Read Last Page for Instructions!

1. (24) Solve the following initial value problems.

(a) $$\begin{cases} \dfrac{dy}{dx} + \sin(x)y^2 = 0 \\ y(0) = 1. \end{cases}$$

(b) $$\begin{cases} \dfrac{dy}{dx} + \sin(x)y^2 = 0 \\ y(0) = 0. \end{cases}$$

(c) $$\begin{cases} \dfrac{dy}{dx} + \dfrac{2}{10+x}y = 3 \\ y(0) = 1. \end{cases}$$

(d) $$\begin{cases} \dfrac{dy}{dx} + \dfrac{2}{10+x}y = 3 \\ y(0) = 0. \end{cases}$$

2. (24) Consider the second order differential equation

$$(*) \qquad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0.$$

(a) Find the general solution to $(*)$.

(b) Find the solution which satisfies the initial conditions $y(0) = 2$ and $y'(0) = 3$.

(c) Find one solution to the second order differential equation

$$(\dagger) \qquad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 2x^2.$$

(d) Find the general solution to $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 2x^2$.

3. (7) Find the MacLaurin series for

$$f(x) = x^2 \cos(2x).$$

To get full credit you must provide at least four nonzero terms and a formula for the general term of the series.

4. (16) Let $f(x) = \cos(x)$.

(a) Find the Taylor polynomial of degree 2 for f about $x = 0$.

(b) Use your answer from part (a) to estimate $\cos(\frac{1}{2})$.

(c) Find the best upper bound you can for the absolute value of the error in your estimate in part (b). (That is, you should find the smallest number which you can show is larger than the absolute value of the difference between $\cos(\frac{1}{2})$ and your estimate in part (b).)

(Problem 4 continued on next page)

(d) (This part of problem 4 is a bit subtle. You may want to come back to it after you've completed the rest of the exam.) Find the shortest interval which you can guarantee contains the exact value of $\cos(\frac{1}{2})$. (If you wish, you may use the facts that $\frac{1}{2} < \frac{\pi}{6}$, that $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ and that $\sin(\frac{\pi}{6}) = \frac{1}{2}$.)

5. (11) A large tank contains 10 pounds of salt disolved in 100 gallons of pure water. If water containing 1 pound of salt per gallon is added at a rate of one gallon per minute while the well-mixed solution is drawn off at the same rate, then how many minutes will it take until there is 55 pounds of disolved salt in the tank. (Hint: first find a formula for the number of pounds $A(t)$ of salt in the tank at time t by setting up a first order differential equation and solving it.) *To get full credit you must clearly indicate (a) the differential equation to be solved, (b) your solution of the differential equation and (c) your value for the time t required.*)

6. (18) MULTIPLE CHOICE. Circle the best response. No partial credit will be given on this problem and you do not need to justify your answers.

(a) The Taylor polynomial of degree 2 for $f(x) = e^{2x}$ about $x = 0$ is

A. $1 + e^{2x}x + \frac{e^{2x}}{x}x^2$ **B**. $1 - 2x^2$ **C**. $1 + 2x + 2x^2$

D. $1 + 2x + 4x^2$ **E**. None of These

(b) The power series $\sum_{n=0}^{\infty}(3x-2)^n$ converges for all x in

A. $(0, 1)$ **B**. $(\frac{1}{3}, 1)$ **C**. $(-\frac{1}{3}, -\frac{1}{3})$ **D**. $(1, 3)$

E. None of These

(c) The radius of convergence of $\sum_{n=0}^{\infty} 3^n x^{2n}$ is

A. infinite **B**. $\sqrt{3}$ **C**. $\frac{1}{3}$ **D**. $\frac{1}{\sqrt{3}}$ **E**. None of These

(d) The solution to $\frac{dy}{dx} = y^2 + x$ passing through $(1, -1)$ is most likely to pass closest to which of the following points?

A. $(1.5, -2)$ **B**. $(1.5, -1.5)$ **C**. $(1.5, -3)$ **D**. $(1.5, 0)$

E. $(1.5, -5)$

(e) The sum of the series $\frac{3}{2} - \frac{3}{4} + \frac{3}{2^3} - \frac{3}{2^4} + \dots$ is

A. $\frac{2}{3}$ **B**. $\frac{3}{2}$ **C**. 1 **D**. 2 **E**. None of These

(f) The general solution to $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ is

A. $Ae^{2x} + Be^{-2x}$ **B**. $Ae^{2x} + Bxe^{2x}$ **C**. $Ae^{2x} + Be^{2x}$

D. $Ae^{-2x} + Bxe^{-2x}$ **E**. None of These

NAME : _____

Math 8

14 April 2001
Sample Hour Exam I

INSTRUCTIONS:

- Please *print* your name in the space provided.
- A well-prepared student should be able to finish this exam in one hour and fifteen minutes. However, it would be tight.
- Calculators are not allowed. The use of a calculator is a violation of the Honor Code.
- Except on the multiple choice problem (#6), you must show your work and justify assertions to receive full credit.

Problem	Points	Score
1	24	
2	24	
3	7	
4	16	
5	11	
6	18	
Total	100	