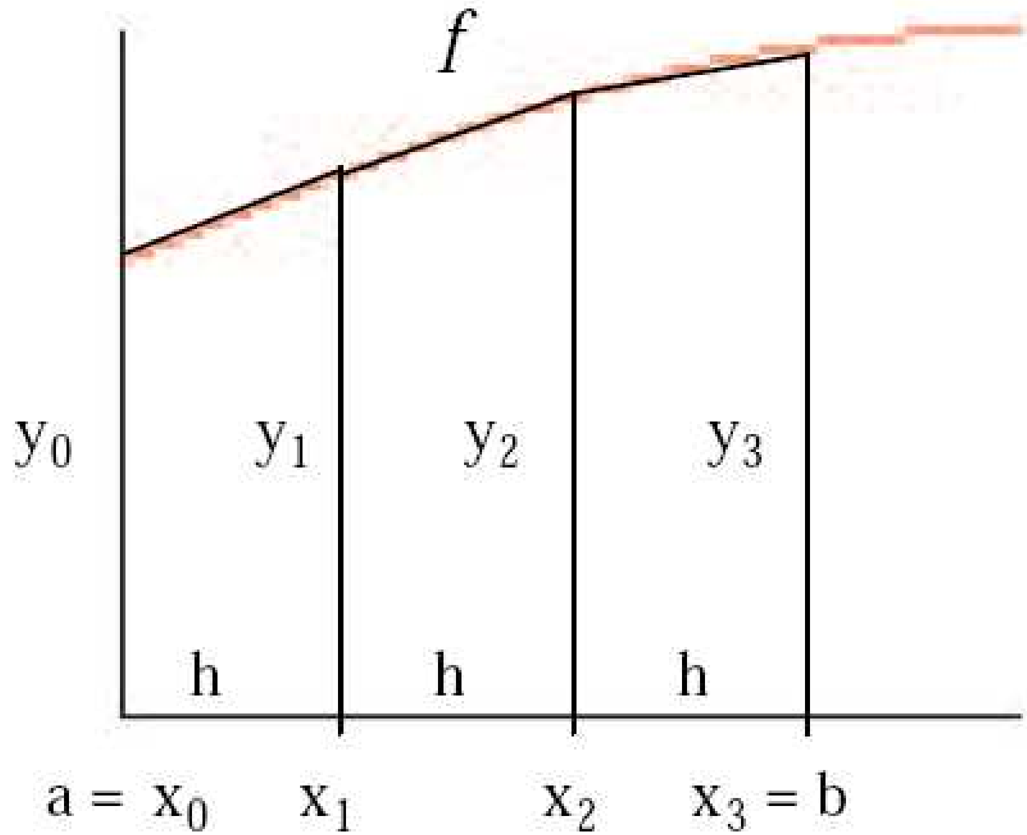
Trapezoid Rule

11/13/2005

- If we can find an antiderivative for the integrand, then we can evaluate the integral fairly easily.

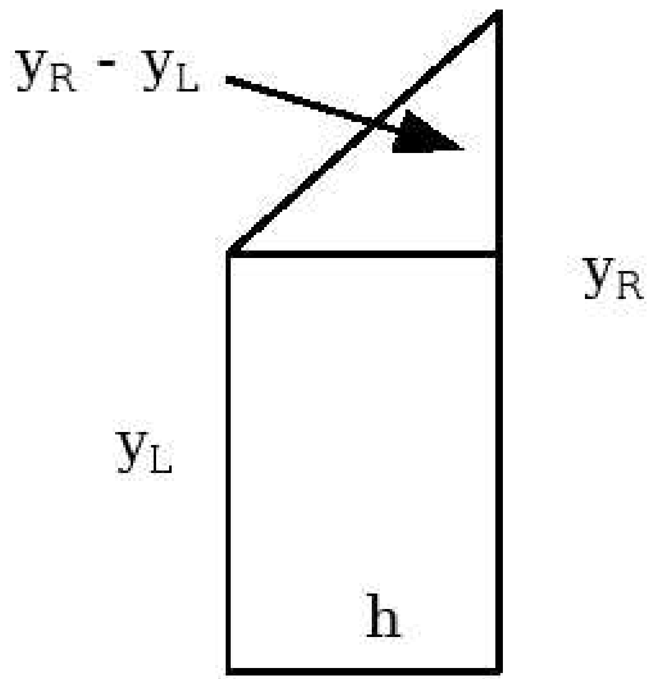
- When we cannot, we turn to numerical methods.

