MATH 20C: FUNDAMENTALS OF CALCULUS II
EXAM #1

Problem 1.

(a) Evaluate the integral

$$\int (4x^3 + x^2)\,dx.$$

(b) Evaluate the integral

$$\int x\left(x^{1.5} - x^{-1} + \frac{3}{x^2}\right)dx.$$

Solution. For (a), we have

$$\int (4x^3 + x^2)\,dx = x^4 + \frac{x^3}{3} + C.$$

For (b), we multiply out to obtain

$$\int x\left(x^{1.5} - x^{-1} + \frac{3}{x^2}\right)dx = \int (x^{2.5} - 1 + 3x^{-1})\,dx = \frac{x^{3.5}}{3.5} - x + 3\ln|x| + C = \frac{2}{7}x^{3.5} - x + 3\ln|x| + C.$$

Problem 2. The slope of the function $f(x)$ at the point $(x, f(x))$ is equal to $9 - e^x$ and $f(0) = 1$. Find the function $f(x)$.

Solution. We are given that $f'(x) = 9 - e^x$, since the derivative is the slope, so $f(x) = 9x - e^x + C$. Since $f(0) = -1 + C = 1$, we have $C = 2$, so $f(x) = 9x - e^x + 2$.

Problem 3. Evaluate the integral

$$\int \frac{x^2}{(x^3 - 7)^{0.7}}\,dx.$$

Solution. We make the subtitution $u = x^3 - 7$, so $du = (3x^2)\,dx$ or $x^2\,dx = du/3$. Then

$$\int \frac{x^2}{(x^3 - 7)^{0.7}}\,dx = \int \frac{1}{u^{0.7}}\frac{du}{3} = \frac{1}{3}\int u^{-0.7}\,du = \frac{1}{3}\frac{u^{0.3}}{0.3} + C = \frac{10}{9}(x^3 - 7)^{0.3} + C.$$

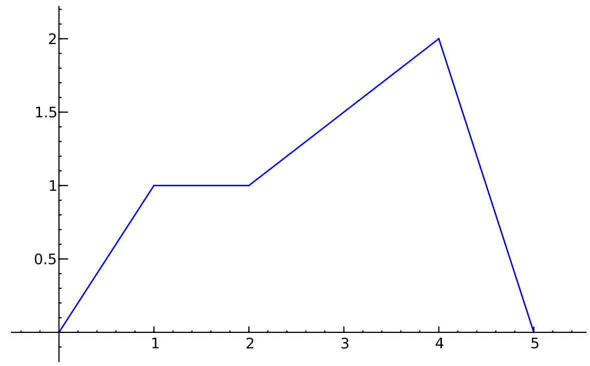
Problem 4. Evaluate the integral

$$\int (2x - 3)e^{2x^2 - 6x}\,dx.$$

Solution. We make the substitution $u = 2x^2 - 6x$, so that $du = (4x - 6)\,dx = 2(2x - 3)\,dx$, or $(2x - 3)\,dx = du/2$. Thus

$$\int (2x - 3)e^{2x^2 - 6x}\,dx = \int e^u \frac{du}{2} = \frac{1}{2}e^u + C = \frac{1}{2}e^{2x^2 - 6x} + C.$$

Problem 5. Use the following graph of $f(x)$ to compute $\int_0^5 f(x)\,dx$.



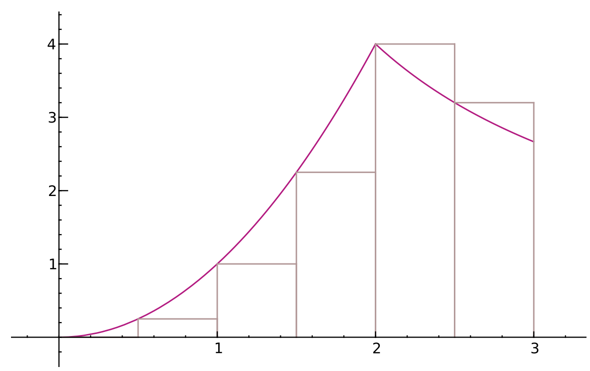
Solution. The integral is just the area under the curve, which is $1/2 + 1 + 2 + 1 + 1 = 11/2$.

Problem 6(a). Calculate the left Riemann sum to approximate $\int_0^3 \frac{1}{1+2x}\,dx$ using $n = 3$ subdivisions.

Solution. Let $f(x) = 1/(1+2x)$. We have $a = 3$ and $b = 0$ so $\Delta x = (3-0)/3 = 1$. Thus the Riemann sum is simply

$$1\,(f(0) + f(1) + f(2)) = 1 + 1/3 + 1/5 = 23/15.$$

Problem 6(b). Draw the rectangles representing the left Riemann sum for the following function $f(x)$ on the interval $[0, 3]$ using 6 subdivisions.



Problem 7. Compute the area under the graph of $f(x) = x(x^2-1)^4$ between $x = 0$ and $x = 1$.

Solution. We need to compute

$$\int_0^1 x(x^2-1)^4\,dx.$$

We make the substitution $u = x^2 - 1$, so that $du = 2x\,dx$. If $x = 0$ then $u = -1$ and if $x = 1$ then $u = 0$. Thus

$$\int_0^1 x(x^2-1)^4\,dx = \int_{-1}^0 u^4 \frac{du}{2} = \frac{u^5}{10}\bigg|_{-1}^0 = -(-1)^5/10 = 1/10.$$

Problem 8. Evaluate the definite integral

$$\int_1^e \left(2x + \frac{2}{x}\right)\,dx.$$

Solution. We have

$$\int_1^e \left(2x + \frac{2}{x}\right)\,dx = \left(x^2 + 2\ln|x|\right)\big|_1^e = (e^2+2) - (1+0) = e^2 + 1.$$

Problem 9. A book publisher declares that the marginal cost to produce x books is

$$C'(x) = 10 - 500\frac{x}{(x+1)^3}$$

dollars, and that the fixed cost is 500 dollars. What is the cost function $C(x)$?

Solution. The marginal cost is the derivative of the total cost, so

$$C(x) = \int \left(10 - 500\frac{x}{(x+1)^3}\right) dx = 10x - 500\int \frac{x}{(x+1)^3}\, dx.$$

We now make the substitution $u = x + 1$, so $du = dx$ and $x = u - 1$. Thus

$$\int \frac{x}{(x+1)^3}\, dx = \int \frac{u-1}{u^3}\, du = \int (u^{-2} - u^{-3})\, du = \frac{u^{-1}}{-1} - \frac{u^{-2}}{-2} + K = -\frac{1}{x+1} + \frac{1}{2(x+1)^2} + K.$$

So

$$C(x) = 10x + 500\left(\frac{1}{x+1} - \frac{1}{2(x+1)^2}\right) + K.$$

The fixed cost is $C(0) = 500(1/2) + K = 250 + K = 500$, so $K = 250$. Thus

$$C(x) = 10x + 500\left(\frac{1}{x+1} - \frac{1}{2(x+1)^2}\right) + 250.$$