MATH 20C: FUNDAMENTALS OF CALCULUS II
WORKSHEET, DAY #17

Problem 1. Find the total value of the income stream $R(t) = 40000$ on the interval $0 \le t \le 5$ and find its future value at the end of the interval using the interest rate 10%.

Solution. We have the total value is ($a = 0$, $b = 5$)

$$TV = \int_a^b R(t)\,dt = \int_0^5 40000\,dt = 40000t\Big|_0^5 = \$200000.$$

To compute the future value, we have $r = 0.1$, so

$$FV = \int_a^b R(t)e^{r(b-t)}\,dt = \int_0^5 40000e^{0.1(5-t)}\,dt = -\frac{40000}{0.1}e^{0.1(5-t)}\Big|_0^5 = \$259.488.51.$$

Problem 2. Find the total value of the income stream $R(t) = 50000 + 2000t$ on the interval $0 \le t \le 10$ and find its present value at the beginning of the interval using the interest rate 5%.

Solution. The total value is

$$TV = \int_0^{10} (50000 + 2000t)\,dt = (50000t + 1000t^2)\Big|_0^{10} = \$600000.$$

The present value is ($r = 0.05$)

$$PV = \int_a^b R(t)e^{r(a-t)}\,dt = \int_0^{10} (50000 + 2000t)e^{-0.05t}\,dt.$$

Use integration by parts with $u = 50000 + 2000t$ and $v = e^{-0.05t}$ to obtain

$$\int_0^{10} (50000 + 2000t)e^{-0.05t}\,dt = \left((-1000000 + 40000t)e^{-0.05t} - 800000e^{-0.05t}\right)\Big|_0^{10} = \$465632.55.$$

Problem 3. You begin saving for your retirement by investing \$700 per month in an annuity with a guaranteed interest rate of 6% per year. You increase the amount you invest at the rate of 3% per year. With continuous investment and compounding, how much will you have accumulated in the annuity by the time you retire in 45 years?

Solution. The revenue stream is $R(t) = 12 \times 700e^{0.03t} = 8400e^{0.03t}$ since you do it for each month. So the future value is

$$\int_0^{45} (8400e^{0.03t})e^{0.06(45-t)}\,dt = 8400e^{2.7}\int_0^{45} e^{-0.03t}\,dt = -280000e^{2.7}e^{-0.03t}\Big|_0^{45} = \$3086245.73.$$

See why you should start saving now?

Date: Friday, 10 October 2008.