# Math 112: Geometric Group Theory
# Fall 2015 - Assignment 2

Return date: Tuesday 09/29/15

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**keywords:** *Cayley graphs, trees*

**exercise 2.** Draw a Cayley graph of the following groups.

a) $D_5$ with two generators.
**Note:** Let $P_5 \subset \mathbb{R}^2$ be a regular pentagon. The dihedral group $D_5$ is the subgroup of isometries of $\mathbb{R}^2$ that maps $P_5$ onto itself. This group consists of 5 rotations and 5 reflections.

b) Draw the Cayley graph of $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$ with the generators $(\bar{1}, 0)$ and $(\bar{0}, 1)$.

**exercise 3.** Find two groups $G_1$, $G_2$ such that the corresponding undirected Cayley graphs $\Gamma_1$ and $\Gamma_2$ are isomorphic, but $G_1$ and $G_2$ are not isomorphic as groups.

**exercise 4.** Let $T = (V, E, \delta)$ be a tree with a finite number of edges and vertices. A vertex $v \in V$ is called *leaf* of $T$, if $\mathrm{val}(v) = 1$.

a) Show that $\#V = \#E + 1$.

b) Let $\#V \geq 2$. Let furthermore $L = (v_i, \mathrm{val}(v_i))_{i=1,..,m}$ be a list of vertices $v_i$ of $T$ with valence $\mathrm{val}(v_i) \geq 3$. Determine a formula that calculates the number of leaves of $T$ form the list $L$.

**exercise 5.** Show that any connected undirected graph with countably many vertices and edges contains a spanning tree. A *spanning tree* is a subgraph which is a tree that contains all vertices of the graph itself.