XXV.

De amplificatione campi apparentis in telescopiis.

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1. Duae res potissimum ad perfectionem telescopiorum, quae quidem objecta clare ac distincte repraesentent, requiruntur, quarum altera ad lentem, quam vocare solent objectivam, spectat, cujus perfectio in eo consistit, ut pro apertura quantumvis magna repraesentatio imaginis nulla confusione inquinetur, atque insuper ejus ope confusio a reliquis lentibus oriunda destrui possit: hoc quippe modo lente objectiva uti licebit, cujus distantia foci non est adeo magna, sicque tota telescopii longitudo maxime contrahitur, ex quo utique summum commodum in praxin promanat. Quemadmodum autem hujusmodi lentes objectivas ex duabus lentibus, altera convexa, altera concava, componi oporteat, alio loco fusius exposui, ubi simul ostendi hujusmodi lentes composita in combinatione cum quotcunque aliis instar simplicium considerari posse, et quia exiguum lentium illarum intervallum variationem quampiam admittit, hoc modo confusionem a cunctis lentibus oriundam ad nihilum redigere licet.

2. Altera autem res in amplitudine campi per telescopium conspicui sita est, quae quo fuerit major, eo perfectius merito telescopium existimatur, quin etiamsi haec instrumenta a nimia longitudine liberari non possent, insignis campi apparentis amplificatio hoc incommodum largiter compensare censeretur. In navigatione certe usus majorum telescopiorum non tam ob ingentem eorum longitudinem excluditur, quam ideo, quod nimis exiguum campum complectuntur, quo fit, ut vel levissima facta agitatione objecta subito dispareant, quod non eveniret, si campum satis magnum suggererent. Verum sine dubio summus perfectionis gradus attingetur, si cum insigni campo modica ejus instrumenti longitudo fuerit conjuncta, ubi quidem telescopium ad datam multiplicationem producendam accomodatam intelligi debet.

3. In vulgaribus quidem telescopiis, quae duabus lentibus convexis sunt instructa, notum est campi conspicui amplitudinem ab apertura lentis ocularis pendere, quae cum certos limites per ejus naturam praescriptos transgredi nequeat, hinc terminus campo statuitur. Foci quidem distantia per

multiplicationem determinatur, et cum pro eodem foco innumerabiles lentes exhiberi queant, maxime aperturae est capax, quae utrinque est aequaliter convexa, unde hoc praeceptum haud exigui momenti derivatur, ut lentes oculares utrinque aequaliter convexae conficiantur. Tum vero aliae rationes exigunt, ut apertura nullos arcus 30 gradibus majores comprehendat, ex quo sequitur diametrum aperturae cujusque lentis ocularis semissem ejus distantiae foci superare non debere. Hinc autem si objecta in ratione $m:1$ secundum diametrum multiplicentur, sequitur diametrum campi apparentis fore $= \frac{1}{2(m+1)}$ in partibus radii, seu tot $\frac{1718}{m+1}$ minutorum primorum.

4. Quamdiu autem unica lente oculari utimur, campum apparentem ultra hunc terminum augere non licet. Jam pridem igitur lente oculari geminata uti coeperunt, dum duas lentes aequales, quarum utriusque distantia foci duplo esset major, junxerunt, hocque modo fere campum duplo majorem obtinuerunt. Quodsi distantia inter lentes revera pro nihilo haberi posset, binae lentes quarum utrius distantia foci esset $= 2p$, hoc modo junctae lenti simplici aequivalerent, cujus distantia foci foret $= p$, et quoniam duplo majorem aperturam admitterent, etiam campum duplo majorem essent exhibiturae. Simili modo etiam tres lentes aequales, distantia focali uniuscujusque existente $= 3p$, sibi immediate junctae loco simplicis, cujus distantia focalis est p, substitui possent, sicque campum triplo latiorem largirentur, siquidem singulae utrinque conficerentur aequaliter convexae. Hocque modo ulterius progrediendo campum, quousque libuerit, amplificare liceret.

5. Verum plurimum abest, quominus distantia binarum lentium pro nihilo haberi queat; dum enim hae lentes aperturam admittere debent, cujus diameter semissi distantiae focalis aequetur, earum crassitiem partem decimam sextam distantiae focalis superare necesse est; quo fit ut, cum vera binarum lentium distantia ex intervallo earum quasi centrorum in medie crassitie sitorum aestimari debeat, nunquam ea tanquam nulla spectari possit. Neque ergo binae lentes junctae pro simplici haberi, neque exacte duplo majorem campum aperient, ac multo minus haec campi multiplicatio succedet pluribus lentibus inter se conjungendis, cum earum crassities spatium satis notabile esset completura. Interim tamen negari nequit, hac ratione duabus pluribusve lentibus conjungendis campum amplificari, verum lucrum propemodum obscuritate et confusione ex tot lentibus enata iterum deletur. Imprimis autem tali lentium multiplicatione colores iridis visioni se admiscere solent, quod incommodum distinctae repraesentationi maxime nocet; haecque sine dubio vera est causa, quod hoc artificium campum augendi vix ad usum adhibeatur.

6. Hic igitur ostendere constitui, quomodo multiplicatione lentium ocularium campus visionis non solum amplificari, sed etiam repraesentatio a coloribus illis vagis liberari queat. Ad hoc autem lentes non immediate inter se jungi convenit; experientia enim jam constat tubis astronomicis, qui duabus tantum lentibus constant, insigni cum utilitate tertiam lentem ultra focum ocularis inseri posse, qua non solum campus duplicetur, sed etiam colores illi vagi penitus deleantur. Imprimis autem in dissertatione mea de instrumentorum dioptricorum perfectione Vol. XIII Actorum Academiae Regiae inserta formulas exhibui generales tam pro campo amplificando, quam pro coloribus illis destruendis. Inde ergo subsidia necessaria depromam, unde numerum lentium ocularium augendo campus quoque sine ullo incommodo ampliari possit. Semper scilicet ejusmodi binas lentes oculares

licet, quibus diameter campi apparentis duplicetur; tum vero etiam ternis lentibus combi-natis campi diameter triplicari, quaternis autem quadruplicari potest et ita porro: unde secundum ordinem progrediendo primo constructionem hujusmodi lentium ocularium duplicatarum, tum triplicatarum, deinde etiam quadruplicatarum docebo, quas investigationes quousque libuerit ulterius continuare licet.

De lentibus ocularibus duplicatis, quibus campus duplicatur.

7. Sit α distantia focalis lentis objectivae in A, sive ea sit simplex, sive composita ad confusionem superandam, et $m:1$ denotet rationem, qua objecta secundum diametrum multiplicari debeant, unde statim apertura lentis objectivae definitur, ut sufficientem luminis copiam excipiat. Binae lentes oculares positae sint in B et C, pro quibus sint litterae in dissertatione mea adhibitae:

$$B = \frac{-b}{1+b}, \quad \mathfrak{B} = \frac{B}{1+B} = -b, \quad C = \infty \quad \text{et} \quad \mathfrak{C} = 1.$$

Porro autem cum sit campi apparentis semidiameter $\varphi = \frac{\pi - \pi^{I}}{m+1}$, ubi fractiones π, π^{I} certum terminum, veluti $\frac{1}{4}$, pro quo autem hic generaliter scribam ω, superare nequeunt, quare ut campus reddatur maximus, ponamus $\pi = \omega$ et $\pi^{I} = -\omega$, fietque $\varphi = \frac{2\omega}{m+1}$, seu posito brevitatis gratia $\frac{2}{m+1} = M$, erit $\varphi = M\omega$, hincque $\mathfrak{B}\pi - \varphi = -(b+M)\omega$ et $\mathfrak{C}\pi^{I} - \pi + \varphi = -(2-M)\omega$.

