

Solutions

Daily Homework # 1
Due Wednesday, January 6th

1. Let $f(x) = x^3 - x$. Calculate f' and f'' . Graph f , f' and f'' . Remember to label your axes.

$$f'(x) = 3x^2 - 1$$

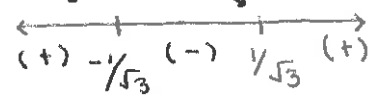
$$f''(x) = 6x$$

NOTE:
local max/min? inflection pts?

when do we have $f'(x) = 0$?

$$x = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$$

LOCAL MAX LOCAL MIN



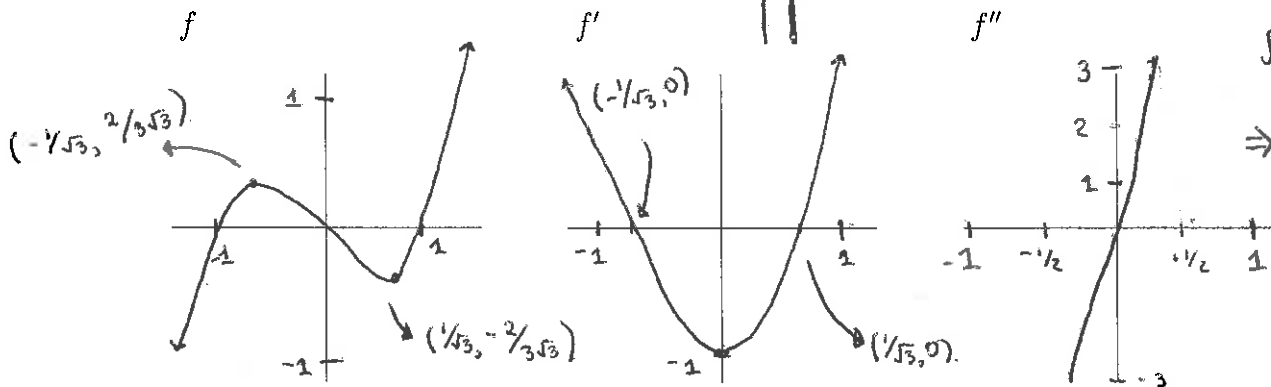
~~Inflection pt?~~ Inflection pt?

$f'(x) = 0$ when $x = 0$, $f''(x) < 0$ if $x < 0$,

$f''(x) > 0$ if $x > 0$

$\Rightarrow x = 0$ is inflection point

f has local max at $x = -1/\sqrt{3}$, local min at $x = 1/\sqrt{3}$ and inflection pt at $x = 0$



Using your graph or calculations, answer the following questions. Use interval notation.

Note: A number $x \in \mathbb{R}$ is positive if $x > 0$. A number $x \in \mathbb{R}$ is negative if $x < 0$.

- Where is f positive? on intervals $(-1, 0)$ and $(1, \infty)$.
- Where is f increasing? on intervals $(-\infty, -1/\sqrt{3}]$ and $[1/\sqrt{3}, \infty)$.
- Where is f' positive? on intervals $(-\infty, -1/\sqrt{3})$ and $(1/\sqrt{3}, \infty)$
- Where is f' increasing? on interval $[0, \infty)$
- Where is f'' positive? on interval $(0, \infty)$.

Idea to ponder: How are the questions above related?

(write some ideas that you have, this question will not be graded).

If f' is positive on an interval, then f is increasing on that interval.

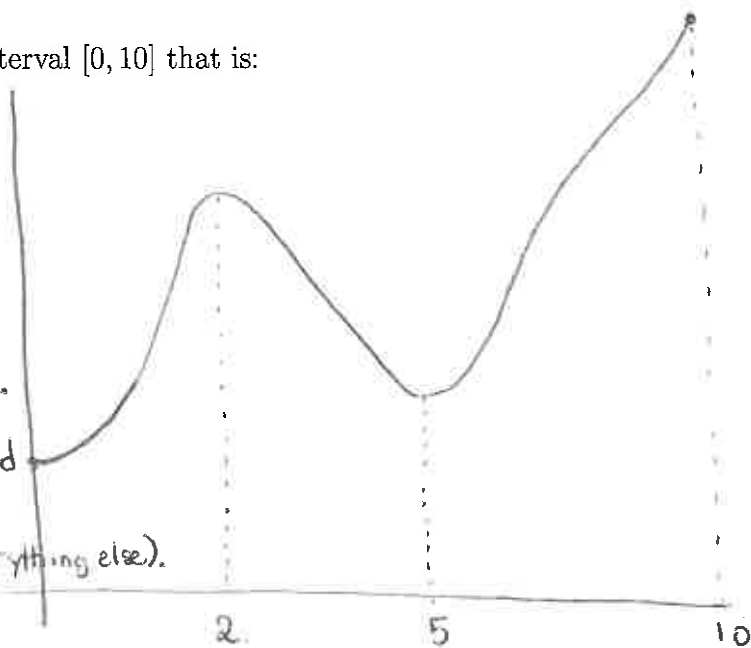
If f is increasing on an interval and f' is defined everywhere on that interval, then $f'(x) \geq 0$, (for $x \in I$ the interval)

2. Draw a continuous function defined on the interval $[0, 10]$ that is:

- I) increasing on the interval $(0, 2)$
- II) decreasing on the interval $(2, 5)$
- III) increasing on the interval $(5, 10)$.

* Remember endpoints are not considered local max/min.

Note: In this picture, $x=10$ is considered the ABSOLUTE MAX, because $f(10) \geq f(x)$ for all $x \in [0, 10]$ (it's bigger than everything else). (we'll get to this later).

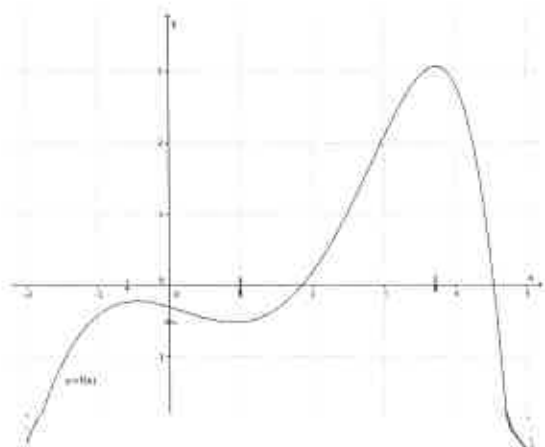


Question: Where are the local maxima and minima of your function? (Give the x coordinates.)

local minimum at $x = 5$.

local maximum at $x = 2$.

3. For the function below, answer the following questions.



* all these numbers are just approximations.

a) Where is f increasing? on intervals $[-2, -0.5]$ and $[3.8, 5]$

b) Where is f decreasing? on intervals $[-0.5, 1]$ and $[1, 3.8]$

c) Where are the local minima? local minimum at $x = 1$, pt: $(1, -0.5)$

d) Where are the local maxima? at $x = -0.5$ pt: $(-0.5, -0.25)$

$x = 3.8$ pt: $(3.8, 3.1)$