Principles of Calculus Modeling: An Interactive Approach by Donald Kreider, Dwight Lahr, and Susan Diesel
Exercises for Section 4.2

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1. (1 pt)
Expand the sum $\sum_{i=1}^{4} i^4$. Use only those answer boxes that you need; leave the rest blank.
first term = ________________
second term = ________________
third term = ________________
fourth term = ________________
fifth term = ________________

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2. (1 pt)
Which of the following represents the sum
$5^8 + 6^8 + \cdots + 10^8$
in sigma notation?
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A. $\sum_{i=5}^{10} i^8$
B. $\sum_{i=6}^{10} i^8$
C. $\sum_{i=0}^{n} i^8$
D. $\sum_{i=1}^{n} i^8$
E. $\sum_{i=5}^{n} i^8$

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3. (1 pt)
Write the sum
$\frac{1}{3} + \frac{16}{9} + \frac{81}{27} + \cdots + \frac{n^4}{3^n}$
in sigma notation.
lower limit: $i =$ ________________
upper limit: $i =$ ________________
$f(i) =$ ________________

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4. (1 pt)
Let P_7 denote the partition of the interval [0,3] into n = 7 subintervals of equal length. If $f(x) = x$, evaluate $L(f, P_7)$ and $U(f, P_7)$.
$L(f, P_7)$ = ____________________________
$U(f, P_7)$ = ____________________________

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5. (1 pt)
Let P_5 denote the partition of the interval [-1,1] into n = 5 subintervals of equal length. If $f(x) = e^x$, evaluate $L(f, P_5)$ and $U(f, P_5)$.
$L(f, P_5)$ = ____________________________
$U(f, P_5)$ = ____________________________

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6. (1 pt)
Is the function $f(x) = \begin{cases} x-1 & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1 \end{cases}$ Riemann integrable on $[-1, 1]$ **(yes/no)**?
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7. (1 pt)
What is $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5^2}{n^2} \sqrt{n^2 - i^2}$?
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A. One fourth of the area of a circle of radius 3
B. π
C. ∞
D. None of the above.

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8. (1 pt)
Which formula is not equivalent to the others?
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A. $\sum_{j=-1}^{1} \frac{(-1)^j}{j+2}$
B. $\sum_{k=0}^{2} \frac{(-1)^k}{k+1}$
C. $\sum_{k=1}^{3} \frac{(-1)^k}{k}$
D. $\sum_{k=2}^{4} \frac{(-1)^{k-1}}{k-1}$

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9. (1 pt)
Express the limit
$\lim_{\|P\|\to 0} \sum_{i=1}^{n} {c_i}^3 \Delta x_i$, where P is a partition of [2, 10] and c_i is a number in the ith subinterval of this partition
as a definite integral $\int_a^b f(x)dx$, with $a < b$.
$a =$ ________________
$b =$ ________________
$f(x) =$ ________________

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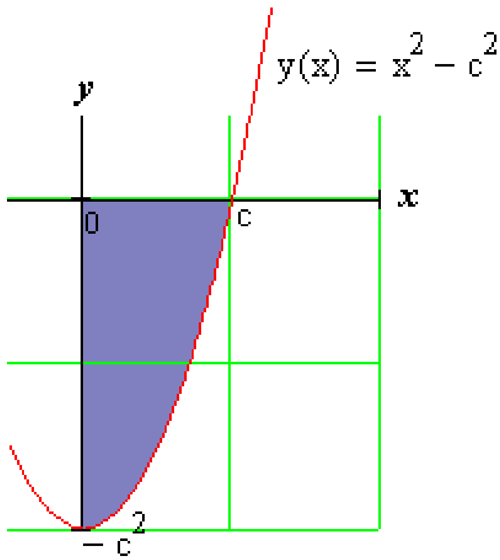
10. (1 pt)
Express the area of the shaded region as an integral $\int_a^b f(x)dx$, with $a < b$.



$a =$ ____________________

$b =$ ____________________

$f(x) =$ ____________________

11. (1 pt)
Express the following sum in sigma notation, using i as your index and beginning at i = 1.

$\frac{1}{2^2} - 11 + \frac{2}{2^2} - 22 + \frac{3}{2^2} - 33 + \frac{4}{2^2} - 44 + \ldots + \frac{n}{2^2} - 11n$

Let L be the upper limit of i and E be the expression inside the summation.

L = ____________________

E = ____________________

12. (1 pt)
Which of the following functions are Riemann integrable over the interval [0,1]?

- A. x if x rational, 0 if x irrational
- B. 5 if x rational, 2 if x irrational
- C. $\tan(\pi x)$
- D. $x^{59,844,589}$
- E. $\cos(x)$
- F. All of the above
- G. None of the above

13. (1 pt)
What is the value of $\sum_{i=1}^{7} \frac{i^2}{\cos(i)^4}$?

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14. (1 pt)
Let $f(x) = x^6 + x^7$, and let P_i be the partition of [0,1] constructed by subdividing [0,1] in half i times.

What is $L(P_3, f)$? ____________________

What is $U(P_3, f)$? ____________________

15. (1 pt)
Express the following sum in sigma notation, using j as your index and ending at j = 6.

$\frac{6}{7} - 3^2 + \frac{8}{7} - 4^2 + \frac{10}{7} - 5^2 + \frac{12}{7} - 6^2$

Let L be the lower limit of j and E be the expression inside the summation.

L = ______

E = ____________________

16. (1 pt)
What is the value of $\sum_{i=4}^{10} \ln(11i) - \cos(8i)$?

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17. (1 pt)
What is the value of $\sum_{i=1}^{8} 16i + \frac{8}{i}$?

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18. (1 pt)
Find $\int_0^2 \frac{x}{2}$ using limits of Riemann sums.

$\int_0^2 \frac{x}{2} = \lim_{n\to\infty} \sum_{i=1}^{n}$ ____________________

This limit is equal to ____________________