8. Ex his per formulas meas colligitur distantia focalis lentis in $B = \frac{bM}{b+M}\alpha = p^{I}$, et lentis postremae in $C = \frac{b}{1+b} \cdot \frac{M}{2-M}\alpha = p^{II}$; praeterea vero intervalla:

$$AB = \frac{b}{b+M}\alpha \quad \text{et} \quad BC = \frac{b}{1+b} \cdot \frac{M(2+b)}{(b+M)(2-M)}\alpha,$$

et pro loco oculi distantia $CO = \frac{1}{Mm} \cdot p^{II}$, at ob $Mm = 2 - M$, erit $CO = \frac{b}{1+b} \cdot \frac{M}{(2-M)^2} \cdot \alpha$. Si unica lente oculari uteremur, pro eadem multiplicatione ejus distantia focalis esse deberet $= \frac{\alpha}{m}$, unde binae lentes B et C, quas hic definiemus, aequivalebunt lenti simplici, cujus distantia focalis esset $= \frac{\alpha}{m}$. Quae quo distinctius evolvamus, sit distantia focalis lentis $B = q$, lentis $C = r$ et lentis simplicis ipsis aequivalentis $= k$, habebimusque has determinationes:

$$k = \frac{\alpha}{m}, \qquad q = \frac{bM}{b+M}\alpha, \qquad r = \frac{b}{1+b} \cdot \frac{M}{2-M}\alpha,$$

ac distantias:

$$AB = \frac{b}{b+M}\alpha, \qquad BC = \frac{b}{1+b} \cdot \frac{M(2+b)}{(b+M)(2-M)}\alpha, \qquad CO = \frac{b}{1+b} \cdot \frac{M}{(2-M)^2}\alpha,$$

existente $M = \frac{2}{m+1}$. Unde colligimus fore:

$$BC = \frac{q(k-r)}{k} + r \quad \text{et} \quad m = \frac{q(2k-r)}{r(2k-q)},$$

ita ut haec convenientia simul a multiplicatione m pendeat.

9. Verum quod hic praecipuum est, originem colorum vagorum destruamus, quod aequatione adimplenda:

$$\frac{\pi}{\mathfrak{B}\pi - \varphi} + \frac{\pi^I}{\mathfrak{C}\pi^I - \pi + \varphi} = 0, \quad \text{seu} \quad \frac{-1}{b + M} + \frac{1}{2 - M} = 0,$$

unde conficitur $b = 2 - 2M$ et $2 + b = 2(2 - M)$. Hoc ergo valore substituto, has pro nostra oculari composita adipiscimur formulas:

$$k = \frac{\alpha}{m}, \quad q = \frac{2M(1-M)}{2-M}\alpha, \quad r = \frac{2-2M}{3-2M} \cdot \frac{M}{2-M}\alpha$$

et: $$AB = \frac{2(1-M)}{2-M}\alpha, \quad BC = \frac{4(1-M)M}{(3-2M)(2-M)}\alpha \quad \text{et} \quad CO = \frac{2-2M}{3-2M} \cdot \frac{M}{(2-M)^2}\alpha.$$

Quae pro M restituto valore $\frac{2}{m+1}$, in sequentes abeunt formas:

$$k = \frac{\alpha}{m}, \quad q = \frac{2(m-1)}{m+1} \cdot \frac{\alpha}{m}, \quad r = \frac{2(m-1)}{3m-1} \cdot \frac{\alpha}{m}$$

et: $$AB = \frac{m-1}{m}\alpha, \quad BC = \frac{4(m-1)}{3m-1} \cdot \frac{\alpha}{m} \quad \text{et} \quad CO = \frac{m+1}{2m} r = \frac{mm-1}{m(3m-1)} \cdot \frac{\alpha}{m}.$$

10. Determinatio ergo hujus lentis ocularis compositae non solum a distantia focali k lentis simplicis, cui aequivalet, pendet, sed etiam a multiplicatione, cui producendae destinatur. Cum autem sit

$$q = \frac{2(m-1)}{m+1}k, \quad r = \frac{2(m-1)}{3m-1}k, \quad BC = 2r \quad \text{et} \quad CO = \frac{m+1}{2m}r,$$

patet pro majoribus multiplicationibus omne discrimen inter haec elementa evanescere, ita ut omnia per solam distantiam focalem lentis simplicis aequivalentis k determinentur. Patet autem distantiam binarum lentium BC semper duplo esse majorem distantia focali lentis postremae r, quae triente fere minor est, quam distantia focalis lentis simplicis aequivalentis, k, at distantia focalis lentis prioris q fere triplo major est quam r. Quo autem pateat quantillam variationem ratio multiplicationis m in haec elementa inferat, ea his formulis vero proximis exprimamus:

$$q = 2k - \frac{4}{m}k, \quad r = \frac{2}{3}k - \frac{4}{9m}k, \quad BC = \frac{4}{3}k - \frac{8}{9m}k, \quad CO = \frac{1}{3}k + \frac{1}{9m}k.$$

11. Proposita ergo lente objectiva, cujus distantia focalis sit $= \alpha$, quae idonea judicetur ad multiplicationem m producendam, inde statim colligitur distantia focalis lentis ocularis simplicis $k = \frac{\alpha}{m}$, ex qua porro constructio lentis ocularis duplicatae cognoscitur, quae duplo majorem campum aperiet, quam si lente simplici uteremur, si modo hae binae lentes B et C utrinque aequaliter convexae conficiantur, ut maximae aperturae, cujus diameter verbi gratia semissi distantiae focalis aequetur, fiant capaces, quod fit si posita ratione refractionis ex aëre in vitrum ut ζ ad η et distantia focali $= p$, utriusque faciei radius statuatur $= \frac{2(\zeta - \eta)}{\eta}p$, unde si ratio refractionis fuerit ut 31 ad 20, hic radius erit $= \frac{11}{10}p$.

