempus T , quo cometa a perihelio A ad locum M pertingit, est directe ut area ASM , et inverse ut radix quadrata ex latere recto seu parametro $4a$, aream ASM indagare oportet, quae est $= \text{area } APM - \Delta SPM$. At area APM ex natura parabolae est

$$= \frac{2}{3} xy, \text{ et } \Delta SPM = \frac{1}{2} y(x - a) = \frac{1}{2} xy - \frac{1}{2} ay;$$

hinc area ASM erit $= \frac{1}{6} xy + \frac{1}{2} ay$. Est vero $x = \frac{a(1 - \cos \varphi)}{1 + \cos \varphi}$ et $y = (a+x) \sin \varphi = \frac{2a \sin \varphi}{1 + \cos \varphi}$.

hinc erit $\frac{1}{6} x + \frac{1}{2} a = \frac{2a + a \cos \varphi}{3(1 + \cos \varphi)}$, ideoque area $ASM = \frac{2aa(2 + \cos \varphi) \sin \varphi}{3(1 + \cos \varphi)^2}$.

Hanc expressionem simpliciorum reddendam ponatur semmissis anguli ASM tangens, seu

$t = \tan \frac{1}{2} \varphi$, erit $\sin \frac{1}{2} \varphi = \frac{t}{\sqrt{1+t^2}}$, $\cos \frac{1}{2} \varphi = \frac{1}{\sqrt{1+t^2}}$, indeque $\sin \varphi = \frac{2t}{1+t^2}$, $\cos \varphi = \frac{1-t^2}{1+t^2}$,

ergo porro $2 + \cos \varphi = \frac{3+t^2}{1+t^2}$ et $1 + \cos \varphi = \frac{2}{1+t^2}$. Fiet itaque

$$\text{area } ASM = \frac{1}{3} aat(3+t^2) = aa(t + \frac{1}{3}t^3).$$

Ponatur jam semiaxis major orbitae terrae, seu distantia media terrae a sole $= c$, atque planeta,

circum solem circulum, cujus radius $= c$, describeret, periodum absoluturus esset uno anno

terreno, hoc est $365^d 6^h 8' 31''$, quod tempus ponamus $= \theta$. Cum igitur hujus circuli area sit πcc ,

habente $1 : \pi$ rationem diametri ad peripheriam, et parameter diametro $2c$ sit aequalis, erit tem-

pus unius revolutionis θ ut area πcc divisa per $\sqrt{2}c$, hoc est ut $\frac{\pi}{\sqrt{2}} c\sqrt{c}$. Simili vero modo est

tempus T , quo cometa ex A in M pertingit, ut area $ASM = aa(t + \frac{1}{3}t^3)$ divisa per $\sqrt{4}a$, hoc

est ut $(t + \frac{1}{3}t^3) \frac{a\sqrt{a}}{2}$; unde haec nascitur analogia $\theta : T = \frac{\pi c\sqrt{c}}{\sqrt{2}} : \frac{a\sqrt{a}}{2}(t + \frac{1}{3}t^3)$, ergo

$$t + \frac{1}{3}t^3 = \frac{\pi T c\sqrt{2c}}{6a\sqrt{a}} = 4,4428829381 \cdot \frac{T}{\theta} \cdot \frac{c\sqrt{c}}{a\sqrt{a}}$$

Quia θ est annus sidereus, et T tempus datum, erit θ ad T ut 360° ad motum terrae medium

tempori T convenientem. Si ergo ponatur motus terrae medius tempori T respondens $= m$, fiet

$\frac{m}{360}$. Ex aequatione ergo cubica

$$t^3 + 3t = 13,3286488144 \cdot \frac{m}{360} \cdot \frac{c\sqrt{c}}{a\sqrt{a}}$$

ponatur valor ipsius t , qui erit tangens semmissis anguli ASM , hincque ad datum tempus vel ante

post transitum cometae per perihelium A assignabitur locus cometae heliocentricus. Q. E. I.

2. Coroll. 1. Si tempus T sit spatium unius diei seu 24 horarum, erit $m = 59' 8''$, unde

calculo subducto fiet $t^3 + 3t = 0,036491289910 \cdot \frac{c\sqrt{c}}{a\sqrt{a}}$. Quare si T sit spatium n dierum, erit

$$t^3 + 3t = 0,036491289910 \cdot \frac{nc\sqrt{c}}{a\sqrt{a}} \text{ seu}$$

$$t + \frac{1}{3}t^3 = 0,012163763303 \cdot \frac{nc\sqrt{c}}{a\sqrt{a}}.$$

3. Coroll. 2. Pro quovis ergo dierum numero n reperitur numerus ipsi $t + \frac{1}{3}t^3$ aequalis,

unde cum difficulter valor ipsius t , ex eoque valor anguli φ reperitur, conveniet tabulam construere,

quae pro singulis valoribus anguli φ exhibeat valores respondententes ipsius $t + \frac{1}{3}t^3$: hujus enim

tabulae ope vicissim ex valore ipsius $t + \frac{1}{3}t^3$ dato angulus φ colligetur.

4. **Coroll. 3.** Si ergo detur tempus, quo cometa in perihelio versatur, atque distantia helii a sole a , ad quodvis tempus, distantia cometae a perihelio ex sole visa determinari poterit tabellae. Scilicet propositum sit tempus n dierum vel ante vel post appulsum cometae ad perihelium, computetur valor $0,012163763303 \cdot \frac{nc\sqrt{c}}{a\sqrt{a}}$; hic valor quaeratur in tabula sub columna t ac respondens valor ipsius φ dabit angulum quaesitum.

5. **Exemplum.** Cometae, qui A. 1680 apparuit, Newtonus statuit latus rectum orbitae $4a = 236,8$, seu $a = 59,2$ existente $c = 10000$, atque istum cometam collegit in perihelio versatum esse A. 1680 decembr. die 8, $0^h 4'$ p. m. Hinc erit $0,012163763303 \cdot \frac{c\sqrt{c}}{a\sqrt{a}} = 26,70458$, atque diebus vel ante vel post appulsum cometae ad perihelium erit $t + \frac{1}{3}t^3 = 26,70458 n$, cui respondens angulus φ ostendet cometae distantiam a perihelio in orbita sua ex sole visam. Uno die tam ante quam post perihelium hic cometa confecit angulum ASM plus quam 152° ; die autem vel praecedente vel antecedente tantum 6 circiter gradus absolvit. Postquam autem cometa ad perihelium transiisset, per spatium 90 dierum adhuc apparuit, toto ergo hoc tempore absolvit angulum ASM circiter 174° .

