Practice Exam 2.
We think this would take a well-prepared student about 75 minutes.

1. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$.

   (a) Circle whichever of the following matrix products that are defined:

   $$A^2 \quad AB \quad BA \quad B^2$$

   (b) Compute the products in part (a) which are defined.

   (c) Compute the inverse of the matrix B or else show it has no inverse.

2. An airplane is flying in the jetstream at a speed of 300 miles per hour. The jetstream is blowing due east at a constant speed of 100 miles per hour. The airplane is now 400 miles south of the place where it will begin it's landing path.

(a) What is the tangent of the angle the plane must make with a north-south axis to arrive at the beginning of its landing path?

(b) When will the plane reach the beginning of its landing path?

3. Find a vector parametric equation for the line of intersection between the planes $x + 2y + 3z = 1$ and $2x + 5y + 4z = 3$.

4. Let A be a 3×4 matrix with row reduced form

$$R = \begin{pmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Find all solutions to $A\mathbf{x} = \mathbf{0}$.

(b) Let $\mathbf{b}$ be a vector in $\mathbb{R}^3$. Suppose that $\mathbf{x}_p = (1, 1, 0, 1)$ is a solution to

$$A\mathbf{x} = \mathbf{b}.$$

Find the general solution to $A\mathbf{x} = \mathbf{b}$.

5. Suppose that r and s are real numbers and that

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & r & s \end{pmatrix}.$$

For which values, if any, of r and s does

$$A\mathbf{x} = \mathbf{b}$$

have a solution for all $\mathbf{b}$ in $\mathbb{R}^3$? For what values, if any, of r and s does

$$A\mathbf{x} = \mathbf{b}$$

have a unique solution for all $\mathbf{b}$ in $\mathbf{R}^3$?

6.

Show that the lines $\mathbf{r}_1 = (2 + t, 4 + 2t, 4 + t)$ and $\mathbf{r}_2 = (2 + s, 3 + 2s, s)$ are parallel and find the equation of the plane which contains both lines.

7. Find the distance from the point $(1, 2, 3)$ to the plane $2x + y - 4z = 5$.

8. MULTIPLE CHOICE. Circle the correct response. There will be no partial credit and you do not have to show your work.

(a) Which of the following points does not lie on the line through $(1, 2, 3)$ and parallel to $\mathbf{v} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$?

A. $(3, 1, 2)$ **B**. $(-1, 3, 4)$ **C**. $(2, 1, 1)$ **D**. $(5, 0, 1)$

E. None of These

(b) Which of the following points does not lie on the plane through $(1, 2, 3)$ with normal vector $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$?

A. $(0, 0, 3)$ **B**. $(4, -1, 0)$ **C**. $(1, 1, 0)$ **D**. $(0, -9, 0)$

E. None of These

(c) Suppose that $\mathbf{u}$ and $\mathbf{v}$ are 3-vectors such that $\mathbf{u} \times \mathbf{v} = 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{u} \bullet \mathbf{u} = 3 = \mathbf{v} \bullet \mathbf{v}$. Then $|\mathbf{u} \bullet \mathbf{v}| =$

A. 0 **B**. 1 **C**. 2 **D**. 3 **E**. None of These

(d) The scalar projection of $\mathbf{u} = (3, 1, 4)$ in the direction of $\mathbf{v} = (2, 2, 1)$ is

A. 1 **B**. 2 **C**. 3 **D**. 4 **E**. None of These

(e) Which of the following is *not* perpendicular to $\mathbf{i} + \mathbf{j} + \mathbf{k}$?

A. $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ **B**. $-\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ **C**. $\mathbf{j} - \mathbf{k}$ **D**. $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

E. None of These

(f) Find the cosine of the angle between the planes $x + 2y + z = 9$ and $-x - y + 2z = 5$. (Note that the angle between the planes is the same as the angle between the normal vectors to the planes.)

A. $-\frac{\pi}{4}$ **B.** $\frac{5\pi}{7}$ **C.** $-\frac{1}{6}$ **D.** $\frac{5}{9}$ **E.** None of These

(g) If $A = \begin{pmatrix} 1 & 0 & 3 & 3 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$, number of parameters in the most general solution to $A\mathbf{x} = \mathbf{0}$ is

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

(h) The area of the triangle with vertices $P_1(1,0,1)$, $P_2(2,2,2)$ and $P_3(3,4,5)$ is

A. 3 **B.** $\sqrt{6}$ **C.** 4 **D.** $\sqrt{3}$ **E.** $\sqrt{5}$