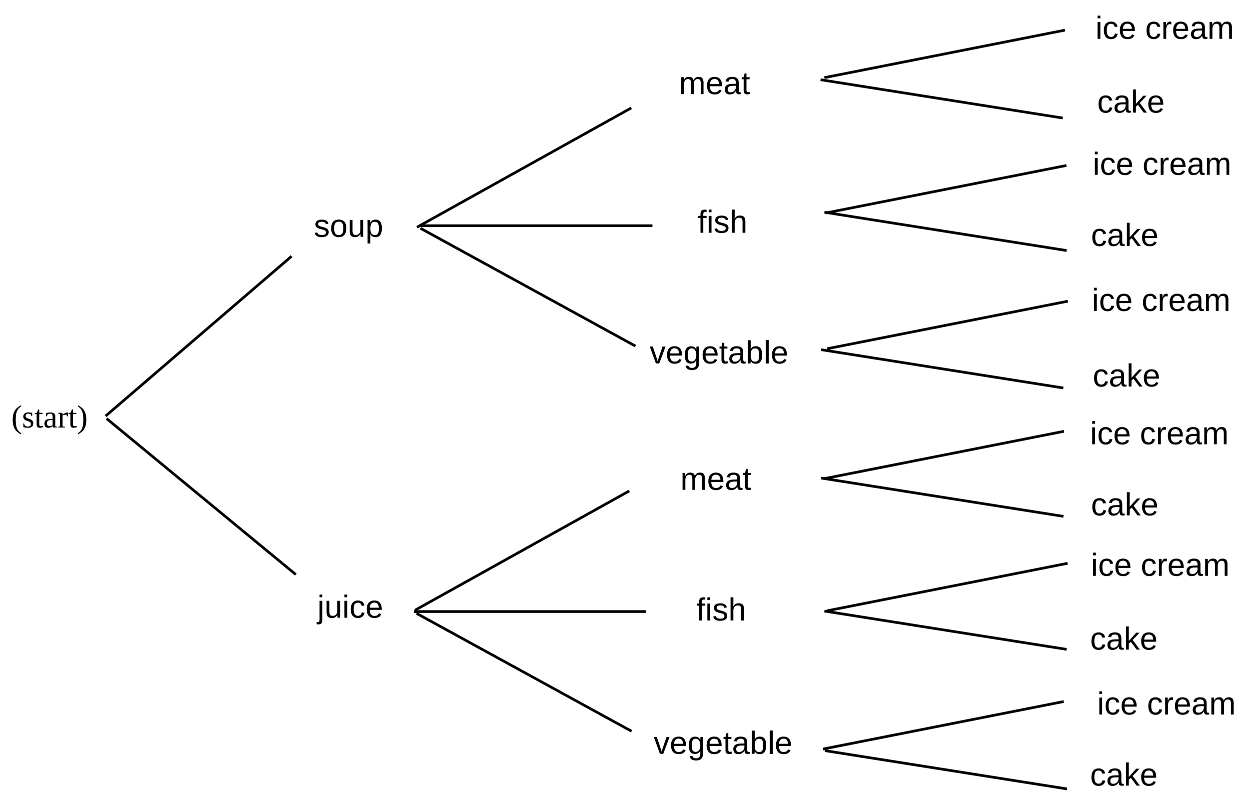
Combinatorics

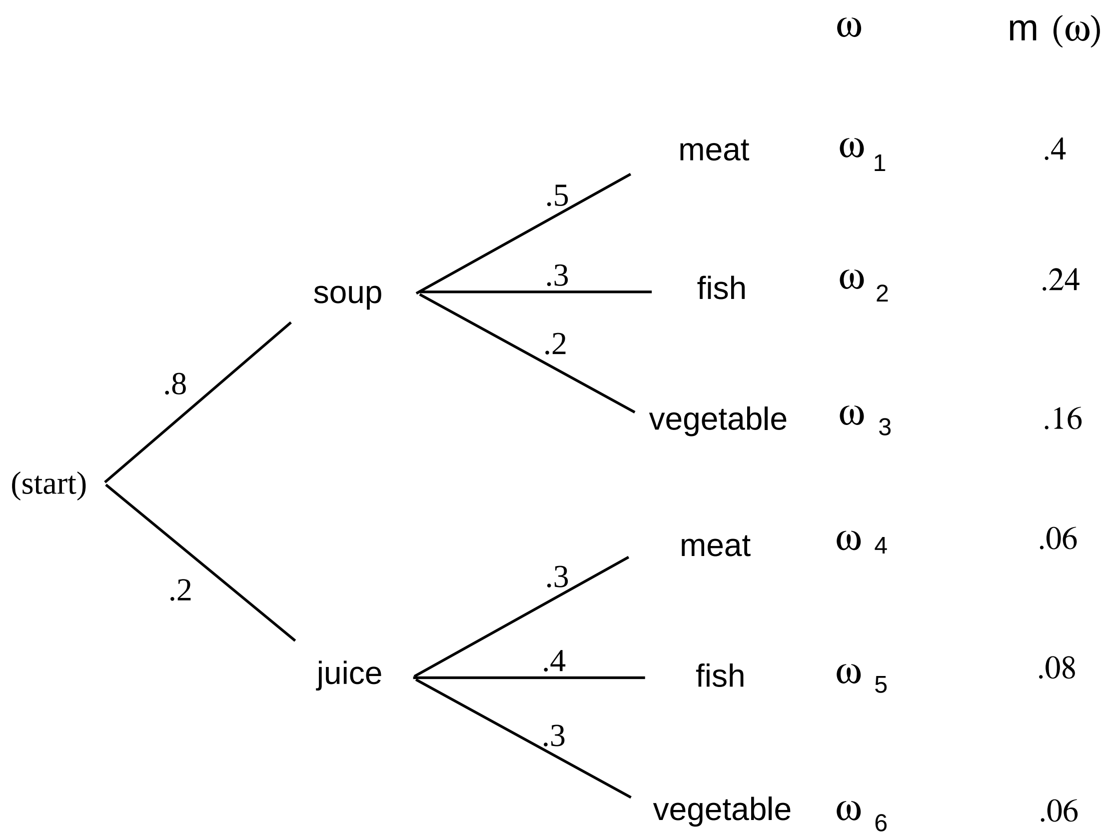
You are eating at Émile's restaurant and the waiter informs you that you have