6. **Coroll. 4.** Cognito angulo $ASM = \varphi$, innotescet cometae a sole distantia SM , quae est $= \frac{2a}{1 + \cos \varphi}$. Posito autem $t = \tan \frac{1}{2} \varphi$, erit $\cos \varphi = \frac{1-tt}{1+tt}$ et $1 + \cos \varphi = \frac{2}{1+tt}$. Hinc erit distantia $SM = a(1+tt) = a \sec^2 \frac{1}{2} \varphi$. Distantia ergo cometae a sole SM confecto angulo $ASM = 174^\circ$ ubi disparere coepit, ob $a = 59,2$ et $t = \tan 87^\circ$ erat 21613; non multum ergo diametrum orbis magni $2c$ excedebat.

7. **Coroll. 5.** Si ducatur tangens curvae in M , in eamque ex S perpendicularum demittatur, erit hoc perpendicularum $= a\sqrt{1+tt}$, et celeritas cometae in puncto M erit ut $\frac{1}{a\sqrt{1+tt}}$, seu $\frac{1}{a}$ constans, ut $\cos \frac{1}{2} \varphi$.

8. **Scholion.** Si numerus, qui pro $t + \frac{1}{3}t^3$ resultat, non exacte reperiatur in tabula, tum per interpolationem consueto more investigabitur angulus φ in minutis primis et secundis, nisi forte numerus ille sit nimis magnus, atque numeri $t + \frac{1}{3}t^3$ angulis φ respondentes nimium a progressionem arithmetica discrepent. Hoc igitur casu peculiari artificio opus erit, ex natura progressionis petito ad angulum φ exactius determinandum. Quaeratur exempli gratia angulus ASM , quem cometa A. 1680 tempore 10 dierum confecerat: erit ergo $t + \frac{1}{3}t^3 = 267,0458$, unde apparet angulum contineri intra 167° et 168° . Primum ergo more solito interpolatio instituatur:

167°:	234,1492	267,0458
168°:	296,6044	234,1492
60':	62,4552	32,8966
	1,0409	1,0409
		32,8968
		1,3158
		296

angulus $\varphi = 167^{\circ} 34'$ et $t = \text{tang } 83^{\circ} 47'$. Ponatur jam $t = \text{tang } (83^{\circ} 47' + m'')$ eritque

$$t = 9,1802838 + 4145 m \quad ut = 0,9628561 + 196 m$$

$$257,8974000 + 350000 m \quad ut^3 = 2,8885683 + 588 m$$

$$\frac{267,0776838 + 354145 m}{\quad} \quad ut = 0,4771213$$

$$l \frac{1}{3} t^3 = 2,4114470 + 588 m$$

$$\text{num.} = 257,8974 + 350 m$$

$267,0776 + 0,0354 m = 267,0458$, hincque $354 m = -318$, ergo m nequidem unum

secundum valet, ita ut vere sit $\varphi = 167^{\circ} 34'$.

Problema 2. Ex datis tribus locis heliocentricis cometæ ejus orbitam determinare.

Solutio. Fig. 215. Sit $LMNA$ orbita cometæ, quæ quaeritur; ac primo quidem planum, in quo existit, sponte innotescit ex duabus observationibus. Observetur primum cometa in directione SL ; secundo in directione SM , et tertio in directione SN . Dantur ergo anguli LSM et LSN , itemque distantiæ temporum inter has observationes. Sit tempus inter observationem primam et secundam $= m$ dierum, inter primam ac tertiam $= n$ dierum. Porro sit tangens semissis anguli $LSM = f$, tangens semissis anguli $LSN = g$. Ponatur tempus, quo cometa ex L in perihelium A perveniet $= z$ dierum; erit tempus inter cometæ loca M et $A = z - m$, et inter loca N et $A = z - n$. Sit tangens semissis anguli $ASL = t$, erit

$$\text{tang } \frac{1}{2} ASM = \frac{t-f}{1+ft} \quad \text{et} \quad \text{tang } \frac{1}{2} ASN = \frac{t-g}{1+gt}$$

Posito jam brevitatis gratia $N = 0,012163763303 \cdot \frac{c\sqrt{a}}{a\sqrt{a}}$, denotante a distantiam SA , et c distantiam mediam terræ a sole. His positis erit

$$t + \frac{1}{3} t^3 = Nz$$

$$\frac{t-f}{1+ft} + \frac{1}{3} \frac{(t-f)^3}{(1+ft)^3} = N(z-m)$$

$$\frac{t-g}{1+gt} + \frac{1}{3} \frac{(t-g)^3}{(1+gt)^3} = N(z-n)$$

Ex quibus tribus aequationibus tres incognitas N , z , et t determinari oportet. Per subtractionem secundæ et tertiæ a prima obtinentur hæc duæ aequationes

$$\frac{f(1+tt)}{1+ft} + \frac{ftt(1+tt) - ft(1-t^4) + \frac{1}{3} f^3(1+t^6)}{(1+ft)^3} = Nm$$

$$\frac{g(1+tt)}{1+gt} + \frac{ggt(1+tt) - ggt(1-t^4) + \frac{1}{3} g^3(1+t^6)}{(1+gt)^3} = Nn$$

$$\frac{f(1+tt)^2 + ft(1+tt)^2 + \frac{1}{3} f^3(1+tt)^3}{(1+ft)^3} = Nm$$

$$\frac{g(1+tt)^2 + ggt(1+tt)^2 + \frac{1}{3} g^3(1+tt)^3}{(1+gt)^3} = Nn$$

quarum una per alteram divisa dabit aequationem incognita N carentem, solamque t involventem

$$\frac{fn(1 + \frac{1}{3}ft + \frac{1}{3}ft^2 + \frac{1}{3}ft^3)}{(1+t)^3} = \frac{gm(1 + \frac{1}{3}gt + \frac{1}{3}gt^2 + \frac{1}{3}gt^3)}{(1+g)^3}$$

$$\text{seu } \frac{fn}{(1+t)^2} + \frac{f^3n(1+tt)}{3(1+t)^3} = \frac{gm}{(1+g)^2} + \frac{g^3m(1+tt)}{3(1+g)^3}$$

in qua aequatione incognita t ad quinque dimensiones ascendit. Quamobrem ut solutio evadat, eligantur tres observationes a se invicem minimum distantes, ita ut f et g quasi infimae parva evadant. Tum vero prima observatio L non tantum a perihelio distet, ut t possit posteriori terminum in utroque membro notabilis quantitatis efficere. Evanescent ergo in utroque termini posteriores, eritque

$$\frac{fn}{(1+t)^2} = \frac{gm}{(1+g)^2}, \text{ unde fit}$$

$$(1+g)\sqrt{fn} = (1+t)\sqrt{gm}, \text{ hincque } t = \frac{\sqrt{gm} - \sqrt{fn}}{g\sqrt{n} - f\sqrt{gm}}$$

