MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #9

Problem 1 (DF 13.1.5). Let $f(x) \in \mathbb{Z}[x]$ be a monic polynomial. Suppose that $f(\alpha) = 0$ for some $\alpha \in \mathbb{Q}$. Show that $\alpha \in \mathbb{Z}$.

Problem 2 (DF 13.2.7). Prove that $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. *[Hint: Consider* $\sqrt{2}+\sqrt{3}$*.]* Conclude that $[\mathbb{Q}(\sqrt{2} + \sqrt{3}) : \mathbb{Q}] = 4$. Find an irreducible polynomial over $\mathbb{Q}$ satisfied by $\sqrt{2} + \sqrt{3}$.

Problem 3 (DF 13.2.14). Prove that if $[F(\alpha) : F]$ is odd then $F(\alpha) = F(\alpha^2)$.

Problem 4 (DF 13.2.21). Let $D \in \mathbb{Z}$ be squarefree and let $K = \mathbb{Q}(\sqrt{D})$. Let $\alpha = a + b\sqrt{D} \in K$.

(a) Show that the "multiplication by α" map

$$\begin{aligned} \phi : K &\to K \\ \beta &\mapsto \phi(\beta) = \alpha\beta \end{aligned}$$

is a linear transformation (of vector spaces over $\mathbb{Q}$).

(b) Compute the matrix of ϕ on the basis $1, \sqrt{D}$ of K.

Date: 6 April 2012; due Friday, 6 April 2012.