1. (a) Find and classify any critical points of $f(x, y) = x^2 + x + 2y^2$ in the region $x^2 + y^2 < 1$.

ANS: ____.

(b) Find the extreme values of $f(x, y) = x^2 + x + 2y^2$ on the region $x^2 + y^2 \leq 1$.

The maximum is ______________ which occurs at the point(s) ______________.

The minimum is ______________ which occurs at the point(s) ______________.

2. (a) Find the general solution to $y'' + 4y' + 4y = 0$.

$y(x) =$ ______________________.

(b) Given that $y_p(x) = x$ is a solution to the nonhomogeneous equation

(†) $$y'' + 4y' + 4y = f(x),$$

find the solution to (†) which satisfies $y(0) = 2$ and $y'(0) = 1$.

$y(x) =$ ______________________.

(c) What is $f(x)$ equal to?

$f(x) =$ ____________________.

3. Suppose that $z = f(x, y)$, $x = uv$ and $y = u + 3v$. When $u = 2$ and $v = 1$, then $\dfrac{\partial z}{\partial u} = -2$ and $\dfrac{\partial z}{\partial v} = -1$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

$\dfrac{\partial z}{\partial x} =$ ________________.

$\dfrac{\partial z}{\partial y} =$ ________________.

4. (a) Find the equation of the plane containing the three points $(1, 1, 1)$, $(2, 3, 2)$ and $(2, 0, 3)$.

ANS: ______________________________

(b) Find parametric equations for the line of intersection of the planes $x + 2y + 2z = 1$ and $-x + y - 8z = 2$.

$x =$ ____________

$y =$ ____________

$z =$ ____________

5. Find all solutions to the system of equations

$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= 1 \\ x_1 - x_3 + 2x_4 &= 1 \\ x_2 + x_3 + x_4 &= 3 \\ 2x_1 + x_3 + 2x_4 &= 5 \end{aligned}$$

What kind of geometric object in four-dimensional space is the set of all solutions to the system of equations? (Be as specific as you can, but be brief!)

6. Find the *minimum* of $f(x, y, z) = x^2 + y^2 + 2z^2$ on the plane $x + y + z = 2$.

The minimun is ______________ which occurs at ______________.

7. The temperature at the point (x, y, z) is given by $T(x, y, z) = xy^2 + yz^2$. You are located at $(1, 1, 1)$.

(a) If you move in the direction $\mathbf{u} = \frac{1}{\sqrt{3}}(1, -1, 1)$, then does your temperature increase, decrease of stay the same? Explain.

ANS: ____________________.

(b) In what direction should you head so that your temperature decreases as rapidly as possible?

ANS: ____________________.

(c) What is your maximum rate of cooling in part (b)?

ANS: ____________________.

8. MULTIPLE CHOICE. Circle the correct response. There will be no partial credit and you do not have to show your work.

(a) The MacLaurin Series for $f(x) = e^x$ is

A. $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}$ **B.** $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ **C.** $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

D. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n!}$ **E.** None of These

(b) The MacLaurin Series for $f(x) = \cos x$ is

A. $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}$ **B.** $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ **C.** $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

D. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n!}$ **E.** None of These

(c) The MacLaurin Series for $f(x) = xe^{-x}$ is

A. $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}$ **B.** $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ **C.** $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

D. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n!}$ **E.** None of These

(d) Suppose that P is parallelogram with vertices at $(1, 1, 1)$, $(2, 3, 4)$ and $(2, 4, 3)$. The fourth vertex is

A. $(0, 0, 0)$ **B.** $(3, 6, 6)$ **C.** $(3, 6, 7)$ **D.** $(4, 5, 6)$

E. None of These

(e) The function $f(x,y) = 2x^3 - 6xy + 3y^2$ has a critical point at $(1,1)$. At $(1,1)$, f has a

A. local minimum **B.** local maximum **C.** saddle point

D. singularity **E.** None of These

(f) The tangent plane to the surface $z = x^2y + xy^2$ at the point $(1,2,6)$ is

A. $8x + 5y = 12$ **B.** $x + 2y - z = 6$ **C.** $8x + 5y - z = 12$

D. $2x + y + z = 6$ **E.** None of These

(g) Suppose that $\frac{dy}{dx} = -\frac{x}{y}$ and $y(0) = 2$. Then $y(1) =$

A. 0 **B.** 1 **C.** $\sqrt{2}$ **D.** $\sqrt{3}$ **E.** 2

(h) The general solution to $y' + 3y = e^x$ is

A. $\frac{1}{4} + Ce^{3x}$ **B.** $\frac{1}{4}e^x + C$ **C.** $\frac{1}{4} + Ce^{-3x}$ **D.** $\frac{1}{4}e^x + Ce^{-3x}$

E. None of These

(i) The radius of convergence of the power series $\sum_{n=0}^{\infty} 2^n x^n$ is

A. 0 **B.** 2 **C.** $\frac{1}{2}$ **D.** ∞ **E.** None of These

(j) Suppose that $A = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 \end{pmatrix}$. Then the number of parameters in a general solution for equation $A\mathbf{x} = \mathbf{0}$ is

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** None of These