MATH 727: QUATERNION ALGEBRAS: ALGORITHMS AND ARITHMETIC

JOHN VOIGHT

This course will introduce the arithmetic theory of quaternion algebras from an algorithmic point of view. Quaternion algebras lie at the crossroads of many areas of mathematics: number theory, Diophantine equations, group theory, noncommutative algebra, automorphic forms—even coding and network theory. We will touch on as many of these aspects as possible, with an effort to be explicit and constructive.

Some of the topics we will cover include:

- Quaternion algebras over fields. Relationship to quadratic forms. Related basic algorithmic problems. Hilbert symbols and recognizing the matrix ring.
- Global arithmetic of quaternion orders (over Dedekind domains). Types of orders. Computing a maximal order.
- Local arithmetic of quaternion orders (over DVRs). Local embedding numbers.
- Ideal theory of orders over Dedekind domains. Ideal classes and their algorithmic enumeration. Global embedding numbers. Strong approximation.
- Unit groups of quaternion orders and geometry. Arithmetic Fuchsian groups. Fundamental domains. Shimura curves and Shimura varieties, explicit methods. Brandt matrices and automorphic forms. Ramanujan graphs.

A course in algebraic number theory is suggested, and some knowledge of algebraic geometry, noncommutative ring theory, and algorithms might prove useful; however, we will define all mathematical objects of study, and so some students may like to take this course as an introduction to the more general theory.

Our first meeting will be an organizational meeting on **Thursday, January 7, 2010** in Burnside 920 at 14:30-15:30.

The image was created using Wordle `http://www.wordle.net/`.