Daily Homework #16

17. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1 - x_i^2}{4 + x_i^2} \Delta x \qquad [2, 6]$

$$= \int_2^6 \frac{1 - x^2}{4 + x^2}\, dx$$

20. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x_i^*}{(x_i^*)^2 + 4} \Delta x \qquad [1, 3] \qquad = \int_1^3 \frac{x}{x^2 + 4}\, dx$

18. $\int \cos^4\theta \sin\theta \, d\theta \qquad = -\int \cos^4\theta (-\sin\theta \, d\theta)$

let $u = \cos\theta$
$du = -\sin\theta \, d\theta$

$$= -\int u^4 \, du$$

$$= -\frac{1}{5} u^5 + C = \boxed{-\frac{1}{5}\cos^5\theta + C}$$

22. $\int \frac{\cos(\pi/x)}{x^2}\, dx \qquad = \frac{-1}{\pi}\int \cos(\pi/x)\left(\frac{-\pi}{x^2}\, dx\right)$

let $u = \pi/x = \pi x^{-1}$
$du = -\pi/x^2 \, dx$

$$= -\frac{1}{\pi}\int \cos(u)\, du$$

$$= -\frac{1}{\pi}\sin u + C$$

$$= \boxed{-\frac{1}{\pi}\sin(\pi/x) + C}$$

Daily Homework #16

28. $\int x^2\sqrt{2+x}\,dx$

$u = 2 + x \qquad du = dx$

$x = u - 2$

$$\int (u-2)^2\sqrt{u}\,du = \int (u^2 - 4u + 4)\sqrt{u}\,du$$

$$= \int u^{5/2} - 4u^{3/2} + 4u^{1/2}\,du$$

$$= \frac{2}{7}u^{7/2} - \frac{4\cdot 2}{5}u^{5/2} + \frac{4\cdot 2}{3}u^{3/2} + C$$

$$= \frac{2}{7}(x+2)^{7/2} - \frac{8}{5}(x+2)^{5/2} + \frac{8}{3}(x+2)^{3/2} + C.$$

36. $\int_0^1 (3t-1)^{50}\,dt$

let $u = 3t - 1$

$du = 3dt$

$t=1$

$$= \frac{1}{3}\int_0^1 (3t-1)^{50}(3dt)$$

remember to change the bounds!

$u=2$

$$= \frac{1}{3}\int_{-1}^{2} u^{50}\,du$$

$$= \frac{1}{3}\left(\frac{1}{51}u^{51}\Big|_{-1}^{2}\right) = \frac{1}{153}\left(2^{51} + 1\right)$$

48. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$

let $u = 1 + 2x$

$du = 2dx$

$x = \frac{u-1}{2}$

$$= \frac{1}{2}\int_1^9 \frac{\frac{u-1}{2}}{\sqrt{u}}\,du$$

$$= \frac{1}{4}\int_1^9 u^{1/2} - u^{-1/2}\,du$$

$$= \frac{1}{4}\left(\frac{2}{3}u^{3/2} - 2u^{1/2}\Big|_1^9\right) = \frac{1}{4}\left(\frac{2}{3}(9)^{3/2} - 2(9)^{1/2} - \frac{2}{3}(1)^{3/2} + 2(1)^{1/2}\right)$$

$$= \frac{1}{4}\left(\frac{2}{3}(27) - 2(3) - \frac{2}{3} + 2\right)$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$