Hoc modo quidem tantum vero proxime valor ipsius t invenitur, quia minimus error in observationibus commissus ingentem aberrationem parit. Verum si hoc modo valor ipsius t prope verum inventus, tum aliae duae quaecunque observationes cum prima L conjungantur, atque tum aequatio, etsi quinque est dimensionum, tamen ob valorem ipsius t prope verum cognitum, verus valor non difficulter eruetur. Sit θ valor prope verus ipsius t , ponaturque $t = \theta + \psi$, ita ut ψ prae θ valde parvum, eritque

$$\frac{fn}{(1+f\theta)^2} + \frac{f^3n(1+\theta\theta)}{3(1+f\theta)^3} - \frac{2fn\psi}{(1+f\theta)^3} - \frac{f^3n\psi(3f-2\theta+f\theta\theta)}{3(1+f\theta)^4} =$$

$$\frac{gm}{(1+g\theta)^2} + \frac{g^3m(1+\theta\theta)}{3(1+g\theta)^3} - \frac{2gm\psi}{(1+g\theta)^3} - \frac{g^3m\psi(3g-2\theta+g\theta\theta)}{3(1+g\theta)^4}$$

ex qua aequatione ψ inventum dabit verum valorem tangentis $t = \theta + \psi$, et anguli ipsi respondentis duplum monstrabit angulum LSA , ideoque praebabit positionem axis AS parabolae quaesitae. Invento autem t erit

$$N = \frac{(1+tt)^2(f+ft + \frac{1}{3}f^3(1+tt))}{m(1+t)^3} = 0,0121637 \cdot \frac{c\sqrt{c}}{a\sqrt{a}}$$

unde elicitur distantia $AS = a$, ac parabolae latus rectum $4a$. Denique colligetur tempus, quo cometa in perihelium perveniet, quod post observationem primam L eveniet z diebus, $z = \frac{3t+tt^3}{3N}$. Cognitis ergo positione axis parabolae AS , ejus parametro $4a$, seu valore litterae una cum tempore, quo cometa perihelium attingit, ad quodvis tempus locus cometae in orbita ejusque distantia a sole per problema praecedens definietur. Q. E. I.

10. **Coroll. I.** Ex cognito tempore, quo cometa ad perihelium appellit, una cum valore numeri N ad datum tempus, distantia cometae a perihelio ex sole visa determinabitur; scilicet si haec distantia desideretur n diebus vel ante vel post appulsum ad perihelium, multiplicetur numerus N per n , ac productum quaeratur in tabula sub columna $t + \frac{1}{3}t^3$, cui respondebit angulus θ , distantiam cometae a perihelio e sole visam indicans.

Coroll. 2. Si valor numeri N inventus dividatur per 0,012163763303, quotus dabitur reperietur distantia perihelii a sole $SA = a$, seu potius ejus ratio ad distantiam mediam a sole. Hinc autem in quovis loco vera cometæ a sole distantia colligetur. (6).

Scholion. Maxima difficultas posita est in inventione tangentis t ex aequatione

$$\frac{fn}{(1+ft)^2} + \frac{f^3 n (1+tt)}{3(1+ft)^3} = \frac{gm}{(1+gt)^2} + \frac{g^3 m (1+tt)}{3(1+gt)^3};$$

quæ præcepimus ejusmodi observationes adhibere, in quibus f et g fiant vehementer parvae, et n et m proxime tantum innotescat. At quoniam minimus error in observationibus nimium vicinis maxime nocentem a veritate conclusionem producere potest, conveniet binas posteriores observationes nimis vicinas, neque nimis remotas a prima accipi, quo f et g neque sint vehementer parvae, neque ad unitatem appropinquent, quod eveniet, dummodo angulus LSN minor sit 60° . Tales observationes si eligantur, tum aequatio $\frac{fn}{(1+ft)^2} = \frac{gm}{(1+gt)^2}$ valorem ipsius t a vero aberrantem quidem, non multum, præbebit. Sit iste valor $t = \theta$, ita ut sit $\frac{fn}{(1+f\theta)^2} = \frac{gm}{(1+g\theta)^2}$, ac statuatur verus valor $t = \theta + \psi$, ubi ψ instar quantitatis vehementer parvae tractare licebit. Pervenietur autem, in solutione, ad hanc aequationem

$$\frac{f^3 n (1 + g\theta)}{3(1 + f\theta)^3} - \frac{fn\psi(6 + 3ff + 4f\theta + ff\theta\theta)}{3(1 + f\theta)^4} = \frac{g^3 m (1 + f\theta)}{3(1 + g\theta)^3} - \frac{gm\psi(6 + 3gg + 4g\theta + gg\theta\theta)}{3(1 + g\theta)^4}$$

quæ propter $fn : gm = (1 + f\theta)^2 : (1 + g\theta)^2$, abit in

$$\frac{ff(1 + g\theta)}{1 + f\theta} - \frac{f\psi(6 + 3ff + 4f\theta + ff\theta\theta)}{(1 + f\theta)^2} = \frac{gg(1 + f\theta)}{1 + g\theta} - \frac{g\psi(6 + 3gg + 4g\theta + gg\theta\theta)}{(1 + g\theta)^2}$$

ex qua aequatione si erutum fuerit ψ , habebitur satis prope $t = \theta + \psi$, qui tamen pari modo valens corrigi potest. Denique consultum erit tres observationes a se invicem maxime remotas adhibere, atque per aequationem quintae potestatis, qua t determinatur, exactissime valorem ipsius determinare, id quod non difficulter præstabitur, cum valor ipsius t jam proxime sit notus. Sicque multiplicato observationum numero, orbita cometæ continuo exactius cognoscetur.

13. Problema 3. Cognita cometæ orbita, una cum temporis momento, quo in perihelio versatur, ad quodvis tempus cometæ longitudinem ac latitudinem heliocentricam definire.