- The numerical method we will discuss here is called the *Trapezoid Rule*.



f
y0
y1
y2
y3
h
h
h
a = x0
x1
x2
x3 = b

The Area of a Trapezoid



- Area$=h(y_L + y_R)/2$.

Definition

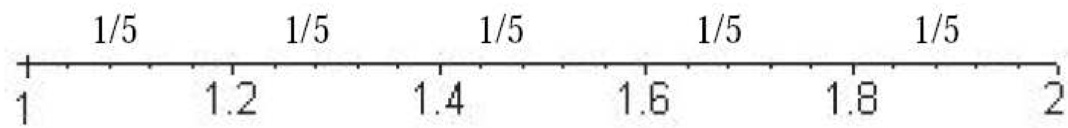
- The n-subinterval trapezoid approximation to $\int_a^b f(x)dx$ is given by

$$
\begin{aligned}
T_n &= \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + y_n\right) \\
&= \frac{h}{2}\left(y_0 + y_n + 2\sum_{j=1}^{n-1} y_j\right)
\end{aligned}
$$

Example

- Find T_5 for $\int_1^2 1/x dx$.

- $f(x) = 1/x, h = 1/5$(so $h/2 = 1/10$), and $x_j = 1 + j/5,\ 0 \leq j \leq 5$.



- $$T_5 = \frac{1}{10}\left(1 + \frac{1}{2} + 2\left(\frac{5}{6} + \frac{5}{7} + \frac{5}{8} + \frac{5}{9}\right)\right) \approx .0696$$

- Find T_5 for $\int_o^1 \sqrt{1-x^2}dx$.

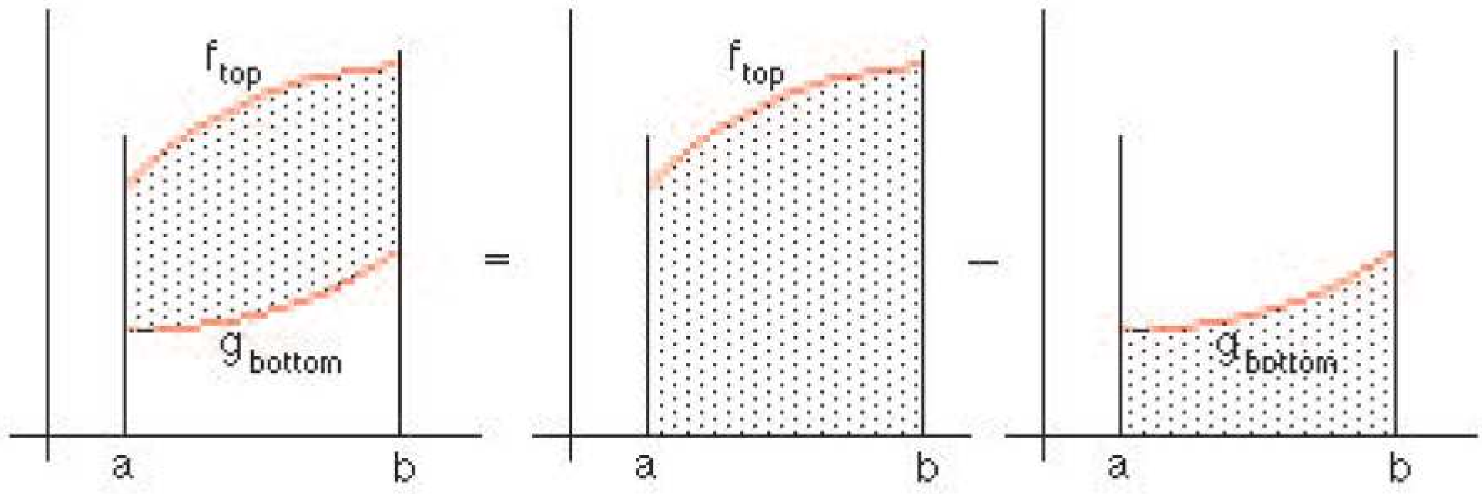
$$T_5 = \frac{1}{10}\left(1 + 2\sum_{j=1}^{4}\sqrt{1-\frac{j^2}{15}}\right) \simeq .75926.$$

Areas Between Curves

- We know that if f is a continuous nonnegative function on the interval $[a, b]$, then $\int_a^b f(x)dx$ is the area under the graph of f and above the interval.

