

**MATH 052: INTRODUCTION TO PROOFS
REVIEW, EXAM #1**

Problem 1. Give the truth table for $Q \wedge (P \vee \sim Q)$.

Problem 2. What is the negation of the statement “There exists $x \in \mathbb{R}$ such that $x^2 < x$ ”? Write this in symbols and in words. Is this statement or its negation true?

Problem 3. I got an A+ in every nuclear physics I took. What is the easiest way for this statement to be true?

Problem 4. List the elements of the sets

$$A = \{n \in \mathbb{N} : n^3 < 100\} \text{ and } B = \{x \in \mathbb{R} : x^2 + 1 = 0\}.$$

Problem 5. Let $n \in \mathbb{Z}$. Prove that $(n + 1)^2 - 1$ is even if and only if n is even.

Problem 6. Let $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}$.

- (a) Determine which of the following are elements of A : \emptyset , $\{\emptyset\}$, $\{\emptyset, \{\emptyset\}\}$. Which are subsets of A ?
- (b) How many elements are in A ?
- (c) Determine the sets $\{\emptyset\} \cap A$ and $\emptyset \cup A$.

Problem 7. Consider the statement

“If a series converges, then its terms go to zero.”

This statement is in the form $P \Rightarrow Q$. Write each of the corresponding statements and determine their truth value.

- (a) $Q \Rightarrow P$.
- (b) $\sim Q \Rightarrow \sim P$.
- (c) $\sim P \Rightarrow \sim Q$.
- (d) $P \Leftrightarrow Q$.
- (e) $\sim(P \Rightarrow Q)$.

Which of these are logically equivalent to the original statement?

Problem 8. Let A, B be sets. Prove that $A \cup B = A$ if and only if $B \subseteq A$.

Problem 9. The statement

“If $x < 1 + \epsilon$ for all $\epsilon > 0$, then $x < 1$ ”

is false.

- (a) Give the negation of the statement.
- (b) Prove that the negation of the statement is true.

Problem 10. Is the following sentence true or false? “This sentence is false.”