12. Plurimum autem quoque interest nosse partem confusionis, quae ab his duabus lentibus perturbandam repraesentationem nascitur, et quae pendet a numeris λ^I et λ^{II} ad has lentes relaquae cum sint utrinque aequaliter convexae, posita ratione refractionis ex aëre in vitrum ut 31 (§ ... erit $\lambda^I = 1 + 0,62979 \left(\frac{B-1}{B+1}\right)^2$ et $\lambda^{II} = 1,62979$; at ob $B = \frac{-b}{1+b}$ erit $\frac{B-1}{B+1} = -2b-1 = (5-4M) = -\frac{(5m-3)}{m+1}$, ideoque $\lambda^I = 1 + 0,62979 \left(\frac{5m-3}{m+1}\right)^2$. Cum igitur posito $\nu = 0,23269$ in genere pro quotcunque lentibus confusio ut haec expressio:

$$\lambda + \frac{(\lambda^I + \nu \mathfrak{B}(1-\mathfrak{B}))}{\mathfrak{B}^4} \cdot \frac{q}{a} + \frac{\lambda^{II} + \nu \mathfrak{C}(1-\mathfrak{C})}{B^4 \mathfrak{C}^4} \cdot \frac{r}{a} + \frac{\lambda^{III} + \nu \mathfrak{D}(1-\mathfrak{D})}{B^4 C^4 \mathfrak{D}^4} \cdot \frac{s}{a} + \text{ etc.}$$

ob

$$\mathfrak{B} = -b, \text{ et } B = \frac{-b}{1+b}; \ \mathfrak{C} = 1, \ \frac{q}{a} = \frac{bM}{b+M} \text{ et } \frac{r}{a} = \frac{b}{1+b} \cdot \frac{M}{2-M},$$

pro nostro casu ista expressio hanc induet formam:

$$\lambda + \frac{\lambda^I - \nu b(1+b)}{b^3} \cdot \frac{M}{b+M} + \frac{\lambda^{II}(1+b)^3}{b^3} \cdot \frac{M}{2-M},$$

quae ob $b = 2-2M$ abit in hanc:

$$\lambda + \frac{\lambda^I - 2\nu(1-M)(3-2M)}{8(1-M)^3} \cdot \frac{M}{2-M} + \frac{\lambda^{II}(3-2M)^3}{8(1-M)^3} \cdot \frac{M}{2-M},$$

et ob $M = \frac{2}{m+1}$ in hanc:

$$\lambda + \frac{1}{m}\left(\frac{\lambda^I(m+1)^3 - 2\nu(m+1)(m-1)(3m-1)}{8(m-1)^3} + \frac{\lambda^{II}(3m-1)^3}{8(m-1)^3}\right),$$

quae denique ad istam formam reducitur:

$$\lambda + \frac{1}{8m(m-1)^3}\left((m+1)^3 + 0,62979(m+1)(5m-3)^2 - 0,46538(m+1)(m-1)(3m-1) + (3m-1)^3 + 0,62979(3m-1)^3\right),$$

cujus valor, si multiplicatio est maxima, fit $\lambda + \frac{7,41912}{m}$. Confusio ergo ex binis lentibus ocularibus fere quinquies major est censenda, quam si lente oculari simplici uteremur, cum haec foret $\lambda + \frac{1,62979}{m}$.

De lentibus ocularibus triplicatis, quibus campus triplicatur.

13. Sit a semper distantia focalis lentis objectivae A, et $m:1$ ratio multiplicationis, unde lentis ocularis simplicis distantia focalis foret $k = \frac{a}{m}$; ejus autem loco hic tres lentes B, C, D introducamus, pro quarum aperturis sint indices π, π^I, π^{II}. Cum igitur sit campi semidiameter $\varphi = \frac{\pi - \pi^I + \pi^{II}}{m+1}$, ut is maximus reddatur, statuatur $\pi = \omega$, $\pi^I = -\omega$ et $\pi^{II} = \omega$, existente ..., si lentes utrinque fuerint convexae et aperturam admittant, cujus semidiameter parti quartae distantiae focalis aequetur. Erit ergo $\varphi = \frac{3\omega}{m+1}$; ponamus autem $M = \frac{3}{m+1}$, ut sit $\varphi = M\omega$.

Tum vero pro his tribus lentibus ponamus $B = \frac{-b}{1+b}$, $\mathfrak{B} = -b$, $C = -1$, $\mathfrak{C} = \infty$, $D = \infty$ et $\mathfrak{D} = 1$, unde nanciscimur:

$$\mathfrak{B}\pi - \varphi = -(b+M)\,\omega, \quad \mathfrak{C}\pi^{I} - \pi + \varphi = -\mathfrak{C}\omega \quad \text{et} \quad \mathfrak{D}\pi^{II} - \pi^{I} + \pi - \varphi = (3-M)\,\omega$$

et destructio colorum praebet hanc aequationem:

$$-\frac{1}{b+M} + \frac{1}{\infty} + \frac{1}{3-M} = 0 \quad \text{hincque} \quad b+M = 3-M \quad \text{et} \quad b = 3-2M.$$

14. Quodsi porro distantias focales trium lentium B, C, D designemus litteris q, r, s, erit:

$$q = \frac{bM}{b+M}\,\alpha, \quad r = \frac{b}{1+b}\cdot M\alpha, \quad s = \frac{b}{1+b}\cdot\frac{M}{3-M}\,\alpha,$$

et lentium intervalla:

$$AB = \frac{b}{b+M}\,\alpha, \quad BC = \frac{b}{1+b}\cdot\frac{M}{b+M}\,\alpha, \quad CD = \frac{b}{1+b}\cdot\frac{M}{3-M}\,\alpha,$$

at pro loco oculi O est distantia $DO = \frac{s}{3-M}$. Substituamus ergo loco b valorem inventum $3-2M$, ac reperiemus has determinationes:

$$q = \frac{(3-2M)M}{3-M}\,\alpha, \quad r = \frac{(3-2M)M}{2(2-M)}\,\alpha, \quad s = \frac{(3-2M)M}{2(2-M)(3-M)}\,\alpha \quad \text{et}$$

$$AB = \frac{3-2M}{3-M}\,\alpha, \quad BC = \frac{(3-2M)M}{2(2-M)(3-M)}\,\alpha, \quad CD = \frac{(3-2M)M}{2(2-M)(3-M)}\,\alpha,$$

existente $k = \frac{\alpha}{m}$.

15. Cum nunc sit $M = \frac{3}{m+1}$, ideoque $3 - M = Mm$, erit:

$$q = \frac{3(m-1)}{m+1}\cdot\frac{\alpha}{m}, \quad r = \frac{9(m-1)}{2(m+1)(2m-1)}\,\alpha, \quad s = \frac{3(m-1)}{2(2m-1)}\cdot\frac{\alpha}{m},$$

$$AB = \frac{m-1}{m}\,\alpha, \quad BC = \frac{3(m-1)}{2(2m-1)}\cdot\frac{\alpha}{m}, \quad CD = \frac{3(m-1)}{2(2m-1)}\cdot\frac{\alpha}{m},$$

et pro oculo $DO = \frac{(m+1)}{3m}\,s = \frac{mm-1}{2m(2m-1)}\cdot\frac{\alpha}{m}$. Ac si introducamus distantiam focalem k lentis simplicis aequivalentis, habebimus:

$$q = \frac{3(m-1)}{m+1}\,k, \quad r = \frac{9m(m-1)}{2(m+1)(2m-1)}\,k, \quad s = \frac{3(m-1)}{2(2m-1)}\,k, \quad \text{ac} \quad BC = CD = s,$$

quae formulae, si multiplicatio m sit satis magna, abeunt in:

$$q = 3k - \frac{6}{m}\,k, \quad r = \frac{9}{4}\,k - \frac{27}{8m}\,k. \quad s = \frac{3}{4}\,k - \frac{3}{8m}\,k,$$

quas tres lentes ita jungi oportet, ut bina intervalla BC et CD sint distantiae focali lentis postremae s aequalia.

