THE VALUE OF MATHEMATICAL STORYTELLING: OUR PERSPECTIVE ON GIVING TALKS

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What is the purpose of giving a mathematical talk? As speakers, we hope to impart informative, intriguing, and sometimes entertaining mathematical ideas, perspective, and intuition. So for example a talk about a paper should be *complementary* to the main body of the paper (and go beyond what may be set up in its introduction). After all, a reader has the freedom to jump around a paper, but a listener wants to be meaningfully led. And some things are just too complicated for a talk, even if they are necessary in a paper. (If you make a definition that no one understands, does it make a sound?) Talks should be audience-focused, seeking to impart digestible material in real time.

We have found that one of the best ways to achieve this goal is to frame talks as a form of *mathematical storytelling*. Indeed, the art of storytelling—an innate human talent—provides a natural, clarifying lens when preparing a talk. People excel at following and remembering stories—we do it all the time! Our understanding is more robust when notions are stitched into context. If we were to give one broad piece of advice about giving a good talk,it would be this: focus on the story.

In what follows, we will expand on this advice based on our (mostly US-based) experience. The usual caveats apply: the advice is not meant to delineate a "one size fits all" recipe, the details are always under revision, and the advice reflects our trials and attempts. Sometimes—maybe often—we fail! But we do hope that something we suggest here will be useful to you as you reflect on how to give a stimulating math talk.

This note is organized as follows. We begin in section 1 by surveying some of the basic aspects of mathematical storytelling. In section 2, we elaborate further and propose storyboarding as an implementation. We conclude in sections 3 and 4 with further things to keep in mind and a few final remarks.

1. Elements of mathematical storytelling

So what makes a good mathematical story anyway? We don't mean *literally* a story, like a crime drama involving elliptic curves (*Law and Order: ECU*?). We just mean that the essential elements that underlie a good narrative are also what makes for a good math talk.

Story arc and key moments. Every good story has a thoughtful *arc*, the path that a story follows. For a mathematical story, this arc might have *constant slope*: start at the beginning with the definitions, establish preliminaries like laying bricks one after the other, and state your theorem at the end. This can work as a talk—indeed, it is quite common—but we contend that it is not always the best way to tell a memorable mathematical story.

In fact, there are many possibilities for a good arc. The best choice will depend on the specific topic, your intentions, and your audience. One *mountain trek* arc might be as follows:

(1) Kickoff with something to grab the audience's attention: a tantalizing question, a quip, a puzzle or riddle, or an old but enduringly potent observation.
(2) Head on a gondola up the mountain (i.e., with good pacing and not expecting your audience to do all the work) to some kind of vista: state the motivating conjecture, give the bigger picture or larger context, or paint a broad theme or principle you are trying to address.
(3) Follow along the ridge to the statement of your main results or key points.
(4) Enjoy and comment on the view. Maybe look back on the path up the mountain by giving some ideas of the proof. Point out the dangerous or intense parts, or highlight an ingenious shortcut. Or instead discuss a side

trip where there is another interesting vista nearby (e.g., examples or applications).

(Further elaboration and some other arc possibilities are provided in §2.)

The denouement or key dramatic moment in the talk is probably in the presentation and digestion of your theorem. But each step along the way can provide an essential or memorable part of the story. Being explicit about these key steps helps to ensure your audience knows where they are on the trip. (Otherwise, they might start murmuring from the back of the car: "Are we there yet?")

The arc clarifies what is essential to your story. A good storyteller is mindful about what to include and what to exclude and a story arc lays out a minimal path that captures the essential elements. More is not always more. A short story is not a compressed version of a novel to be read at top speed, and the same is true for mathematical talks. The best storytellers have their own rhythm and pace, balancing between keeping the audience interested, but not so much that it overwhelms—and adapting as needed, based on how the audience responds.

Present your perspective. Just as a story is told in the voice of the author, so too for a mathematical story, you should showcase your mathematical voice. After all, it was you who proved your theorem! In addition to appreciating the statement itself, the audience surely wants to hear how you think about the problem and came to its resolution. So do that! In particular, you should feel empowered to present material in your own way. If you prefer an equivalent definition to a more "standard" choice because it is more useful for your context, then use your preferred one. If you have a cartoon image or some visual way to understand how the proof works, sketch it. If there's a relevant anecdote which can help provide a bridge across some more technical material, share it.

