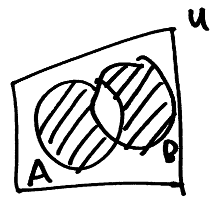
MATH 052: FUNDAMENTALS OF MATHEMATICS
EXAM #1

Problem 1. For (a), the implication is true, because the hypothesis is false. For (b), there are 4 elements– no repetitions in sets are allowed. For (c), the negation of an implication $P \Rightarrow Q$, which is defined to be $\sim(P \wedge (\sim Q))$, is $P \wedge (\sim Q)$. So the negation is: “You earned a passing grade on the midterm exam and did not get your allowance.” For (d), we have:



$(A \cup B) \setminus (A \cap B)$

Problem 2. For (a), for example, $A = \{x \in \mathbb{Z} : -2 \leq x \leq 3 \text{ and } x \neq 0\}$. For (b), $D = \{3, 12\}$, $E = \{1, 12, 20, 35\}$, and $F = \{1, 3, 12, 20, 35\}$; 1 is not a prime number. Since 12 is an element of each of D, E, F, no pair of these sets is disjoint. For (c), for example, $A = C = \{\emptyset\}$ and $B = \emptyset$.

Problem 3. For (a), $P(1)$ is the statement “15 is odd”. For (b), since $5(6)/2 = 15$ and $6(7)/2 = 21$ are odd but $7(8)/2 = 28$ is even, we see that $P(n)$ is T for $n = 1, 2$ and F for $n = 3$. Since $2^0 = 1 < 3$ and $2^1 = 2 < 3$ but $2^2 > 3$, we see that $Q(n)$ is F for $n = 1, 2$ and T for $n = 3$. So there is no value $n \in S$ such that $P(n) \Leftrightarrow Q(n)$. For (c), we have $\sim(P(n) \vee Q(n)) \cong (\sim P(n)) \wedge (\sim Q(n))$ by de Morgan’s laws, and this reads: “$(n + 4)(n + 5)/2$ is even and $2^{n-1} \leq 3$”.

Problem 4. For (a), fun with truth tables!

P	Q	R	$P \Rightarrow$	$(\sim Q) \Rightarrow R$	$(P \Rightarrow R) \vee ((\sim Q) \Rightarrow R$	$P \vee (\sim Q)$	$(P \vee (\sim Q)) \Rightarrow R$
F	F	F	T	F	T	T	F
F	F	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	T	T	T	T	T	F	T
T	F	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	T	F	F	T	T	T	F
T	T	T	T	T	T	T	T

Since columns 6 and 8 do not have the same truth table, the sentential forms are not logically equivalent.

For (b), a tautology is a sentential form which is true no matter what the values of the propositions. For example, $P \vee (\sim P)$ is a tautology.

Problem 5. For (a), $\mathcal{P}(A) = \{\emptyset, \{0\}, \{2\}, \{-6\}, \{0, 2\}, \{0, -6\}, \{2, -6\}, \{0, 2, -6\}\}$. For (b), S_1 is a partition and is the only one. S_2 has $\emptyset \in S_2$, S_3 has 5 repeated, and S_4 is missing 6. For (c), True: this is the definition of a partition. For (d), $A \times B = \{(-1, u), (-1, v), (0, u), (0, v), (1, u), (1, v)\}$. For (e), we have $\bigcup_{x \in \{1,2,3\}} [x, 2x] = [1, 2] \cup [2, 4] \cup [3, 6] = [1, 6]$.

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