Solutio. Fig. 216. Quoniam cometa in plano per solem transeunte movetur, apparebit in circulo maximo incedere. Sit igitur sol in centro coeli siderei S , atque ΩMAS circulus maximus, in quo cometa ingredi cernitur, secundum ordinem litterarum ΩMAS . Sit porro ΩmaS ecliptica secundum signorum ordinem distributa, et O polus eclipticæ borealis, erit Ω nodus ascendens orbitæ cometæ, m nodus descendens. Sit longitudo nodi ascendentis Ω a prima stella arietis computata $= q$, et inclinatio orbitæ cometæ ΩMAS ad eclipticam, seu angulus $M\Omega m = s$, qui si recto fuerit minor, motus cometæ secundum signorum seriem fieri cernitur; contra, si angulus s recto sit major, motus cometæ contra signorum seriem perficietur. Sit deinde A locus perihelii, per quem ducatur circulus maximus $O A a$, erit a longitudo perihelii, cujus distantia a prima stella arietis

sit $= p$, erit arcus $\Omega a = p - q$. Ex triangulo sphaerico $A\Omega a$ ad a rectangulo innotescit latitudo borealis perihelii Aa , erit nempe $\text{tang } Aa = \text{tang } s \cdot \sin(p - q)$ et $\text{tang } \Omega A = \frac{\text{tang}(p - q)}{\cos s}$. Sit a distantia cometae in perihelio a sole, et c distantia media terrae a sole, ac ponatur

$$N = 0,012163763303 \cdot \frac{c\sqrt{c}}{a\sqrt{a}}.$$

Jam quaeratur k diebus, antequam cometa ad perihelium appellit, locus cometae, qui sit in M quem inveniendum sit angulus MSA seu arcus $MA = \varphi$ et $\text{tang } \frac{1}{2}\varphi = t$, erit $t + \frac{1}{2} \frac{dM}{ds} = N$ atque ope tabulae computatae ex numero Nk reperietur angulus φ . Sit arcus $\Omega A = A$, ita $\text{tang } A = \frac{\text{tang}(p - q)}{\cos s}$; erit arcus $\Omega M = A - \varphi$. Hinc in triangulo $\Omega m M$ ad m rectangulo $\sin Mm = \sin(A - \varphi) \sin s$ et $\text{tang } \Omega m = \text{tang}(A - \varphi) \cos s$. Erit ergo Mm latitudo cometae heliocentrica, et si ad Ωm addatur q longitudo nodi, prodibit longitudo cometae a prima stella arctica computata. Q. E. I.

14. **Coroll. 1.** In triangulo sphaerico $M\Omega m$ latera Mm et Ωm et angulus $M\Omega m = s$ invicem pendent, ut sit $\text{tang } Mm = \sin \Omega m \text{ tang } s$. Haecque aequatio ex duabus inventis resultare debet, si arcus $A - \varphi$ eliminetur.

15. **Coroll. 2.** Quodsi autem ex duabus aequationibus inventis $\sin Mm = \sin(A - \varphi) \sin s$ et $\text{tang } \Omega m = \text{tang}(A - \varphi) \cos s$ eliminetur angulus s , orietur haec aequatio

$$\cos(A - \varphi) = \cos Mm \cdot \cos \Omega m.$$

16. **Coroll. 3.** Quatuor ergo habentur aequationes, quae autem duabus tantum aequivalent istae,

$$\begin{aligned} \sin Mm &= \sin(A - \varphi) \sin s, & \text{tang } \Omega m &= \text{tang}(A - \varphi) \cos s \\ \text{tang } Mm &= \sin \Omega m \text{ tang } s, & \cos(A - \varphi) &= \cos Mm \cos \Omega m \end{aligned}$$

ex quibus binae, prouti commodum visum fuerit, ad usum adhiberi poterunt.

17. **Coroll. 4.** Data igitur, ex quibus longitudo ac latitudo heliocentrica cometae ad datum tempus assignatur, sunt primo ex orbita cometae desumpta. Nempe distantia nodi Ω a perihelio ex sole visa $= A$; deinde inclinatio orbitae cometae ad planum eclipticae, seu angulus $M\Omega m = s$. Porro, differentia inter tempus propositum et tempus, quo cometa perihelium attingit, quae in diebus expressa sit $= k$ dierum, ex qua ope tabulae ante datae sequitur angulus φ .

18. **Coroll. 5.** Quoniam cometae a sole distantia SM est $= a \sec^2 \frac{1}{2}\varphi$, erit ejus distantia a terra curtata, seu in ecliptica sumta $= a \sec^2 \frac{1}{2}\varphi \cdot \cos Mm$.

19. **Coroll. 6.** Si longitudo heliocentrica ponatur $= f$, et latitudo heliocentrica $= g$, fiet $f = \Omega m + q$, unde fiet

$$\begin{aligned} \text{I. } \sin g &= \sin(A - \varphi) \sin s, & \text{II. } \text{tang}(f - q) &= \text{tang}(A - \varphi) \cos s \\ \text{III. } \text{tang } g &= \sin(f - q) \text{ tang } s, & \text{IV. } \cos(A - \varphi) &= \cos g \cos(f - q). \end{aligned}$$

20. **Problema 4.** Fig. 217. Ex datis duobus cometæ locis heliocentricis invenire inclinationem orbitæ cometæ ad eclipticam, atque positionem nodorum Ω et \mathcal{S} .

Solutio. Observetur primum cometa in L , sitque longitudo heliocentrica $= f$ et latitudo borealis $= g$; præterea vero observetur cometa in M , sitque longitudo ejus $= f'$ et latitudo borealis $= g'$. Ponatur longitudo nodi ascendentis $\Omega = q$, et inclinatio orbitæ ad eclipticam, seu angulus $L\Omega l = s$. Erit igitur differentia longitudinum $lm = f' - f$. Ex æquatione ergo tertia (19) nascuntur hæc duæ æquationes

$$\text{tang } g = \sin(f - q) \text{ tang } s \quad \text{et} \quad \text{tang } g' = \sin(f' - q) \text{ tang } s$$

quarum hæc per illam divisa dabit

$$\frac{\text{tang } g'}{\text{tang } g} = \frac{\sin(f' - q)}{\sin(f - q)} = \frac{\sin(f - q + lm)}{\sin(f - q)}$$

Quæ autem sit $\sin(f - q + lm) = \sin(f - q) \cos lm + \cos(f - q) \sin lm$, fiet

$$\frac{\text{tang } g'}{\text{tang } g} = \cos lm + \frac{\sin lm}{\text{tang}(f - q)}$$

Hinc reperitur

$$\text{tang}(f - q) = \frac{\text{tang } g \sin lm}{\text{tang } g' - \text{tang } g \cos lm}$$

Innotescit ergo differentia longitudinum $\Omega l = f - q$, unde ob longitudinem puncti l datam definietur longitudo nodi ascendentis Ω ; qua cognita ob arcum $f - q$ datum erit $\text{tang } s = \frac{\text{tang } g}{\sin(f - q)}$, sicque inclinatio orbitæ cometæ ad planum eclipticæ, seu angulus $L\Omega l = s$ invenitur. Q: E. I.