16. Quod ad confusionem attinet ex his tribus lentibus ocularibus natam, ex formulis supra datis ponendo $\mu = 0,62979$, uti est $\nu = 0,23269$, habemus:

$$\lambda^I = 1 + \mu\left(\frac{B-1}{B+1}\right)^2 = 1 + \mu(2b+1)^2 = 1 + \mu\left(\frac{7m-5}{m+1}\right)^2, \quad \lambda^{II} = 1 + \mu\left(\frac{C-1}{C+1}\right)^2 = 1 + \mu(2\mathfrak{C}-1)^2.$$

Qui valor ob $\mathfrak{C} = \infty$ quidem est infinitus, sed quia hoc membrum per biquadratum $\mathfrak{C}^4$ dividitur totum evanescit. Pro ultima lente est $\lambda^{III} = 1 + \mu$. Cum igitur sit:

$$\frac{q}{\alpha} = \frac{3(m-1)}{m(m+1)} \quad \text{et} \quad \frac{s}{\alpha} = \frac{3(m-1)}{m(4m-2)},$$

expressio confusionem exhibens reperitur:

$$\lambda + \frac{1}{27m(m-1)^3}\left((m+1)^3 + \mu(m+1)(7m-5)^2 - 6\nu(m+1)(m-1)(2m-1) + (4m-2)^3 + \mu(4m-2)^3\right),$$

quae si m ut numerus infinitus spectetur, fit:

$$\lambda + \frac{1}{m}\cdot\frac{65 + 113\mu - 12\nu}{27} = \lambda + \frac{133,3740}{27m},$$

sicque pars confusionis ex lentibus ocularibus nata est $\frac{4,939}{m}$, quae ergo multo minor est quam casu praecedente.

De lentibus ocularibus quadruplicatis, quibus campus quadruplicatur.

17. Positis aperturae indicibus pro his quatuor lentibus B, C, D et E, π, π^I, π^{II} et π^{III}, cum sit semidiameter campi apparentis $\varphi = \frac{\pi - \pi^I + \pi^{II} - \pi^{III}}{m+1}$, ut is maximus evadat, statuamus $\pi = \omega$, $\pi^I = -\omega$, $\pi^{II} = \omega$ et $\pi^{III} = -\omega$, eritque $\varphi = \frac{4\omega}{m+1}$, qui valor quadruplo major est, quam si lente oculari simplici uteremur. Sit autem commoditatis gratia $M = \frac{4}{m+1}$, seu $\varphi = M\omega$, tum vero pro quatuor nostris lentibus ocularibus statuamus:

$$B = \frac{-b}{1+b}, \qquad C = \frac{-c}{1+c}, \qquad D = \frac{-d}{d-1}, \qquad E = \infty,$$

$$\mathfrak{B} = -b, \qquad \mathfrak{C} = -c, \qquad \mathfrak{D} = d, \qquad \mathfrak{E} = 1,$$

earum autem distantiae focales sint q, r, s et t.

18. Cum jam hinc sit:

$$\mathfrak{B}\pi - \varphi = -(b + M)\omega,$$
$$\mathfrak{C}\pi^I - \pi + \varphi = (c - 1 + M)\omega,$$
$$\mathfrak{D}\pi^{II} - \pi^I + \pi - \varphi = (d + 2 - M)\omega,$$
$$\mathfrak{E}\pi^{III} - \pi^{II} + \pi^I - \pi + \varphi = -(4 - M)\omega,$$

tam distantiae focales lentium, quam earum intervalla ita exprimuntur:

$$q = \frac{bM}{b+M}\alpha, \qquad AB = \frac{b}{b+M}\alpha,$$

$$r = \frac{b}{1+b}\cdot\frac{cM}{c-1+M}\alpha, \qquad BC = \frac{b}{1+b}\cdot\frac{M(c-b-1)}{(b+M)(c-1+M)}\alpha,$$

$$s = \frac{bc}{(1+b)(1+c)}\cdot\frac{dM}{d+2-M}\alpha, \qquad CD = \frac{bc}{(1+b)(1+c)}\cdot\frac{M(c+d-1)}{(c-1+M)(d+2-M)}\alpha,$$

$$t = \frac{bcd}{(1+b)(1+c)(d-1)}\cdot\frac{M}{4-M}\alpha, \qquad DE = \frac{bcd}{(1+b)(1+c)(d-1)}\cdot\frac{M(d-2)}{(d+2-M)(4-M)}\alpha.$$

Pro loco oculi autem habemus $EO = \frac{t}{4-M}$. Hinc cum lentium intervalla necessario sint positiva, oportet esse $c > b+1$ et $d > 2$.

19. Consideremus nunc etiam formulam, qua apparitio colorum vagorum iridis tollitur, quae cum in genere sit:

$$\frac{\pi}{\mathfrak{B}\pi-\varphi} + \frac{\pi^{I}}{\mathfrak{C}\pi^{I}-\pi+\varphi} + \frac{\pi^{II}}{\mathfrak{D}\pi^{II}-\pi^{I}+\pi-\varphi} + \frac{\pi^{III}}{\mathfrak{E}\pi^{III}-\pi^{II}+\pi^{I}-\pi+\varphi} + \text{etc.} = 0,$$

pro nostro casu habemus:

$$-\frac{1}{b+M} - \frac{1}{c-1+M} + \frac{1}{d+2-M} + \frac{1}{4-M} = 0.$$

Quia nunc $c-1 > b$ et $d+2 > 4$, ponamus $c-1+M = \frac{b+M}{1-\xi}$ et $d+2-M = \frac{4-M}{1-\eta}$, ubi ξ et η denotant fractiones unitate minores, ac nostra aequatio induet hanc formam:

$$\frac{2-\xi}{b+M} = \frac{2-\eta}{4-M}, \quad \text{ideoque} \quad b+M = \frac{2-\xi}{2-\eta}(4-M),$$

unde litterarum b, c, d hos eruimus valores:

$$b = \frac{4(2-\xi)-(4-\xi-\eta)M}{2-\eta},$$

$$c = \frac{10-6\xi-\eta+\xi\eta-(4-3\xi-\eta+\xi\eta)M}{(1-\xi)(2-\eta)},$$

$$d = \frac{2+2\eta-\eta M}{1-\eta},$$

unde sequitur fore:

$$c-b-1 = \frac{\xi(2-\xi)(4-M)}{(1-\xi)(2-\eta)}, \quad \text{et} \quad b+M = \frac{(2-\xi)(4-M)}{2-\eta},$$

$$c+d+1 = \frac{(4-3\xi-3\eta+2\xi\eta)(4-M)}{(1-\xi)(1-\eta)(2-\eta)}, \qquad c-1+M = \frac{(2-\xi)(4-M)}{(1-\xi)(2-\eta)},$$

$$d-2 = \frac{\eta(4-M)}{1-\eta}, \qquad d+2-M = \frac{4-M}{1-\eta}.$$

20. Quod si hic pro M substituamus valorem $\frac{4}{m+1}$, erit primo $4 - M = \frac{4m}{m+1}$, tum vero:

$$b = \frac{4(2-\zeta)m}{(2-\eta)(m+1)} - \frac{4}{m+1}, \qquad c = \frac{(10-6\zeta-\eta+\zeta\eta)m}{(1-\zeta)(2-\eta)(m+1)} - \frac{3}{m+1}, \qquad d = \frac{2(1+\eta)m}{(1-\eta)(m+1)} + \frac{2}{m+1}.$$