1. two choices for appetizers: soup or juice;
2. three for the main course: a meat, fish, or vegetable dish; and
3. two for dessert: ice cream or cake.

How many possible choices do you have for your complete meal?


(start)
soup
juice
meat
fish
vegetable
meat
fish
vegetable
ice cream
cake
ice cream
cake
ice cream
cake
ice cream
cake
ice cream
cake
ice cream
cake

Assume that the owner of Émile's restaurant has observed that 80 percent of his customers choose the soup for an appetizer and 20 percent choose juice. Of those who choose soup, 50 percent choose meat, 30 percent choose fish, and 20 percent choose the vegetable dish. Of those who choose juice for an appetizer, 30 percent choose meat, 40 percent choose fish, and 30 percent choose the vegetable dish. What is the probability that a customer eats vegetable.


ω
m (ω)
(start)
.8
.2
soup
juice
.5
.3
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.3
.4
.3
meat
fish
vegetable
meat
fish
vegetable
ω_1
ω_2
ω_3
ω_4
ω_5
ω_6
.4
.24
.16
.06
.08
.06

Prove that at least two people in Atlanta, Georgia, have the same initials, assuming no one has more than four initials.

Birthday Problem

How many people do we need to have in a room to make it a favorable bet (probability of success greater than $1/2$) that two people in the room will have the same birthday?

Permutations

Definition. *Let A be any finite set. A* permutation of A *is a one-to-one mapping of A onto itself.*

$$\sigma = \begin{pmatrix} a & b & c & d \\ b & d & c & a \end{pmatrix}.$$

Theorem. *The total number of permutations of a set A of n elements is given by $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$.*

Definition. *Let A be an n-element set, and let k be an integer between 0 and n. Then a k-permutation of A is an ordered listing of a subset of A of size k.*

Theorem. *The total number of k-permutations of a set A of n elements is given by $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-k+1)$.*

Factorials

Definition. *The number* $n \cdot (n-1) \cdot \ldots \cdot 1$ *is called* n factorial.

n	$n!$
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800

Stirling's Formula

Definition. *Let a_n and b_n be two sequences of numbers. We say that a_n is* asymptotically equal *to b_n, and write $a_n \sim b_n$, if*

$$\lim_{n \to \infty} \frac{a_n}{b_n} = 1 \ .$$

Theorem. *The sequence $n!$ is asymptotically equal to*

$$n^n e^{-n} \sqrt{2\pi n} \ .$$

n	$n!$	Approximation	Ratio
1	1	.922	1.084
2	2	1.919	1.042
3	6	5.836	1.028
4	24	23.506	1.021
5	120	118.019	1.016
6	720	710.078	1.013
7	5040	4980.396	1.011
8	40320	39902.395	1.010
9	362880	359536.873	1.009
10	3628800	3598696.619	1.008

The Hat Check Problem

In a restaurant n hats are checked and they are hopelessly scrambled; what is the probability that no one gets his own hat back?

Problems

There are three different routes connecting city A to city B. How many ways can a round trip be made from A to B and back? How many ways if it is desired to take a different route on the way back?

A certain state has license plates showing three numbers and three letters. How many different license plates are possible

1. if the numbers must come before the letters?

2. if there is no restriction on where the letters and numbers appear?

Find a formula for the probability that among a set of n people, at least two have their birthdays in the same month of the year (assuming the months are equally likely for birthdays).

As I was going to St. Ives,
I met a man with seven wives,
Each wife had seven sacks,
Each sack had seven cats,
Each cat had seven kits.
Kits, cats, sacks and wives,
How many were going to St. Ives?