21. **Coroll. 1.** Quoniam est $\text{tang}(f - q) = \frac{\text{tang } f - \text{tang } q}{1 + \text{tang } f \text{ tang } q}$ et $lm = f' - f$, erit longitudo nodi quaesita

$$\text{tang } q = \frac{\sin f \text{ tang } g' - \sin f' \text{ tang } g}{\cos f \text{ tang } g' - \cos f' \text{ tang } g}$$

22. **Coroll. 2.** Si ponatur $\text{tang } g = \alpha$, $\text{tang } g' = \beta$, $\sin lm = \sin(f' - f) = \mu$ et $\cos(f' - f) = \nu$, reperietur

$$\text{tang } s = \frac{\sqrt{(\alpha^2 + \beta^2 - 2\alpha\beta\nu)}}{\mu}$$

ujus fractionis numerator $\sqrt{(\alpha^2 + \beta^2 - 2\alpha\beta\nu)}$ est basis trianguli, cujus latera sunt α et β , angulum $= f' - f$ constituentia.

23. **Problema 5.** Si dato tempore, puta k dierum, vel ante vel post cometæ appulsum ad perihelium, detur distantia cometæ a perihelio e sole visa $= \rho$, invenire eandem distantiam tempore $k + x$ dierum vel ante vel post momentum, quo in perihelio versatur.

Solutio. Ponatur $\text{tang } \frac{1}{2} \rho = t$, erit $t + \frac{1}{3} t^3 = Nk$, seu $\text{tang } \frac{1}{2} \rho + \frac{1}{3} \text{tang}^3 \frac{1}{2} \rho = Nk$. Quæ iam angulus tempori $k + x$ dierum respondens $= \rho + \varphi$, erit posito brevitatis gratia $\text{tang } \frac{1}{2} \rho + \frac{1}{3} \text{tang}^3 \frac{1}{2} \rho = V$, per calculum differentiarum finitarum

$$\operatorname{tang} \frac{1}{2} (\nu + \varphi) + \frac{1}{3} \operatorname{tang}^3 \frac{1}{2} (\nu + \varphi) = 2V + \frac{\varphi dV}{d\nu} + \frac{\varphi^2 ddV}{2d\nu^2} + \frac{\varphi^3 d^3V}{6d\nu^3} + \text{etc.} = N\kappa$$

Cum igitur sit $V = N\kappa$, erit

$$N\kappa = \frac{\varphi dV}{d\nu} + \frac{\varphi\varphi ddV}{2d\nu^2} + \frac{\varphi^3 d^3V}{6d\nu^3} + \text{etc.}$$

$$\text{At est } \frac{dV}{d\nu} = \frac{1}{2 \cos^2 \frac{1}{2} \nu} + \frac{\operatorname{tang}^2 \frac{1}{2} \nu}{2 \cos^2 \frac{1}{2} \nu} = \frac{1}{2 \cos^4 \frac{1}{2} \nu}$$

$$\frac{ddV}{d\nu^2} = \frac{\sin \frac{1}{2} \nu}{\cos^5 \frac{1}{2} \nu}$$

$$\frac{d^3V}{d\nu^3} = \frac{1}{2 \cos^4 \frac{1}{2} \nu} + \frac{5 \sin^2 \frac{1}{2} \nu}{2 \cos^6 \frac{1}{2} \nu} = \frac{5}{2 \cos^6 \frac{1}{2} \nu} - \frac{2}{\cos^4 \frac{1}{2} \nu}$$

etc.

$$\text{Ex his ergo fiet } N\kappa = \frac{\varphi}{2 \cos^4 \frac{1}{2} \nu} + \frac{\varphi^2 \sin \frac{1}{2} \nu}{2 \cos^5 \frac{1}{2} \nu} + \frac{5 \varphi^3}{12 \cos^6 \frac{1}{2} \nu} - \frac{\varphi^3}{3 \cos^4 \frac{1}{2} \nu} + \text{etc.}$$

Ponamus jam esse $\varphi = \alpha N\kappa + \beta N^3 \kappa^2 + \gamma N^5 \kappa^3 + \text{etc.}$, atque facta substitutione habebimus

$$\begin{aligned} N\kappa = & \frac{\alpha N\kappa}{2 \cos^4 \frac{1}{2} \nu} + \frac{\beta N^3 \kappa^2}{2 \cos^4 \frac{1}{2} \nu} + \frac{\gamma N^5 \kappa^3}{2 \cos^4 \frac{1}{2} \nu} + \text{etc.} \\ & + \frac{\alpha^2 N^2 \kappa^2 \sin \frac{1}{2} \nu}{2 \cos^5 \frac{1}{2} \nu} + \frac{\alpha\beta N^3 \kappa^3 \sin \frac{1}{2} \nu}{\cos^5 \frac{1}{2} \nu} + \text{etc.} \\ & + \frac{5 \alpha^3 N^3 \kappa^3}{12 \cos^6 \frac{1}{2} \nu} \\ & - \frac{\alpha^3 N^3 \kappa^3}{3 \cos^4 \frac{1}{2} \nu} \end{aligned}$$

His ad aequalitatem reductis erit

$$\alpha = 2 \cos^4 \frac{1}{2} \nu$$

$$\beta = \frac{-\alpha^2 \sin \frac{1}{2} \nu}{\cos \frac{1}{2} \nu} = -4 \cos^7 \frac{1}{2} \nu \sin \frac{1}{2} \nu$$

$$\gamma = \frac{-2\alpha\beta \sin \frac{1}{2} \nu}{\cos \frac{1}{2} \nu} - \frac{5\alpha^3}{6 \cos^2 \frac{1}{2} \nu} + \frac{2\alpha^3}{3}, \text{ seu } \gamma = \frac{4}{3} \cos^{10} \frac{1}{2} \nu (7 - 8 \cos^2 \frac{1}{2} \nu)$$

etc.

