MATH 251: ABSTRACT ALGEBRA I
HOMEWORK #6

PROBLEMS (FOR ALL)

Problem 1 (DF 3.1.8). Let $\phi : \mathbb{R}^\times \to \mathbb{R}^\times$ be defined by $\phi(x) = |x|$. Prove that ϕ is a homomorphism. Find the kernel and image of ϕ, and describe the fibers of ϕ.

Problem 2. Show that $\langle (1\ 2) \rangle \leq S_3$ is not a normal subgroup of S_3.

Problem 3. Let $H = \{(), (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\} \subset S_4$. Show that H is a subgroup of S_4. What are the left cosets of H in S_4? How many are there, and how many elements are in each coset? Write each coset in the form xH for some $x \in S_4$.

Problem 4. Let $N \leq G$ be a subgroup and suppose that $x^{-1}Nx \subset N$ for all $x \in G$. Prove that $x^{-1}Nx = N$ for all $x \in G$.

Problem 5. Show that every subgroup of an abelian group is normal.