# MATH 727:
# QUATERNION ALGEBRAS:
# ALGORITHMS AND ARITHMETIC

JOHN VOIGHT

## Course Info

- **Lectures**: Monday, Wednesday, Friday, 10:35 a.m. - 11:25 a.m.
- **Room**: 1205 Burnside
- **Instructor**: John Voight
- **Office**: 1209 Burnside
- **E-mail**: jvoight@gmail.com
- **Instructor's Office Hours**: By appointment
- **Course Web Page**: http://www.cems.uvm.edu/~voight/crmquat/
- **Instructor's Web Page**: http://www.cems.uvm.edu/~voight/

- **Prerequisites**: None.
- **Required Text**: None. References and course notes will be provided.
- **Grading**: Weekly homework will count for 100% of the grade.

## Homework

Cooperation on homework is permitted (and encouraged), but if you work together, do not take any paper away with you—in other words, you can share your thoughts (say on a blackboard), but you have to walk away with only your understanding. In particular, write the solution up on your own.

## Description

This course will introduce the arithmetic theory of quaternion algebras from an algorithmic point of view. Quaternion algebras lie at the crossroads of many areas of mathematics: number theory, Diophantine equations, group theory, noncommutative algebra, automorphic forms—even coding and network theory. We will touch on as many of these aspects as possible, with an effort to be explicit and constructive.

A course in algebraic number theory is suggested, and some knowledge of algebraic geometry, noncommutative ring theory, and algorithms might prove useful; however, we will define all mathematical objects of study, and so some students may like to take this course as an introduction to the more general theory.

## Statements

In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see www.mcgill.ca/students/srr/honest/) for more information).

## Syllabus

Although we may deviate from this by adding or skipping topics, the tentative plan for the course is as follows.

- Thu, Jan 7:
  Organizational meeting
- Mon, Jan 11:
  Quaternion algebras over fields
  Wed, Jan 13 - Fri, Jan 15:
  **No class**: *Joint meetings*
- Mon, Jan 18 - Wed, Jan 20 - Fri, Jan 22:
  Semisimple algebras, Hilbert symbol, splitting, quaternion algebras over local fields
- Mon, Jan 25 - Wed, Jan 27 - Fri, Jan 29:
  Quaternion orders (local)
- Mon, Feb 1 - Wed, Feb 3 - Fri, Feb 5:
  Quaternion orders (global)
- Mon, Feb 8 - Wed, Feb 10 - Fri, Feb 12:
  Adelic perspective, embedding problems
- Mon, Feb 15 - Wed, Feb 17 - Fri, Feb 19:
  Quaternion ideals
- Mon, Feb 22 - Wed, Feb 24 - Fri, Feb 26:
  **No class**: *Winter break, p-adic L-functions conference at CRM*
- Mon, Mar 1 - Wed, Mar 3 - Fri, Mar 5:
  Brandt matrices, theta series, modular forms on definite quaternion algebra
- Mon, Mar 8 - Wed, Mar 10 - Fri, Mar 12:
  **No class**: *Graphs and arithmetic conference at CRM*
- Mon, Mar 15 - Wed, Mar 17 - Fri, Mar 19:
  Elliptic curves, Eichler's theorem and strong approximation
- Mon, Mar 22 - Wed, Mar 24 - Fri, Mar 26:
  **No class**: *Computer methods conference at CRM*
- Mon, Mar 29 - Wed, Mar 29 - Fri, Apr 2:
  Fuchsian groups, Shimura curves, Kleinian groups
- Mon, Apr 5 - Wed, Apr 7:
  **No class**: *Easter break, Odlyzko visit to UVM*
  Fri, Apr 9:
  Final discussion?
- Mon, Apr 12 - Wed, Apr 14:
  No class: *Computer security conference at CRM*