MATH 251: ABSTRACT ALGEBRA I
IN CLASS REVIEW, EXAM #3

Problem A. Let R be a ring. The *center* of R is the set

$$Z(R) = \{z \in R : zr = rz \text{ for all } r \in R\}.$$

(a) Prove that the center $Z(R)$ is a subring of R.

(b) Prove that the center of a division ring is a field.

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Problem B. Let R be a ring and let $a \in R$. Let $L(a) = \{x \in R : xa = 0\}$. Show that $L(a)$ is a left ideal of R.

Problem C. Let F be a field. Show that any homomorphism $\phi : F \to R$ where R is a ring must either be injective or zero.