- Suppose we are given two continuous functions, f_{top} and g_{bottom} defined on the interval $[a, b]$, with $g_{bottom}(x) \leq f_{top}(x)$ for all x in the interval.

- How do we find the area bounded by the two functions over that interval?



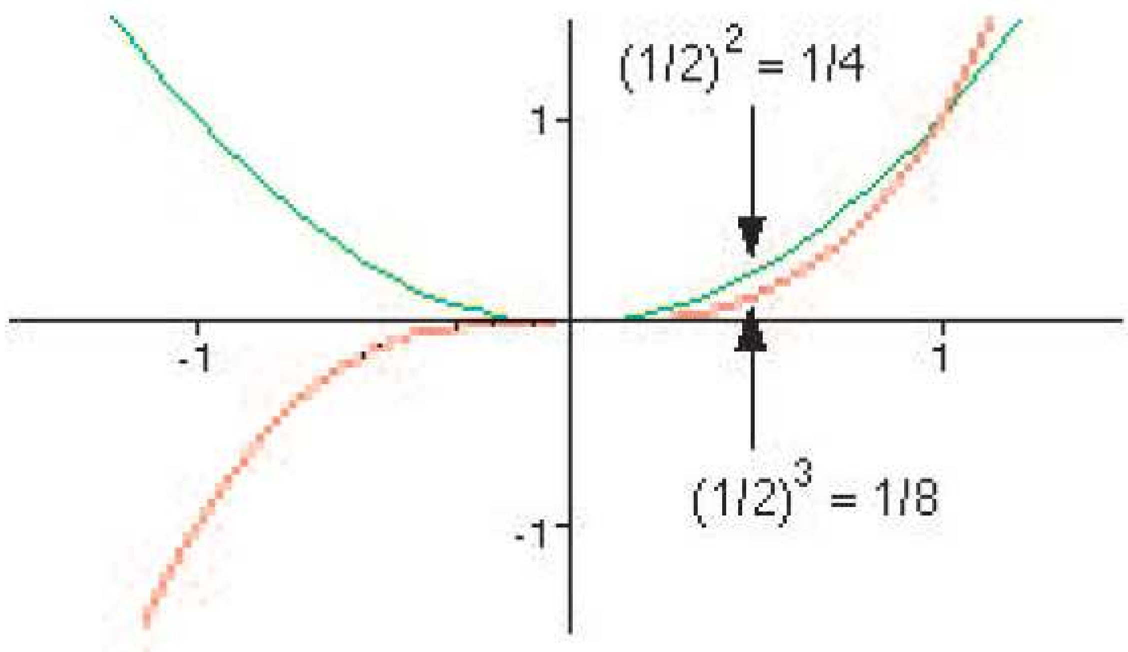
f_{top}
g_{bottom}
a
b
=
f_{top}
a
b
−
g_{bottom}
a
b

