Derivatives of Exponential and Logarithm Functions

10/19/2005

The Derivative of $y = e^x$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \\ &= e^x \lim_{h \to 0} \frac{e^h - 1}{h} \end{aligned}$$

The Derivative of $y = e^x$...

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

The Derivative of $y = e^x$...

$$\frac{dy}{dx} = e^x$$

The Chain Rule

Theorem. *Let u be a function of x. Then*

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}.$$

Examples

- $y = e^{17x}$
- $y = e^{\sin x}$
- $y = e^{\sqrt{x^2+1}}$

The Derivative of $y = \ln x$

- We can find the derivative of $y = \ln x$ by implicit differentiation:

$$\begin{aligned} y = \ln x &\quad \Leftrightarrow \quad e^y = x \\ e^y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{x}. \end{aligned}$$

The Chain Rule

Theorem. *Let u be a function of x. Then*

$$\frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx}.$$

Examples

- $y = \ln x^2$
- $y = \ln(\sin(x^2))$

The Calculus Standards: e^x and $\ln x$

$$\begin{aligned} a^x &= e^{x \ln a} \\ \log_a x &= \frac{\ln x}{\ln a} \end{aligned}$$

Examples

- $y = 2^x$
- $y = x^x$

The Equation $y' = ky$

- Suppose y is a function of x and satisfies the equation

$$y' = ky$$

- If $k = 1$, then $y = e^x$ has this property and thus solves the equation.

- In fact $y = e^{kx}$ solves the equation for any k.

- The equation $y' = ky$ is an example of a *differential equation*.