Ponamus ad abbreviandum:

$$\frac{4(2-\zeta)}{2-\eta} = B, \qquad \frac{10-6\zeta-\eta+\zeta\eta}{(1-\zeta)(2-\eta)} = C, \quad \text{seu} \quad C = 1 + \frac{4(2-\zeta)}{(1-\zeta)(2-\eta)}, \qquad \frac{2(1+\eta)}{1-\eta} = D,$$

ut sit

$$b = \frac{Bm-4}{m+1}, \qquad c = \frac{Cm-3}{m+1}, \quad \text{et} \quad d = \frac{Dm+2}{m+1}, \text{ hinc}$$

$$b+1 = \frac{(B+1)m-3}{m+1}, \qquad c+1 = \frac{(C+1)m-2}{m+1}, \qquad d-1 = \frac{(D-1)m+1}{m+1} \text{ atque}$$

$$b+M = \frac{Bm}{m+1}, \qquad c-1+M = \frac{(C-1)m}{m+1}, \qquad d+2-1 = \frac{(D+2)m}{m+1},$$

$$c-b-1 = \frac{(C-B-1)m}{m+1}, \qquad c+d+1 = \frac{(C+D+1)m}{m+1}, \qquad d-2 = \frac{(D-2)m}{m+1},$$

quibus valoribus substitutis consequimur:

$$q = \frac{4(Bm-4)}{B(m+1)} \cdot \frac{a}{m}, \qquad AB = \frac{(Bm-4)}{B} \cdot \frac{a}{m},$$

$$r = \frac{4(Bm-4)(Cm-3)}{(C-1)(m+1)[(B+1)m-3]} \cdot \frac{a}{m}, \qquad BC = \frac{4(Bm-4)(C-B-1)}{[(B+1)m-3]B(C-1)} \cdot \frac{a}{m},$$

$$s = \frac{4(Bm-4)(Cm-3)(Dm+2)}{(D+2)[(B+1)m-3][(C+1)m-2](m+1)} \cdot \frac{a}{m}, \qquad CD = \frac{4(Bm-4)(Cm-3)(C+D+1)}{[(B+1)m-3][(C+1)m-2](C-1)(D+2)} \cdot \frac{a}{m},$$

$$t = \frac{(Bm-4)(Cm-3)(Dm+2)}{[(B+1)m-3][(C+1)m-2][(D-1)m+1]} \cdot \frac{a}{m}, \qquad DE = \frac{(Bm-4)(Cm-3)(Dm+2)(D-2)}{[(B+1)m-3][(C+1)m-2][(D-1)m+1](D+2)} \cdot \frac{a}{m},$$

et pro loco oculi $EO = \frac{m+1}{4m} t$. Hicque $\frac{a}{m}$ denotat distantiam focalem lentis simplicis aequivalentis, quam posuimus $= k$. Notandum porro est litterae B, C, D ita a se invicem pendere, ut sit:

$$BCD + 6BC - 5BD - 4CD - 14B - 8C + 4D + 8 = 0.$$

21. Fractiones ergo ζ et η unitate minores arbitrio nostro relinquuntur, quas autem ita accipi oportet, ut ne lentium intervalla nimis fiant exigua, quam ut ob crassitiem tam prope sibi invicem adjungi queant. Hinc patet neque ζ neque η evanescere posse, quia illo casu intervallum BC, hoc vero intervallum DE in nihilum abiret: intervallum autem CD non evanescit, nisi utraque fractio ζ et η unitati aequalis accipiatur, quod ergo probe est cavendum. Cum igitur neque utramque nimis parvam, neque ambas simul unitati fere aequales accipere liceat, nonnullos casus, qui ad praxin idonei videntur, evolvamus.

22. Casus 1. Sit ergo primo $\zeta = \eta = \frac{1}{2}$, atque habebimus $B = 4$, $C = 9$ et $D = 6$, hincque pro constructione telescopii ponendo $\frac{a}{m} = k$, determinationes sequentes:

$$q = \frac{4(m-1)}{m+1}k, \qquad AB = \alpha - k,$$

$$r = \frac{6(m-1)(3m-1)}{(m+1)(5m-3)}k, \qquad BC = \frac{2(m-1)}{5m-3}k,$$

$$s = \frac{6(m-1)(3m-1)(3m+1)}{(m+1)(5m-3)(5m-1)}k, \qquad CD = \frac{6(m-1)(3m-1)}{(5m-3)(5m-1)}k,$$

$$t = \frac{12(m-1)(3m-1)(3m+1)}{(5m-3)(5m-1)(5m+1)}k, \qquad DE = \frac{6(m-1)(3m-1)(3m+1)}{(5m-3)(5m-1)(5m+1)}k.$$

Haec intervalla lentium satis sunt magna, ut earum contactus non sit metuendus, unde hic casus ad praxin eo aptior videtur, quo formulae inventae sunt simpliciores.

23. Casus 2. Casus etiam notari meretur, quo $\zeta = 1$, unde fit $B = \frac{4}{2-\eta}$, $C = \infty$ et $D = \frac{2(1+\eta)}{1-\eta}$, quos valores notasse sufficiat, ex quibus habebitur:

$$q = \frac{4(m-2+\eta)}{m+1}k, \qquad AB = \alpha - (2-\eta)k,$$

$$r = \frac{4m(Bm-4)}{(m+1)[(B+1)m-3]}k, \qquad BC = \frac{4(m-2+\eta)}{(B+1)m-3}k,$$

$$s = \frac{(1-\eta)(Bm-4)(Dm+2)}{(m+1)[(B+1)m-3]}k, \qquad CD = \frac{(1-\eta)(Bm-4)}{(B+1)m-3}k,$$

$$t = \frac{(Bm-4)(Dm+2)}{[(B+1)m-3][(D-1)m+1]}k, \qquad DE = \frac{\eta(Bm-4)(Dm+2)}{[(B+1)m-3][(D-1)m+1]}k.$$

Cum crassities cujusque lentis sit fere pars decima sexta distantiae focalis, patet nisi η capiatur infra $\frac{1}{8}$ nihil esse metuendum; commode autem sumetur $\eta = \frac{1}{3}$, eritque $B = \frac{12}{5}$ et $D = 4$, unde fit

$$q = \frac{4(3m-5)}{3(m+1)}k, \qquad AB = \alpha - \frac{5}{3}k,$$

$$r = \frac{16m(3m-5)}{(m+1)(17m-15)}k, \qquad BC = \frac{20(3m-5)}{3(17m-15)}k,$$

$$s = \frac{16(3m-5)(2m+1)}{3(m+1)(17m-15)}k, \qquad CD = \frac{8(3m-5)}{3(17m-15)}k,$$

$$t = \frac{8(3m-5)(2m+1)}{(17m-15)(3m+1)}k, \qquad DE = \frac{8(3m-5)(2m+1)}{3(17m-15)(3m+1)}k.$$

Pro loco oculi est semper $EO = \frac{m+1}{4m}t$.

De lentibus ocularibus quintuplicatis, quibus campus quintuplicatur.

