MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I
EXAM #1

Name ________________________________

Problem 1. Mark each as true or false. Briefly justify your answer.

(a) If (a_n) has a convergent subsequence, then (a_n) is convergent.

(b) If (a_n) is convergent and $(a_n b_n)$ is convergent then (b_n) is convergent.

(c) If (x_n) is a Cauchy sequence, then there exists $N \in \mathbb{N}$ such that for all $n, m \geq N$ we have $|x_{n+1} - x_{m+1}| \leq |x_n - x_m|$.

(d) The set of all functions $f : \mathbb{N} \to \{0, 1\}$ is uncountable.

(e) Every nonempty bounded subset of $\mathbb{R}$ has a maximum.

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Problem 2. Let $A \subset \mathbb{R}$ and $B \subset \mathbb{R}$ be nonempty subsets of the real numbers. Define

$$A + B = \{a + b : a \in A,\ b \in B\}.$$

Prove that if A and B are bounded above, then

$$\sup(A + B) = \sup(A) + \sup(B).$$

Problem 3. Prove using the definition that

$$\lim \frac{\sqrt{n}}{n+1} \to 0.$$

Problem 4. Let (x_n) be a sequence and suppose that $x_n \to x$ and $x_n \to y$. Show that $x = y$.

Problem 5. Suppose $a_n \geq 0$ and $a_n \to 0$. Given any $\epsilon > 0$, show that there is a subsequence (b_n) of (a_n) such that $\sum_{n=1}^{\infty} b_n < \epsilon$.