The Area Between Two Curves

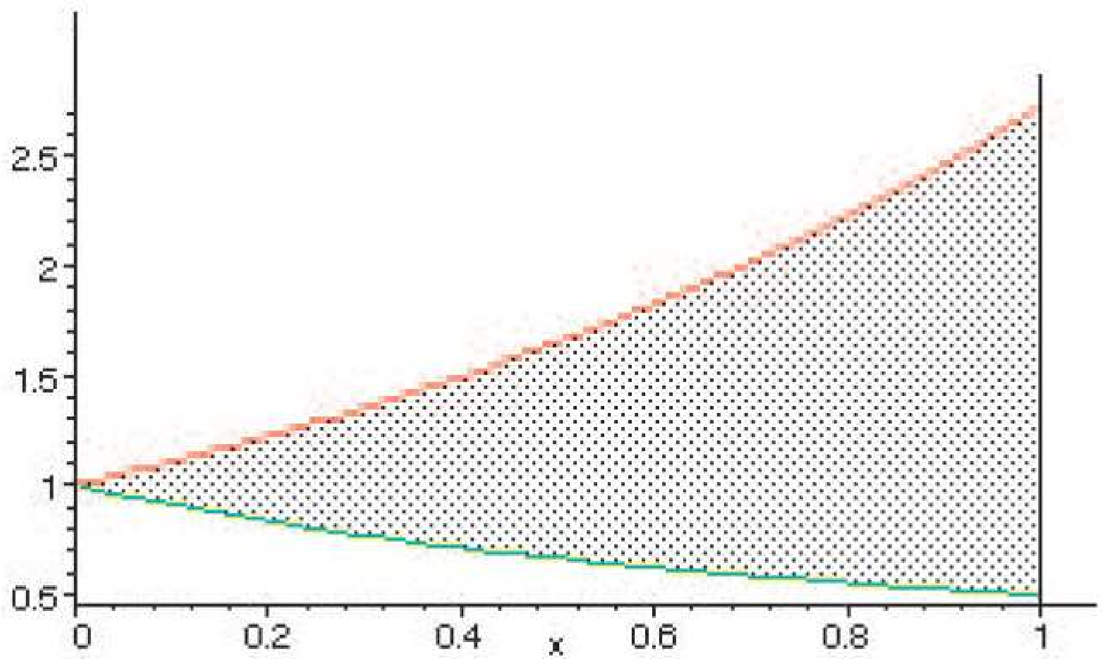
$$\int_a^b f_{top}(x)\,dx - \int_a^b g_{bottom}(x)\,dx = \int_a^b \left(f_{top}(x) - g_{bottom}(x)\right)\,dx$$

Example

- Find the area of the region between the graphs of $y = x^2$ and $y = x^3$ for $0 \leq x \leq 1$.



- Find the area of the region between $y = e^x$ and $y = 1/(1+x)$ on the interval $[0, 1]$.



- Find the area of the region bounded by $y = x^2 - 2x$ and $y = 4 - x^2$.