Ex his igitur reperitur

$$\varphi = 2N\kappa \cos^4 \frac{1}{2} \nu - 4N^3 \kappa^2 \cos^7 \frac{1}{2} \nu \sin \frac{1}{2} \nu + \frac{4}{3} N^5 \kappa^3 \cos^{10} \frac{1}{2} \nu (7 - 8 \cos^2 \frac{1}{2} \nu)$$

quae expressio, nisi differentia temporis κ sit admodum magna, per approximationem satis praebet valorem ipsius φ . Primum enim si cometa a perihelio vehementer distet, angulus $\frac{1}{2} \nu$ multum ab angulo recto differret, ideoque ejus cosinus fractio erit perquam exigua. Hinc terminus secundus multo minor erit primo, ac tertius secundo; ita ut plerumque primus terminus possit ad φ exprimendum, quo invento erit angulus quaesitus $= \nu + \varphi$. Q. E. I.

Coroll. Si igitur cometa tempore k dierum a perihelio movetur per angulum φ , tempore $k + x$ dierum movebitur proxime per angulum $\varphi + 2N\kappa \cos^4 \frac{1}{2} \varphi$; vel si iste angulus propius designatus erit is $\varphi + 2N\kappa \cos^4 \frac{1}{2} \varphi - 4N^2 \kappa^2 \cos^7 \frac{1}{2} \varphi \sin \frac{1}{2} \varphi$, atque tertius terminus

$$\frac{4}{3} N^3 \kappa^3 \cos^{10} \frac{1}{2} \varphi (7 - 8 \cos^2 \frac{1}{2} \varphi)$$

sequentibus semper tuto negligi poterit, dummodo utroque tempore cometa vel ante vel post perihelium versetur.

Scholion 1. Quo haec approximatio magis confirmetur, sumamus exemplum cometae anni 1680 visi, pro quo erat $N = 26,70458$, qui numerus, cum sit multo major unitate, terminos parabolis, valorem ipsius φ exhibentis, crescentes efficere videatur. Cum autem iste cometa uno die per angulum plus quam 152° a perihelio circa solem moveatur, quia cometam diu ante vel post perihelium observari ponimus, erit $\varphi > 152^\circ$ et $\cos \frac{1}{2} \varphi < \sin 14^\circ$ et proinde $\cos \frac{1}{2} \varphi < 0,2419219$, hoc est $< \frac{1}{4}$. Hinc erit $\cos^4 \frac{1}{2} \varphi < \frac{1}{256}$, quo valore terminus $2N\kappa \cos^4 \frac{1}{2} \varphi$ valde redditur exiguus. Sumamus spatium k esse decem dierum, erit (8) $\varphi = 167^\circ 34'$ et $\frac{1}{2} \varphi = 83^\circ 47'$. Si jam hinc quaeramus angulum tempori $k + x$ dierum respondentem, sequens calculus instituitur:

$$\begin{array}{r} l \cos \frac{1}{2} \varphi = (-1),0345825 \\ l \sin \frac{1}{2} \varphi = (-1),9974386 \\ \hline lN = 1,4265857 \end{array} \quad \begin{array}{r} lN = 1,4265857 \\ l2 = 0,3010300 \\ \hline l \cos^4 \frac{1}{2} \varphi = (-4),1383300 \\ \hline (-3),8659457 \end{array}$$

ergo $2N \cos^4 \frac{1}{2} \varphi = 0,007344$.

$lN^2 = 2,8531714$

$l4 = 0,6020600$

$l \cos^7 \frac{1}{2} \varphi = (-7),2420775$

$l \sin \frac{1}{2} \varphi = (-1),9974386$

$(-4),6947475$

ergo $4N^2 \cos^7 \frac{1}{2} \varphi \sin \frac{1}{2} \varphi = 0,000495$.

In termino tertio utrumque membrum seorsim computetur, nempe

$$\begin{array}{r} lN^3 = 4,2797571 \\ l \frac{4}{3} = 0,1249387 \\ l7 = 0,8450980 \\ \hline l \cos^{10} \frac{1}{2} \varphi = (-10),3458250 \\ \hline (-5),5956188 \end{array} \quad \begin{array}{r} lN^3 = 4,2797571 \\ l \frac{4}{3} = 0,1249387 \\ l8 = 0,9030900 \\ \hline l \cos^{12} \frac{1}{2} \varphi = (-12),4149900 \\ \hline (-7),7227758 \end{array}$$

$$\text{ergo } \frac{28}{3} N^3 \cos^{10} \frac{1}{2} \varphi = 0,00003941$$

$$\text{ergo } \frac{32}{3} N^3 \cos^{12} \frac{1}{2} \varphi = 0,0000005282$$

Cometa ergo tempore 10 + x dierum a perihelio movetur per angulum

$$167^\circ 34' + 0,007344 x - 0,0004839 x^2 + 0,00003888 x^3$$

qui termini, nisi x decem dies superet, notabiliter decrescunt. Sit x spatium unius diei, erit

$$+ 0,007344$$

$$- 0,000484$$

$$+ 0,000039$$

$$\varphi = 0,006899$$

$$0,006690$$

$$\sin 23' = 0,006690$$

$$\sin 24' = 0,006981$$

291

$$291 : 60'' = 209 : 43''$$

ergo tempori undecim dierum respondet angulus $167^\circ 57' 43''$.

26. **Scholion 2.** Si ponamus x negativum, tum omnes termini seriei valorem ipsius φ exhibentis iisdem signis erunt affecti, ideoque series eo magis convergit. Quodsi ergo tempore $k - x$ dierum respondeat angulus φ a perihelio sumtus, seu anomalia vera, tempori $k - x$ dierum respon-
debit anomalia vera $\varphi - \varphi$, ita ut sit

$$\varphi = 2N^2 x \cos^4 \frac{1}{2} \varphi + 4N^2 x^2 \cos^7 \frac{1}{2} \varphi \sin \frac{1}{2} \varphi + \frac{4}{3} N^2 x^3 \cos^{10} \frac{1}{2} \varphi (7 - 8 \cos^2 \frac{1}{2} \varphi)$$

quae primo, uti vidimus, vehementer convergit, si angulus $\frac{1}{2} \varphi$ non multum deficiat ab angulo recta hoc est si cometa adhuc longe a perihelio distet, etiamsi hoc casu N sit numerus satis magnus. Quodsi autem N sit numerus multo minor, quod evenit si perihelium cometae longius a sole sit remotum, tum haec series satis convergit, etiamsi cometa non tantopere a perihelio distet.

27. **Problema 6.** Ex datis tribus cometae longitudinibus ac latitudinibus heliocentricae orbitam ipsius determinare.

