

MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #2

Problem 1 (sorta DF 8.2.5). Let $R = \mathbb{Z}[\sqrt{-5}]$.

(a) Show that the ideal $(2, 1 + \sqrt{-5})$ is not a principal ideal.

(b) Let $I = (3, 2 + \sqrt{-5})$ and $J = (3, 2 - \sqrt{-5})$. Show that the product

$$IJ = \{\sum_i x_i y_i : x_i \in I, y_i \in J\} = (3)$$

is principal.

Problem 2 (DF 8.3.5(b)). Let $R = \mathbb{Z}[\sqrt{-n}]$ where $n > 3$ is a squarefree integer. Show that R is not a UFD. [*Hint: Show that either $\sqrt{-n}$ or $1 + \sqrt{-n}$ is not prime.*]

Problem 3. Factor the element 390 into irreducibles in $\mathbb{Z}[i]$. [*Hint: See Proposition 18 in §8.3.*]

Problem 4. Reread your MATH 124 book.