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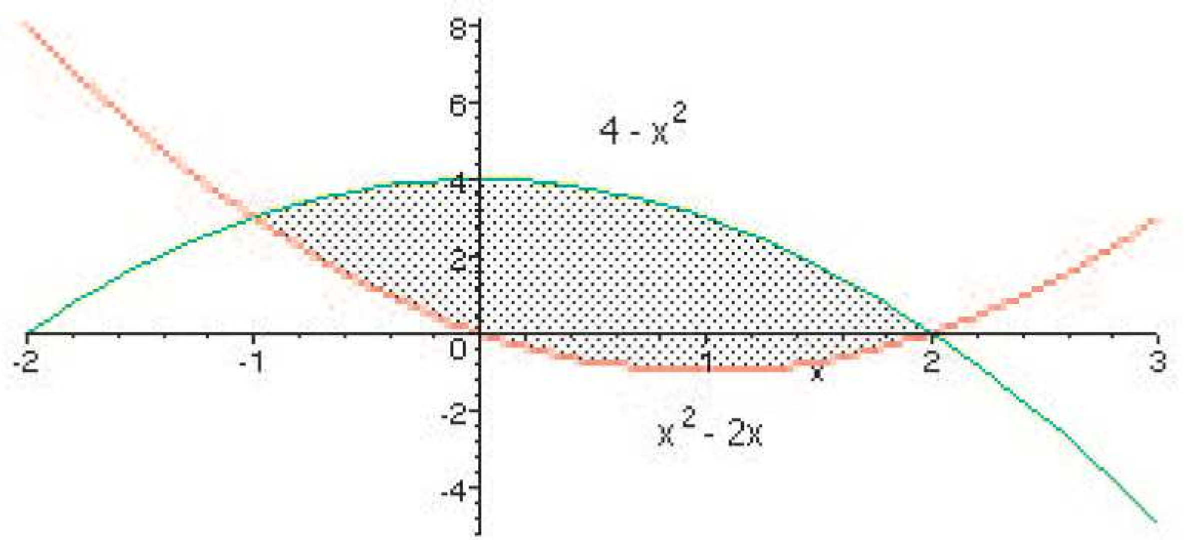
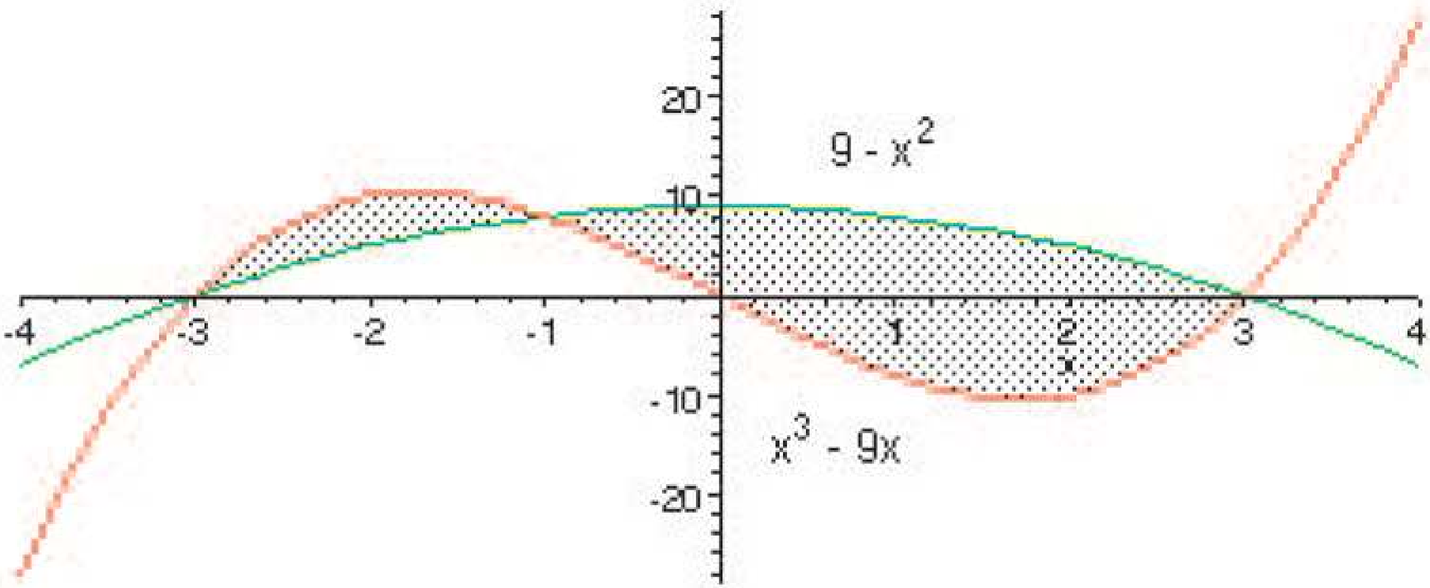
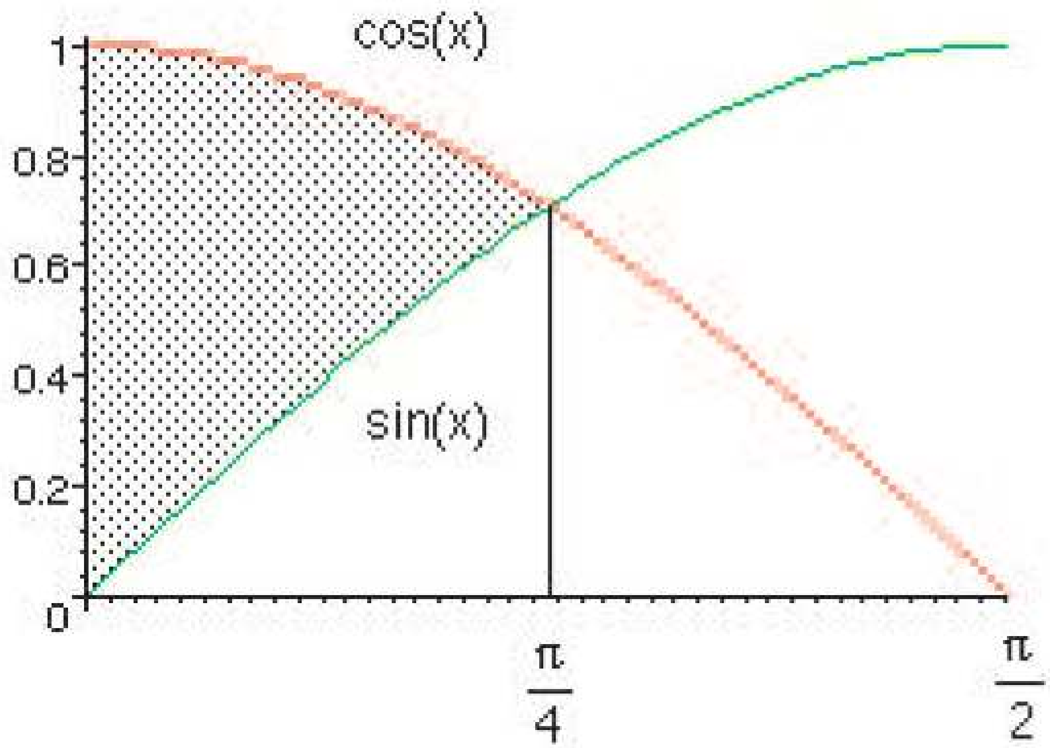