24. Sint lentium B, C, D, E et F distantiae focales q, r, s, t et u, indices autem apertur. π, π^I, π^{II}, π^{III} et π^{IV}, ex quibus cum sit semidiameter campi $\varphi = \frac{\pi - \pi^I + \pi^{II} - \pi^{III} + \pi^{IV}}{m+1}$, qui ut maximus evadat ponamus $\pi = \omega$, $\pi^I = -\omega$, $\pi^{II} = \omega$, $\pi^{III} = -\omega$ et $\pi^{IV} = \omega$ eritque φ

sumposito $M=\frac{5}{m+1}$ fiet $\varphi=M\omega$; tum vero porro pro his lentibus statuatur:

$$B=-\frac{b}{1+b},\quad C=-\frac{c}{1+c},\quad D=-1,\quad E=-\frac{e}{e-1},\quad F=\infty,$$

$$\mathfrak{B}=-b,\quad \mathfrak{C}=-c,\quad \mathfrak{D}=\infty,\quad \mathfrak{E}=e,\quad \mathfrak{F}=1,$$

unde fiet:

$$\mathfrak{B}\pi-\varphi=-(b+M)\omega,\quad \mathfrak{C}\pi^{I}-\pi+\varphi=(c-1+M)\omega,\quad \mathfrak{D}\pi^{II}-\pi^{I}+\pi-\varphi=\mathfrak{D}\omega,$$

$$\mathfrak{E}\pi^{III}-\pi^{II}+\pi^{I}-\pi+\varphi=-(e+3-M)\omega,\quad \mathfrak{F}\pi^{IV}-\pi^{III}+\pi^{II}-\pi^{I}+\pi-\varphi=(5-M)\omega.$$

25. His positis, tam distantiae focales lentium, quam earum intervalla ita determinantur ut sit:

$$q=\frac{bM}{b+M}\alpha,\qquad AB=\frac{b}{b+M}\alpha,$$

$$r=\frac{b}{1+b}\cdot\frac{cM}{c-1+M}\alpha,\qquad BC=\frac{b}{1+b}\cdot\frac{M(c-b-1)}{(b+M)(c-1+M)}\alpha,$$

$$s=\frac{bc}{(1+b)(1+c)}\cdot M\alpha,\qquad CD=\frac{bc}{(1+b)(1+c)}\cdot\frac{M}{c-1+M}\alpha,$$

$$t=\frac{bc}{(1+b)(1+c)}\cdot\frac{eM}{e+3-M}\alpha,\qquad DE=\frac{bc}{(1+b)(1+c)}\cdot\frac{M}{e+3-M}\alpha,$$

$$u=\frac{bce}{(1+b)(1+c)(e-1)}\cdot\frac{M}{5-M}\alpha,\qquad EF=\frac{bce}{(1+b)(1+c)(e-1)}\cdot\frac{M(e-2)}{(e+3-M)(5-M)}\alpha,$$

pro loco autem oculi erit $FO=\frac{m+1}{5m}u$. Patet ergo esse debere $c>b+1$ et $e>2$.

26. Quo autem colores vagi iridis deleantur, satisfieri oportet huic aequationi:

$$-\frac{1}{b+M}-\frac{1}{c-1+M}+\frac{1}{e+3-M}+\frac{1}{5-M}\,0,$$

cui generaliter resolvendae non immoror, cum statim se offerat haec solutio particularis valde simplex: $b+M=5-M$ et $c-1+M=e+3-M=2(5-M)$ unde fit $b=5-2M$, $c=11-3M$ et $e=7-M$, hincque $c-b-1=5-M$ et $e-2=5-M$, ita ut sit:

$$q=\frac{5-2M}{5-M}M\alpha,\qquad AB=\frac{5-2M}{5-M}\alpha,$$

$$r=\frac{b}{1+b}\cdot\frac{11-3M}{2(5-M)}M\alpha,\qquad BC=\frac{b}{1+b}\cdot\frac{M\alpha}{2(5-M)},$$

$$s=\frac{bc}{(1+b)(1+c)}M\alpha,\qquad CD=\frac{bc}{(1+b)(1+c)}\cdot\frac{M\alpha}{2(5-M)},$$

$$t=\frac{bc}{(1+b)(1+c)}\cdot\frac{7-M}{2(5-M)}M\alpha,\qquad DE=\frac{bc}{(1+b)(1+c)}\cdot\frac{M\alpha}{2(5-M)},$$

$$u=\frac{bce}{(1+b)(1+c)(e-1)}\cdot\frac{M}{5-M}\alpha,\qquad EF=\frac{bce}{(1+b)(1+c)(e-1)}\cdot\frac{M\alpha}{2(5-M)}.$$

27. Restituto autem valore $M = \frac{5}{m+1}$, unde fit $5 - M = \frac{5m}{m+1}$ et $\frac{M}{5-M} = \frac{1}{m}$, habebimus

$$b = \frac{5(m-1)}{m+1}, \quad c = \frac{11m-4}{m+1}, \quad e = \frac{7m+2}{m+1},$$

sicque determinationes pro constructione telescopii erunt:

$$q = \frac{5(m-1)}{m+1} k, \qquad AB = \alpha - k,$$

$$r = \frac{5(m-1)}{2(3m-2)} \cdot \frac{11m-4}{2(m+1)} k, \qquad BC = \frac{5(m-1)}{4(3m-2)} k,$$

$$s = \frac{5(m-1)(11m-4)}{2(3m-2)(12m-3)} \cdot \frac{5m}{m+1} k, \qquad CD = \frac{5(m-1)(11m-4)}{4(3m-2)(12m-3)} k,$$

$$t = \frac{5(m-1)(11m-4)}{2(3m-2)(12m-3)} \cdot \frac{7m+2}{2(m+1)} k, \qquad DE = \frac{5(m-1)(11m-4)}{4(3m-2)(12m-3)} k,$$

$$u = \frac{5(m-1)(11m-4)(7m+2)}{2(3m-2)(12m-3)(6m+1)} k, \qquad EF = \frac{5(m-1)(11m-4)(7m+2)}{4(3m-2)(12m-3)(6m+1)} k,$$

et pro loco oculi $FO = \frac{m+1}{5m} u$.

28. Ad plures lentes has investigationes non profero, cum hae jam satis sibi sint propinquae et plures radios intercipiendo, claritati non parum officere videntur, cui tamen malo aperturam lentis objectivae augendo remedium afferri poterit. Praeterea vero campus, cujus diameter quinquies major est, quam in telescopiis communibus astronomicis, jam spatium vicies et quinquies majus spectandum offert, quod adeo sub angulo 103 graduum cernetur, si quidem omnes hae lentes utrinque conficiantur aequaliter convexae, et unicuique apertura tribuatur, cujus diameter semissi distantiae focalis aequetur. Neque vero ob multitudinem harum lentium ocularium major confusio est pertimescenda, cum supra viderimus lentem triplicatam minorem confusionem parere, quam duplicatam. Verum hic imprimis est cavendum, ne hae lentes vitio irregularis confusionis inquinentur.