(It is also a good idea to literally use your voice: modulate your pitch and speed to indicate what is exciting, what you consider background, using pauses for emphasis. No one knows what is important if you speak in a monotone way.)

Introduce your characters. The mathematical objects, definitions, and theorems are the characters of your story. Give them identities, make them memorable. Can anyone remember a slew of characters introduced in the first 5 minutes of a movie, with nothing differentiating them? So let's not introduce a whole lot of new notation all at once without giving the audience a way to understand the objects.

What makes mathematical objects, concepts, or results memorable? Perhaps the relevant objects are mysterious, and your theorem acts as a microscope to see them in detail? Or maybe your theorem says that certain objects are impenetrable or behave randomly and you can't obtain more structural understanding than already exists? Does your theorem serve as a telescope revealing a hint of structure in an unexplored part of the universe? Is there an aspect which is unexpectedly subtle or particularly conniving? Does an object deserve a nickname, perhaps because it is known for behaving a certain way?

2. Strategies for building a story

Now that we've brainstormed some possible aspects of mathematical storytelling, let's discuss some strategies to translate this into action.

Decide on your story arc. Above, we presented the mountain trek arc, but you should think of what is best suited for your story. Here are some story arcs can also be quite effective:

- Your talk could have a 'three act' structure, with Act I an opening, Act II describing conflict or progress towards a goal, and Act III the resolution.
- Some tales are best understood as conquering a foe: the theorem statement is like naming or knowing the enemy (and, in the end, making it a friend). How did you hem in your adversary? Perhaps you want to briefly survey near-miss attempts? When cornered, did it vanish from sight, or did you need just to give it a hug?
- You may have a single thread or idea that you want to explain using three different examples or cases, so you could set out the thematic statement at the start and repeatedly return to it.

- Instead of presenting your theorem towards the end, consider presenting it right up front. You may not be able to define every term, so you might have to substitute rough ideas with precise definitions. In this way, your theorem is the hook. Then, you can flash back ("three weeks earlier") to explain terms, elaborate, and justify.
- A survey talk might situate your work into the larger story of a developing area.
- Perhaps you have a 'tale of two theorems', which explains a result from two different angles, or addresses a tension between two sides (like computational versus theoretical).

Consider adding an interlude for relief by providing an example, a fun tangent, or a compelling bit of history.

Just be sure that your arc is simple enough to fit nicely into the time slot of your talk. We find that key points in the arc might be spaced out approximately every 10 minutes. The choice of a story arc should also be informed by the audience: to know potential interests, points of connection, and their stamina for arduous parts of the journey. Your story arc should compel the audience to want to follow your path. You may not want to start the talk with a detailed outline: not every story starts out by saying exactly what is going to happen and when!

Thematic possibilities. Some themes in mathematics reappear across time and area, and these can be useful as a template to get your story started. For example: the juxtaposition of structure and randomness, or how topic X informs topic Y (the different ways in which analysis and algebra inform arithmetic, for example). Or maybe these are 'axes' on which a talk could land, like problem solving versus theory building, the concrete versus the abstract, general versus specific. Referring to these archetypes can provide a quick connection to your audience, a touchstone which can be particularly useful when giving a talk to a group outside your research specialty.

Storyboard. Now that you have your arc, it's time to flesh out what your audience will take home from the arc. For key steps, your audience might appreciate a 'postcard memory'. For example, in a 50 minute talk, you might have time for four key moments, corresponding to the points in the mountain trek arc. You might try to fill in your talk first in postcard form, then filling four sheets of paper with more details (answering questions like: when is the right time to introduce this character?).

We also find it helpful sometimes to think of *slogans*, i.e., summaries given in a few words that avoid jargon, like "geometry determines arithmetic". What is your guiding philosophy, motivating question, or summary? It can be helpful to lay these out on the storyboard. These slogans should be as free as possible from specialized mathematical jargon—these are the parts of your talk your audience should be able to recount to someone else over lunch. (Well, to whatever extent this is possible, anyway!) If you are a more visual thinker, you could sketch a cartoon panel for each key point; often this exercise can give you great ideas for illustrations to use during your talk.