- Find the area of the region bounded by the two curves $y = x^3 - 9x$ and $y = 9 - x^2$.

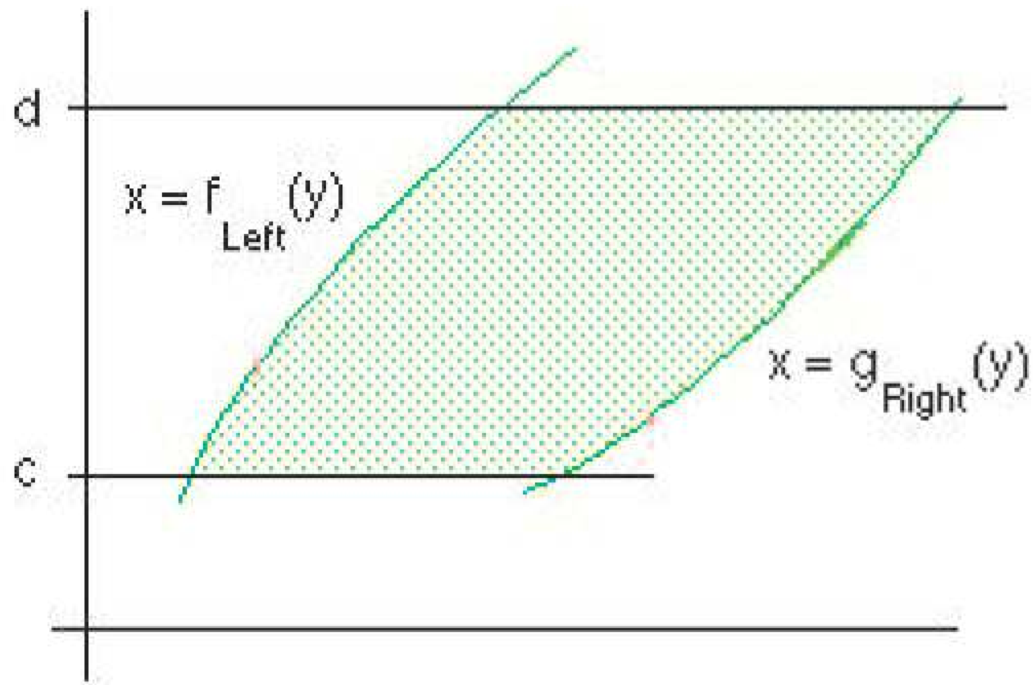


- Find the area between $\sin x$ and $\cos x$ on $[0, \pi/4]$.



Functions of y

- We could just as well consider two functions of y, say, $x = f_{Left}(y)$ and $x = g_{Right}(y)$ defined on the interval $[c, d]$.



Area Between the Two Curves

- Find the area under the graph of $y = \ln x$ and above the interval $[1, e]$ on the x-axis.