# Math 43: Spring 2020
# Lecture 14 Summary

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# Monday

1. Are we recording?
2. We will have a class meeting as normal on Friday.
3. We will have a recorded lecture for Monday, but no class meeting from 10:10 to 11:15.
4. I will have extra office hours Monday from 1:30 to 2:30 as well as office hours as usual TTh 1:30 - 2:30. (Same zoom meeting number.)

# Antiderivatives

**Theorem**

*Suppose that $D$ is a simply connected domain. Then every analytic function $f$ on $D$ has an antiderivative on $D$.*

**Corollary**

*Neither the punctured plane $D' = \mathbf{C} \setminus \{0\}$ nor the annulus $A = \{\, z : 1 < |z| < 2 \,\}$ is simply connected.*

**Theorem (The Jordan Curve Theorem Revisited)**

*Suppose that $\Gamma$ is a simple closed contour. (We may call such a $\Gamma$ a Jordan contour.) Then the interior of $\Gamma$ is simply connected.*

# Example

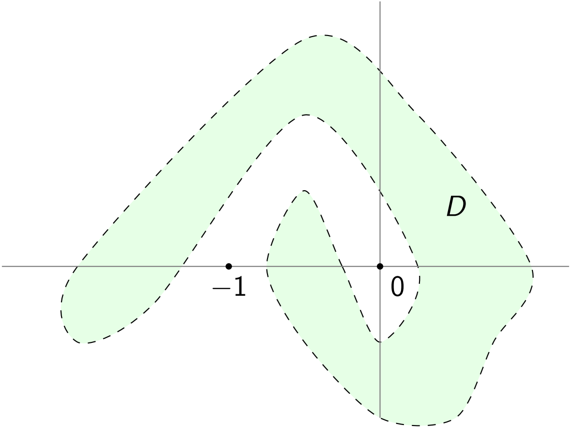



Figure: A Simply Connected Domain $D$

# Branches

**Corollary**

*Suppose that $D$ is a simply connected domain that does not contain $0$. Then there is an analytic branch of $\log(z)$ in $D$.*

**Corollary**

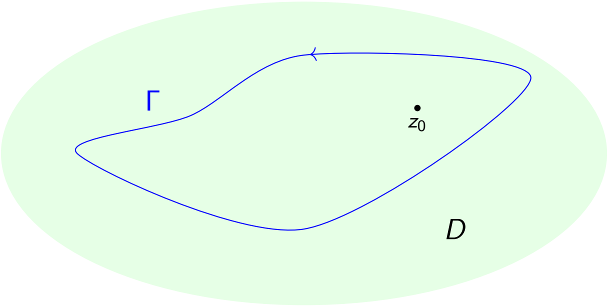
*Suppose that $u$ is harmonic in a simply connected domain $D$. Then $u$ has a harmonic conjugate in $D$.*

# The Cauchy Integral Formula

**Theorem (Cauchy Integral Formula)**

*Suppose that $\Gamma$ is a positively oriented simple closed contour in a simply connected domain $D$. If $z_0$ lies inside of $\Gamma$ and if $f$ is analytic in $D$, then*

$$f(z_0) = \frac{1}{2\pi i} \int_\Gamma \frac{f(w)}{w - z_0}\, dw.$$



# Some Shortcuts

## Remark (Just Between Us)

The authors like the phase, "suppose that $f$ is analytic on and inside a simply closed contour $\Gamma$". This means that $f$ is analytic on the domain that forms the interior of $\Gamma$ and at each point of $\Gamma$. Recall the technicality that we say $f$ is analytic at a point only if it is analytic in a neighborhood of that point. It follows that if "$f$ is analytic on and inside a simply closed contour $\Gamma$", then $f$ is analytic in domain that contains $\Gamma$. We are going to accept the highly nontrivial fact that we can take this domain to be simply connected. This is what we did explicitly in the previous example where $f(z) = \cos(z^2)/z^2$ was clearly analytic on and inside $|z - 3| = 2$.

# NC-17 Versions

## Theorem (Cauchy NC-17)

*Suppose that $f$ is analytic on and inside a simple closed contour $\Gamma$.*

1. *[Cauchy Integral Theorem] Then*

$$\int_\Gamma f(z)\,dz = 0.$$

2. *[Cauchy Integral Formula] If $z_0$ lies inside of $\Gamma$, then*

$$f(z_0) = \frac{1}{2\pi i}\int_\Gamma \frac{f(w)}{w - z_0}\,dw.$$