29. Si cum his lentibus ocularibus ejusmodi lentes objectivae conjungantur, quibus omnis confusio e medio tolli queat, nullum est dubium, quin haec telescopia repraesentationem objectorum inversam exhibentia ad summum perfectionis gradum evehantur. Quare haud abs re fore arbitror, si constructionem hujusmodi lentium objectivarum alio loco explicatam hic quoque docuero, ubi quidem solam refractionis rationem 1,54 : 1 utpote in plerisque vitri speciebus observatam assumam. Quodsi ergo distantia focalis lentis objectivae debeat esse $= p$ (quam hic posui $= \alpha$) multiplicationis ratio $= m : 1$, et ex lentibus ocularibus nascatur confusio $= M$, quae pro duplicatis supra erat $= \frac{7}{m}$, pro triplicatis vero $= \frac{4}{m}$, lens objectiva ex hujusmodi binis lentibus erit conficienda:

Pro lente priori:

$$\text{radius faciei} \begin{cases} \text{anterioris} = 0{,}51467p, \\ \text{posterioris} = 4{,}05851p. \end{cases}$$

Pro lente posteriori:

$$\text{radius faciei} \begin{cases} \text{anterioris} = -0{,}73978p + 0{,}7319\,Mp, \\ \text{posterioris} = +1{,}01897p - 1{,}3885\,Mp, \end{cases}$$

quas ad tantum intervallum a se invicem disponi oportet, donec confusio evanescat, et quia hoc intervallum optime per experientiam definitur, non opus est, ut de vero valore litterae illius M nimis simus solliciti.

Evolutio telescopii objecta quinquagies aucta repraesentantis.

30. Cum hic sit $m = 50$, pro lente objectiva semidiametrum aperturae ad minimum esse oportet $= \frac{3m}{200}$ seu $\frac{3}{4}$ dig., quoniam vero pluribus lentibus uti volumus, ne inde obscuritas nascatur, conveniet aperturae semidiametrum sumi unius digiti. Quod autem ad distantiam focalem lentis objectivae attinet, si ea fuerit simplex, cujusmodi vulgo in usum adhiberi solent, distantia focalis, quam hic littera α vel p indico vix infra 10 pedes accipi posset, unde ob $\alpha = 120$ dig. prodiret lentis ocularis distantia focalis $k = \frac{120}{50} = 2,4$ dig. Sin autem lente objectiva duplicata uti velimus, qualem modo descripsi, ejus distantiam focalem p vehementer imminuere liceret, neque tamen eam ulterius minuere consultum videtur, quam ut k fierit $\frac{1}{2}$ dig. ac proinde $p = 25$ dig., quia alioquin distantia oculi ab ultima lente nimis fieret exigua. Expediet fortasse ipsi p valorem duplo majorem tribui, quo k fiat integri digiti, atque confusio multo minus sit metuenda.

31. Quo autem constructio hujusmodi lentis objectivae duplicatae facilius succedat, quicunque valor litterae p tribuatur, quantitatem litterae M, qua portio confusionis a lentibus ocularibus oriunda denotatur, justo majorem accipi conveniet, etsi enim forma lentis posterioris concavae inde pendet, tamen recordandum est, ejus distantiam a priori convexa per experientiam definiri debere, quo fit, ut si valor ipsius M nimis magnus fuerit assumtus, distantiam lentium aliquantillum augeri oportere, cum contra minui deberet, ac fortasse plus quam lentium crassities permitteret. Hanc ob rationem ponam $M = \frac{10}{m}$, nostroque casu $M = \frac{1}{5}$, sicque pro distantia focali $= p$ et ratione refractionis 1,54 : 1 constructio utriusque lentis ita se habebit:

$$\text{Lentis prioris radius faciei} \ldots \begin{cases} \text{anterioris} = 0{,}51467p, \\ \text{posterioris} = 4{,}05851p, \end{cases}$$

$$\text{Lentis posterioris radius faciei} \begin{cases} \text{anterioris} = -\,0{,}59340p, \\ \text{posterioris} = +\,0{,}74127p. \end{cases}$$

32. Quomodocunque autem lens objectiva fuerit constituta, ex ejus distantia focali α vel p definitur lentis ocularis, siquidem simplex adhiberetur, distantia focalis $k = \frac{\alpha}{50}$, hacque utrinque aequaliter facta convexa, ut aperturam capiat, cujus diameter $= \frac{1}{2}k$, campus detegetur, cujus semidiameter sit $= \frac{1}{4(m+1)}$, ac propterea ob $m = 50$ ipse diameter campi apparentis $= \frac{1}{2.51}$ seu $\frac{3437}{102}$ min. seu $33\frac{2}{3}$ min., neque haec repraesentatio a contagio colorum prorsus erit immunis. Quodsi vero lentibus ocularibus multiplicatis supra descriptis utamur, non solum hi vagi colores evanescent, sed etiam diameter campi in eadem ratione augebitur, singulas igitur lentis ocularis species percurramus, ubi quidem monendum, omnes has lentes utrinque aequaliter convexas fieri et unicuique aperturam, cujus diameter semissi ipsius distantiae focalis aequetur, tribui debere.

33. *Si lens ocularis adhibeatur duplicata, ejus constructio ita erit comparata:*

Distantiae focales: $q = \frac{98}{51} k = 1{,}9215 k,$ distantiae lentium: $AB = \alpha - k = 49 k,$

$r = \frac{98}{149} k = 0{,}6577 k,$ $BC = \frac{196}{149} k = 1{,}3154 k,$

et pro loco oculi distantia $CO = \frac{51}{100} r = 0{,}3354 k,$ sicque tota tubi longitudo $AO = 50{,}6508 k$ et campi apparentis diameter $1^0 7\frac{1}{3}'$.

34. *Si lens ocularis adhibeatur triplicata,* ejus constructio erit:

Distantiae foci: $q = \frac{147}{51} k = 2{,}8822 k,$ intervalla lentium: $AB = \alpha - k = 49 k,$

$r = \frac{450.49}{102.99} k = 2{,}1836 k,$ $BC = \frac{147}{198} k = 0{,}7424 k,$

$s = \frac{147}{198} k = 0{,}7424 k,$ $CD = \frac{147}{198} k = 0{,}7424 k,$

pro oculo erit distantia $DO = \frac{51}{150} s = 0{,}2524 k,$

sicque longitudo tubi tota $AO = 50{,}7362 k.$

et campi apparentis diameter $= 1^0 41'$.

35. *Si lens ocularis adhibeatur quadruplicata,* eam ita confici oportebit secundum § 22:

Distantiae foci: $q = \frac{4.49}{51} k = 3{,}8431 k,$ intervalla lentium: $AB = \alpha - k = 49 k,$

$r = \frac{6.49.149}{51.247} k = 3{,}4775 k,$ $BC = \frac{2.49}{247} k = 0{,}3968 k,$

$s = \frac{6.49.149.151}{51.247.249} k = 2{,}1088 k,$ $CD = \frac{6.49.149}{247.249} k = 0{,}7123 k,$

$t = \frac{12.49.149.151}{247.249.251} k = 0{,}8570 k,$ $DE = \frac{6.49.149.151}{247.249.251} k = 0{,}4285 k,$

pro loco oculi distantia $EO = \frac{51}{200} t = 0{,}2185 k,$

unde oritur tota tubi longitudo $AO = 50{,}7561 k,$

et campi apparentis diameter erit $= 2^0 14\frac{2}{3}'$.