Solutio. Sit longitudo perihelii = p , distantia perihelii a sole = a , distantia terrae a sole = c , ac ponatur $N = 0,012163763303 \cdot \frac{c \sqrt{a}}{a \sqrt{a}}$. Sit longitudo nodi ascendentis = q , inclinatio orbitae cometae ad eclipticam = s ; capiatur angulus r , ut sit $\text{tang } r = \frac{\text{tang } (p - q)}{\cos s}$, erit r distantia perihelii a nodo. Sint tres observationes sumtae diu antequam cometa ad perihelium pertingit, seu statim atque apparere incipit. Sit pro observatione

	I.	II.	III.
longitudo cometae heliocentrica	= f	f'	f''
latitudo cometae heliocentrica	= g	g'	g''
tempus inter observationem I. et II.	= x dierum		
inter I. et III.	= λ dierum.		

primam observationem ponamus cometam ad perihelium pertingere spatio k dierum, et sit
 omnia vera cometæ tempore primæ observationis $= \varphi$, tempore secundæ $= \varphi - \varphi$, tempore
 tertiæ $= \varphi - \psi$, erit, uti vidimus,

$$\varphi = 2N\kappa \cos^4 \frac{1}{2}\varphi + 4N^2\kappa^2 \cos^7 \frac{1}{2}\varphi \sin \frac{1}{2}\varphi + \text{etc.}$$

$$\psi = 2N\lambda \cos^4 \frac{1}{2}\varphi + 4N^2\lambda^2 \cos^7 \frac{1}{2}\varphi \sin \frac{1}{2}\varphi + \text{etc.}$$

jam erit per (19), posito r loco A

$$1) \sin g = \sin(r - \varphi) \sin s$$

$$2) \text{tang}(f - q) = \text{tang}(r - \varphi) \cos s$$

$$\sin g' = \sin(r - \varphi + \varphi) \sin s$$

$$\text{tang}(f' - q) = \text{tang}(r - \varphi + \varphi) \cos s$$

$$\sin g'' = \sin(r - \varphi + \psi) \sin s$$

$$\text{tang}(f'' - q) = \text{tang}(r - \varphi + \psi) \cos s$$

itemque

$$3) \text{tang} g = \sin(f - q) \text{tang} s$$

$$4) \cos(r - \varphi) = \cos g \cos(f - q)$$

$$\text{tang} g' = \sin(f' - q) \text{tang} s$$

$$\cos(r - \varphi + \varphi) = \cos g' \cos(f' - q)$$

$$\text{tang} g'' = \sin(f'' - q) \text{tang} s$$

$$\cos(r - \varphi + \psi) = \cos g'' \cos(f'' - q)$$

æquationibus N° 3 sequitur

$$\frac{\sin(f' - q)}{\sin(f - q)} = \frac{\text{tang} g'}{\text{tang} g} = \frac{\sin f' \cos q - \cos f' \sin q}{\sin f \cos q - \cos f \sin q}$$

undeque

$$\text{tang} q = \frac{\text{tang} g' \sin f - \text{tang} g \sin f'}{\text{tang} g' \cos f - \text{tang} g \cos f'}$$

Item valor pro longitudine nodi q prodire debet ex binis quibusvis aliis æquationibus ejusdem ordi-
 nis, siquidem observationes omni cura sunt institutæ; erit ergo pariter

$$\text{tang} q = \frac{\text{tang} g'' \sin f' - \text{tang} g' \sin f''}{\text{tang} g'' \cos f' - \text{tang} g' \cos f''}$$

Inventa autem longitudine nodi ascendentis q , simul innotescit inclinatio orbitæ cometæ ad eclipti-

nam s ex æquatione $\text{tang} s = \frac{\text{tang} g}{\sin(f - q)}$. Quia porro φ et ψ sunt anguli perquam exigui, erit

$$\sin(r - \varphi + \varphi) = \sin(r - \varphi) + \varphi \cos(r - \varphi) \quad \text{et} \quad \sin(r - \varphi + \psi) = \sin(r - \varphi) + \psi \cos(r - \varphi)$$

unde ex ordine æquationum primo habebitur

$$\frac{\sin g'}{\sin g} = 1 + \varphi \cot(r - \varphi) \quad \text{et} \quad \frac{\sin g''}{\sin g} = 1 + \psi \cot(r - \varphi)$$

$$\text{unde} \quad \frac{\sin g'' - \sin g}{\sin g' - \sin g} = \frac{\psi}{\varphi} = \frac{\lambda + 2N\lambda^2 \cos^3 \frac{1}{2}\varphi \sin \frac{1}{2}\varphi}{\kappa + 2N\kappa^2 \cos^3 \frac{1}{2}\varphi \sin \frac{1}{2}\varphi}$$

Præterea vero cum sit $\sin(r - \varphi) = \frac{\sin g}{\sin s}$, dabitur quoque $\cot(r - \varphi)$, unde erit

$$\frac{\sin g' - \sin g}{\sin g \cot(r - \varphi)} = \varphi = 2N\kappa \cos^4 \frac{1}{2}\varphi + 4N^2\kappa^2 \cos^7 \frac{1}{2}\varphi \sin \frac{1}{2}\varphi$$

$$\text{et} \quad \frac{\sin g'' - \sin g}{\sin g \cot(r - \varphi)} = \psi = 2N\lambda \cos^4 \frac{1}{2}\varphi + 4N^2\lambda^2 \cos^7 \frac{1}{2}\varphi \sin \frac{1}{2}\varphi$$

Ex his ergo aequationibus reperietur et valor numeri N , ex quo distantia perihelii a sole innotescit, et anomalia vera ν pro prima observatione, ex qua tempus k , quo cometa perihelium innotescit. Deinde vero ex cognito ν innotescit angulus r , hincque tandem longitudo perihelii Q. E. I.

28. **Coroll. 1.** Quoniam invenimus

$$\varphi = \frac{\sin g' - \sin g}{\sin g \cot(r - \nu)} \quad \text{et} \quad \psi = \frac{\sin g'' - \sin g}{\sin g \cot(r - \nu)},$$

atque angulus $r - \nu$ datus est ex aequatione $\sin(r - \nu) = \frac{\sin g}{\sin s}$, dabuntur decremēta anomaliae verae ν in observatione secunda et tertia, quae sunt φ et ψ .