A key part of making this storyboard is experimenting with your definitions and theorems. Mathematicians often state theorems and definitions to prioritize ease of use over comprehension. For example, while the definition of a one-to-one function ("if $f(a) = f(b)$ then $a = b$") makes it very easy to prove that the composition of two one-to-one functions is one-to-one, we have found that some students better grasp the concept when it is introduced with the slogan "*not* many-to-one" (from the contrapositive). We encourage you to try out several different ways of stating your theorems and definitions, keeping your audience in mind. For example, consider writing out or explaining your results in special cases. Would a pictorial or diagrammatic representation help? What about a slogan that gets at the key reason why your theorem is surprising or powerful or useful, without being bogged down by mathematical symbols and unnecessary terminology? Remember that the exact, strongest, precise version of your theorem can be found in your paper (which, when finalized, can be made available on `arXiv.org`) and does not need to be reproduced verbatim in

your talk. Although you can, and should, mention that there is a stronger or more general version.

3. Things to keep in mind

When trying out mathematical storytelling, we have found a few broad suggestions to be helpful.

Focus on the why. It is very easy for a talk to primarily answer 'what' questions. What is the background needed? What has been proved before? What is the definition of this object? What is the main theorem? But for someone who is not already an expert in the area—and sometimes even for experts!—the answers to these questions may feel like a jumble of facts *without context*, which can be very challenging to remember.

It can be helpful to focus on answering 'why' questions. Why is this question interesting? Why is this problem difficult? Why is this answer expected or unexpected? Why is this hypothesis necessary? Why is the theorem true? Why is it believable in the first place? A proof shows *that* a theorem is true, but it does not always make clear *why* the theorem is true. Conversely, the reason why we expect a theorem to be true does not always yield a proof. In that case, it is also valuable to explain why the heuristic falls short.

What should the audience remember? It is easy to think of a talk as accomplishing the important but limited objective: what do I need to tell the audience so that they will understand and appreciate my theorem? In our experience, that can often lead to a talk that is jam-packed with technical definitions and background.

A more helpful prompt might be: What do I want my audience to remember after the talk? These can inform the key moments in your storyboard. Imagine an audience member leaving your talk and running into a colleague who could not attend. How would the audience member, who is probably not an expert in the area, recount your talk?

Show, don't tell. Instead of saying that a hypothesis is necessary, provide an example where the argument or theorem fails. Instead of saying that a question is central or difficult, give a simple example that illustrates its subtlety or an intriguing special case that remains open. Capture your audience's sense of wonder and surprise instead of asserting that your result is surprising. Use a heuristic explanation which gives a good reason to believe your theorem, even if it is not actually used in the proof.

4. Concluding remarks

Almost everything we have discussed here is most useful *before* we even start writing notes or preparing slides. It may seem like a lot, but we have found that taking these steps actually makes the writing or slide preparation easier and more efficient, because we have greater clarity about what is essential and what can be omitted.

Of course, there are many good places to read about "tips and tricks" for preparing good slides and for giving good presentations. We suggest considering these *after* thinking about the story, but they are still very important! There are several Early Career articles [Ker19, Dev19, Kra22, Leh20] and online resources [Ell05, Tao, Raw] that give advice about giving talks, and we have also compiled a (long) list of our favorite tips [VV].

Two final caveats. First, trying to improve all aspects of your talk at once can be counterproductive (and paralyzing). After deciding on your mathematical story, we suggest choosing at most three presentation-related aspects to focus on improving (e.g., remembering to pause before advancing to the next slide). Once you have made improvement on one, you can always pick another. Second, our advice reflects our personalities and cultural upbringing. You may have a different style, so interpret our advice in a way that works for you!

Storytelling can be a powerful technique for developing mathematical talks. Before preparing a talk, decide on story arc; then flesh it out using a storyboard, keeping in mind the need for economy and focusing on what the audience should remember from the talk. We hope this will be helpful in sharing your mathematical insights and results in a meaningful and memorable way.

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