MATH 115: ELEMENTARY NUMBER THEORY
HOMEWORKS #3–4

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Homework #3 (Due July 12):

- §4.2: 1(a)–(c), 6, 10
- §4.3: 4(a)–(c), 12, 14, 15, 16;
  4.3A: Let $f(x) \in \mathbb{Z}[x]$ be a polynomial with integral coefficients. For $m \in \mathbb{Z}_{>1}$, let $\#X(\mathbb{Z}/m\mathbb{Z})$ denote the number of solutions in $\mathbb{Z}/m\mathbb{Z}$ of the congruence
  $$f(x) \equiv 0 \pmod{m}.$$
  (a) Prove that if $m = m_1 m_2$, where $\gcd(m_1, m_2) = 1$, then
  $$\#X(\mathbb{Z}/m\mathbb{Z}) = \#X(\mathbb{Z}/m_1\mathbb{Z}) \cdot \#X(\mathbb{Z}/m_2\mathbb{Z}).$$
  (b) What can you conclude if $\gcd(m_1, m_2) > 1$?
- §4.4: 1, 2, 10;
  4.4A: Let $k \in \mathbb{Z}_{>0}$.
  (a) Show that the product of any k consecutive integers is divisible by $k!$. *[Hint: Use a binomial coefficient.]*
  (b) Let $f(x) \in \mathbb{Z}[x]$ be a polynomial with integer coefficients, let $r \in \mathbb{Z}$. Let $f^{(k)}(x)$ denote the kth derivative of $f(x)$. Show that each coefficient of $f^{(k)}(x)$ is divisible by $k!$. Conclude that for any $r \in \mathbb{Z}$, $f^{(k)}(r)/k!$ is an integer.
- §6.1: 3, 12, 14, 16, 25, 33
- §6.3: 2, 5, 9

Homework #4 (Due July 19):

- §2.1: 1, 17, 28, 29
- §5.1: 1(c), 2(c), 3(b), 4(d), 11, 12
- §6.2: 1, 3, 18