29. **Coroll. 2.** Quoniam ergo dantur φ et ψ , erit ex (23)

$$N\kappa = \frac{\varphi}{2 \cos^4 \frac{1}{2} \nu} + \frac{\varphi \varphi \sin \frac{1}{2} \nu}{2 \cos^5 \frac{1}{2} \nu} + \text{etc.}$$

$$N\lambda = \frac{\psi}{2 \cos^4 \frac{1}{2} \nu} + \frac{\psi \psi \sin \frac{1}{2} \nu}{2 \cos^5 \frac{1}{2} \nu} + \text{etc.}$$

hincque eliminando numerum N erit

$$\frac{\lambda}{\kappa} = \frac{\psi \cos \frac{1}{2} \nu + \psi^2 \sin \frac{1}{2} \nu}{\varphi \cos \frac{1}{2} \nu + \varphi^2 \sin \frac{1}{2} \nu}, \quad \text{seu} \quad \lambda \varphi + \lambda \varphi^2 \tan \frac{1}{2} \nu = \kappa \psi + \kappa \psi^2 \tan \frac{1}{2} \nu,$$

ex qua expedite reperitur anomalia vera ν , cum sit

$$\tan \frac{1}{2} \nu = \frac{\kappa \psi - \lambda \varphi}{\lambda \varphi^2 - \kappa \psi^2}.$$

30. **Coroll. 3.** Invento ergo hoc modo angulo ν , ex eo statim angulus r , hincque longitudo perihelii p innotescit per aequationes $\sin(r - \nu) = \frac{\sin g}{\sin s}$ et $\tan(p - q) = \tan r \cos s$. Tum vero etiam numerus N definitur ex aequatione

$$N = \frac{\varphi}{2\kappa \cos^4 \frac{1}{2} \nu} + \frac{\varphi^2 \sin \frac{1}{2} \nu}{2\kappa \cos \frac{1}{2} \nu}$$

ex quo porro distantia perihelii a sole a determinatur.

31. **Scholion.** Si ob cosinum anguli $\frac{1}{2} \nu$ valde parvum, series, qua numerus N definitur parum convergat, calculus sine approximatione, postquam φ et ψ sunt inventa, institui poterit modo: Cum sit $N\kappa = \tan \frac{1}{2} \nu + \frac{1}{3} \tan^3 \frac{1}{2} \nu$, erit

$$N(\kappa - \kappa) = \tan \frac{\nu - \varphi}{2} + \frac{1}{3} \tan^3 \frac{\nu - \varphi}{2}$$

hincque $N\kappa = \tan \frac{1}{2} \nu - \tan \frac{\nu - \varphi}{2} + \frac{1}{3} \tan^3 \frac{1}{2} \nu - \frac{1}{3} \tan^3 \frac{\nu - \varphi}{2}$. Sit $\tan \frac{1}{2} \nu = t$ et

$\tan \frac{1}{2} \varphi = \mu$, itemque $\tan \frac{1}{2} \psi = \nu$, erit $\tan \frac{\nu - \varphi}{2} = \frac{t - \mu}{1 + \mu t}$ et $\tan \frac{\nu - \psi}{2} = \frac{t - \nu}{1 + \nu t}$. His sub-

stitutis erit

$$N\alpha = \frac{\mu(1+tt)}{1+\mu t} + \frac{\mu t(1+tt) - \mu^2 t(1-t^4) + \frac{1}{3}\mu^3(1+t^6)}{(1+\mu t)^3}$$

$$\text{et } N\lambda = \frac{\nu(1+tt)}{1+\nu t} + \frac{\nu t(1+tt) - \nu^2 t(1-t^4) + \frac{1}{3}\nu^3(1+t^6)}{(1+\nu t)^3}$$

his eliminando N prodibit ista aequatio

$$\frac{\lambda(1+\nu t)^3}{\alpha(1+\mu t)^3} = \frac{\nu(1+\nu t)^2 + \nu t t - \nu^2 t(1-tt) + \frac{1}{3}\nu^3(1-tt+t^4)}{\mu(1+\mu t)^2 + \mu t t - \mu^2 t(1-tt) + \frac{1}{3}\mu^3(1-tt+t^4)}$$

$$\text{sive } \frac{\lambda(1+\nu t)^3}{\alpha(1+\mu t)^3} = \frac{\nu + \nu^2 t + \frac{1}{3}\nu^3(1+tt)}{\mu + \mu^2 t + \frac{1}{3}\mu^3(1+tt)}$$

haec igitur aequatione etsi quinti gradus, si methodo praecedente jam prope valor ipsius t innotuit, satis cito verus valor ipsius t colligi poterit. Quo invento tam numerus N quam tempus k determinatum assignabitur.

32. **Scholion 2.** Quanquam ante valores φ et ψ tantum per approximationem invenimus, observationes tres invicem proximas assumentes, tamen inveniri quoque possunt exacte, etiamsi observationes maxime a se invicem distent. Inventis enim g et s modo praescripto, qui nulla approximatione nitentur, statim innotescit angulus $r - \nu$, cum sit

$$\text{vel } \sin(r - \nu) = \frac{\sin g}{\sin s}, \quad \text{vel } \tan g(r - \nu) = \frac{\tan(g - s)}{\cos s}.$$

hinc porro innotescit angulus φ ex aequatione $\sin(r - \nu + \varphi) = \frac{\sin g'}{\sin s}$, et angulus ψ ex aequatione $\sin(r - \nu + \psi) = \frac{\sin g''}{\sin s}$. Inventis ergo φ et ψ methodo in Scholio praecedente exhibita reperientur N et c . Ex datis ergo tribus quibuscunque locis cometae heliocentricis, longitudinibus scilicet ac latitudinibus, orbita cometae exactissime determinari poterit. Praestat tamen antequam huiusmodi modus adhibeatur, ex tribus observationibus invicem proximis et a perihelio longe remotis orbitam cometae vero proxime determinare; quo aequatio superior quinque dimensionum facilius resolvi poterit. Cum enim hoc casu fiant μ et ν valde parva, erit proxime

$$\frac{\lambda(1+\nu t)^3}{\alpha(1+\mu t)^3} = \frac{\nu}{\mu} \quad \text{et} \quad \frac{1+\nu t}{1+\mu t} = \frac{\sqrt[3]{\nu\lambda}}{\sqrt[3]{\lambda\mu}}, \quad \text{unde } t = \frac{\sqrt[3]{\lambda\mu} - \sqrt[3]{\nu\lambda}}{\mu\sqrt[3]{\nu\lambda} - \nu\sqrt[3]{\lambda\mu}};$$

quo valore prope vero facile valor exactior elicietur.