36. *Si lens ocularis adhibeatur quintuplicata,* ejus constructio secundum haec praecepta institui debet:

Dist. foci: $q = \frac{5.49}{51} k = 4{,}8039 k,$ interv. lentium: $AB = \alpha - k = 49 k,$

$r = \frac{5.49.546}{2.148.102} k = 4{,}4307 k,$ $BC = \frac{5.49}{4.148} k = 0{,}4139 k,$

$s = \frac{5.49.546.250}{2.148.597.51} k = 3{,}7108 k,$ $CD = \frac{5.49.546}{4.148.597} k = 0{,}3785 k,$

$t = \frac{5.49.546.352}{2.148.597.102} k = 2{,}6124 k,$ $DE = \frac{5.49.546}{4.148.597} k = 0{,}3785 k,$

$u = \frac{5.49.546.352}{2.148.597.301} k = 0{,}8853 k,$ $EF = \frac{5.49.546.352}{4.148.597.301} k = 0{,}4426 k,$

pro loco oculi est distantia $FO = \frac{51}{250} u = 0{,}1806 k,$

hincque tota tubi longitudo $AO = 50{,}7941 k,$

campi autem apparentis diameter est $2^0 48\frac{1}{3}'$. Unde patet aucto campo apparente longitudinem tubi vix sensibile incrementum capere.

Evolutio telescopii objecta centies aucta repraesentantis.

37. Haec multiplicatio secundum diametros fieri est intelligenda, ut sit $m = 100$, quo casu diameter aperturae lentis objectivae ad minimum sumi debet 3 digitorum, ut in contemplandis objectis sufficiente lumine fruamur. At si lente oculari multiplicata uti velimus, sine dubio ipsi majorem aperturam, veluti $3\frac{1}{2}$ dig. tribui conveniet. Foci autem distantia hujus lentis, si simplex adhibeatur, per regulas vulgares 30 pedes superare debet, quanquam si omnis cura ad eam formandam afferatur, haud parum minor accipi posse videatur. Verum si lentem compositam in usum vocare velimus, qua omni confusioni occurri queat, ejus distantia focalis p multo minor accipi poterit, neque tamen infra 50 dig. minuenda videtur, ne lentes oculares nimium fiant exiguae.

38. Ut igitur hujusmodi lentis objectivae compositae structuram exponam, quae ex lente convexa ac menisco componitur, intervallo per experientiam determinando, partem confusionis ex lentibus ocularibus natam, quam littera M denotavi, iterum $= \frac{10}{m}$ statuam, ut sit pro casu praesente $M = \frac{1}{10}$, ex quo posita ratione refractionis ex aëre in vitrum ut 1,54 ad 1, utriusque lentis constructio ita se habebit, distantia focali compositae existente $= p$:

$$\text{lentis prioris radius faciei} \ldots \begin{cases} \text{anterioris} & = 0{,}51467p, \\ \text{posterioris} & = 4{,}05851p, \end{cases}$$

$$\text{lentis posterioris radius faciei} \begin{cases} \text{anterioris} & = -0{,}66658p, \\ \text{posterioris} & = +0{,}88012p, \end{cases}$$

ubi notandum est p designare distantiam foci post lentem posteriorem, intervallo lentium vix notabili existente.

39. Lente autem objectiva constituta, cujus distantia foci jam sit $= \alpha$, lentis ocularis vel ipsius simplicis vel multiplicatis aequivalentis distantia focalis esse debet $= k$, ut sit $k = \frac{\alpha}{100}$, quae si fuerit simplex et aperturam habeat, cujus diameter est semissi distantiae foci aequalis, diameter campi apparentis erit $= \frac{1}{2(m+1)}$ seu $\frac{3437}{202}$ min. $= 17'$, quem autem lentibus ocularibus multiplicatis adhibendis simili ratione multiplicare licet, dummodo singulae utrinque aequaliter convexae formentur, ut cuilibet apertura, cujus diameter semissi distantiae focalis ejus aequetur, tribui possit, ex quo singulas lentis ocularis multiplicationes ut ante persequamur.

40. *Si lens ocularis adhibeatur duplicata*, eam ita confici oportet:

Distantiae focales: $q = \frac{2.99}{101} k = 1{,}9604\,k$, Intervalla lentium: $AB = \alpha - k = 99\,k$,

$r = \frac{2.99}{299} k = 0{,}6578\,k$, $BC = 2r = 1{,}3156\,k$,

pro loco oculi $CO = \frac{101}{200} r = 0{,}3322\,k$,

hinc tota tubi longitudo $AO = 100{,}6478\,k$;

campi autem apparentis diameter erit $34'$, ita ut hujusmodi telescopium totum solem complectatur. Ac si constructio lentis objectivae descriptae successu non careret, ut capi posset $\alpha = 50$ dig. et $k = \frac{1}{2}$ dig., hoc instrumentum foret $50\frac{1}{3}$ dig.

41. *Si lens ocularis adhibeatur triplicata*, haec praecepta in ejus constructione sequi convenit:

Distantiae focales: $q = \frac{3.99}{101} k = 2{,}9405 k$, Intervalla lentium: $AB = \alpha - k = 99k$,

$r = \frac{9.100.99}{2.101.199} k = 2{,}2165 k$, $BC = s = 0{,}7462 k$,

$s = \frac{3.99}{2.199} k = 0{,}7462 k$, $CD = s = 0{,}7462 k$,

pro loco oculi distantia $DO = \frac{101}{300} s = 0{,}2512 k$,

hinc tota tubi longitudo $AO = 100{,}7496 k$,

campi autem apparentis diameter erit $51'$.

42. *Si lens ocularis adhibeatur quadruplicata*, ejus constructio ita erit instituenda:

Distantiae focales: $q = \frac{4.99}{101} k = 3{,}9208 k$, Interv. lentium: $AB = \alpha - k = 99k$,

$r = \frac{6.99.299}{101.497} k = 3{,}5381 k$, $BC = \frac{2.99}{497} k = 0{,}398$[illegible]$k$,

$s = \frac{6.99.299.301}{101.497.499} k = 2{,}1342 k$, $CD = \frac{6.99.299}{497.499} k = 0{,}7162 k$,

$t = \frac{12.99.299.301}{497.499.501} k = 0{,}8605 k$, $DE = \frac{6.99.299.301}{497.499.501} k = 0{,}4302 k$,

pro loco oculi distantia $EO = \frac{101}{400} t = 0{,}2172 k$,

hinc tota tubi longitudo $AO = 100{,}7620 k$,

campi autem apparentis diameter erit $1^{\circ} 8'$.

43. *Si lens ocularis adhibeatur quintuplicata*, his mensuris ejus constructio continetur:

Dist. focales: $q = \frac{5.99}{101} k = 4{,}9010 k$, Interv. lentium: $AB = \alpha - k = 99k$,

$r = \frac{5.99.1096}{4.298.101} k = 4{,}5063 k$, $BC = \frac{5.99}{4.298} k = 0{,}4153 k$,

$s = \frac{5.99.1096.1000}{4.298.1197.101} k = 3{,}7647 k$, $CD = \frac{5.99.1096}{4.298.1197} k = 0{,}3802 k$,

$t = \frac{5.99.1096.702}{4.298.1197.101} k = 2{,}6428 k$, $DE = \frac{5.99.1096}{4.298.1197} k = 0{,}3802 k$,

$u = \frac{5.99.1096.702}{2.298.1197.601} k = 0{,}8882 k$, $EF = \frac{5.99.1096.702}{4.298.1197.601} k = 0{,}4$[illegible]$k$,

pro loco oculi distantia $FO = \frac{101}{500} u = 0{,}179$[illegible]$k$

et tota tubi longitudo seu distantia $AO = 100{,}7992 k$,

campi vero apparentis diameter erit $1^{\circ} 25'$.