4.1 #2, 4, 19   4.2 #34.

2. a)

(i) L_6

$A_1 = b_1 h_1 = 2 \cdot 9$ $h_1 = f(0)$

$A_2 = b_2 h_2 = 2 \cdot (8.75)$ $h_2 = f(2) \ldots$

$A_3 = b_3 h_3 = 2 \cdot (8.25)$

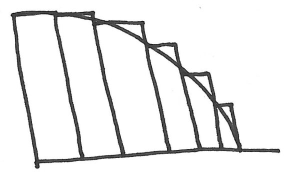
$A_4 = b_4 h_4 = 2 \cdot (7.25)$

$A_5 = b_5 h_5 = 2 \cdot (6)$

$A_6 = b_6 h_6 = 2 \cdot 4$ $L_6 = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 86.5$

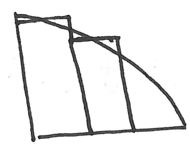
* All of these numbers are my best guess *

b) L_6 is an overestimate



c) R_6 is an underestimate.

d) M_6 gives the best estimate because there is a small bit of overestimation and a small piece of underestimation, and these little errors balance eachother out somewhat.



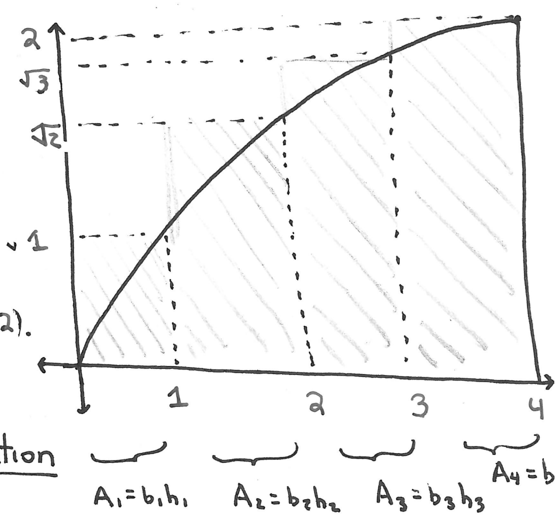
4. a)

$$R_4 = \sum_{i=1}^{4} A_i = \sum_{i=1}^{4} b_i h_i$$

$$= 1(f(1)) + 1(f(2)) + 1(f(3)) + 1 f(2).$$

$$= 1 + \sqrt{2} + \sqrt{3} + 2$$

$$= 3 + \sqrt{2} + \sqrt{3} \approx 6.146.$$ overestimation

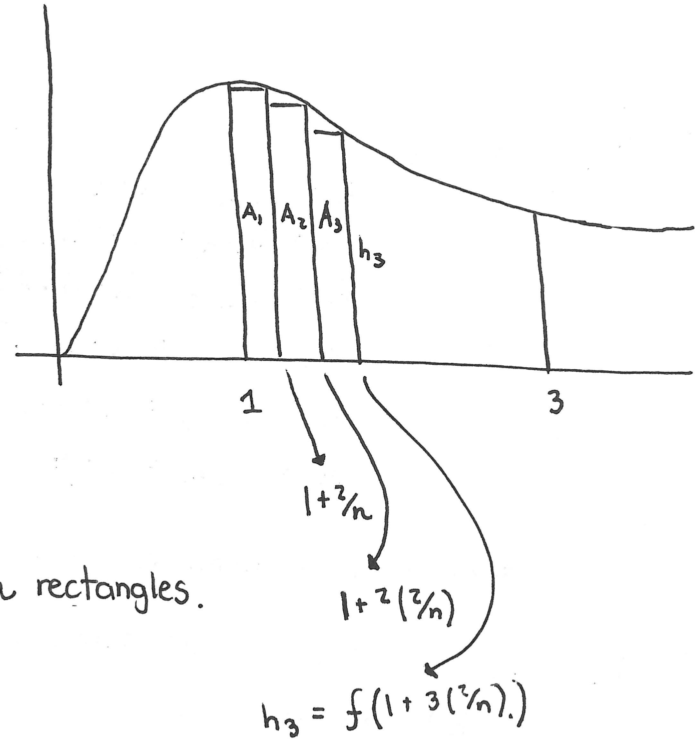


b) $$L_4 = \sum_{i=1}^{4} A_i = \sum_{i=1}^{4} b_i h_i \quad k=1$$

$$= f(0) + f(1) + f(2) + f(3)$$

$$= 0 + 1 + \sqrt{2} + \sqrt{3} \approx 4.146.$$ underestimation

19. $f(x) = \dfrac{2x}{x^2+1}$ $\quad 1 \le x \le 3$



Find a formula for the right endpoint Riemann sum with n rectangles.

What is the width (base) of each rectangle? The interval is of length 2, divided into n rectangles.

So $b_i = 2/n$

What are our sampling points?

$$h_i = f(1 + i \cdot 2/n)$$

Then sum over all areas...

$$R_n = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \underbrace{(2/n)}_{b_i}\Big(\underbrace{f(1 + i\,2/n)}_{h_i}\Big)$$

$$= \frac{2}{n} \sum_{i=1}^{n} \frac{2(1 + 2i/n)}{(1 + 2i/n)^2 + 1}.$$

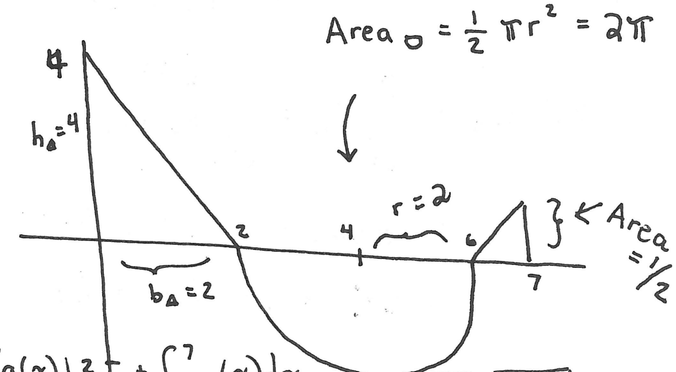
So that

$$\int_1^3 \frac{2x}{x^2+1} = \lim_{n \to \infty} \frac{4}{n} \sum_{i=1}^{n} \frac{1 + \frac{2i}{n}}{(1 + \frac{2i}{n})^2 + 1}.$$ wow.

Area$_{\smile} = \frac{1}{2}\pi r^2 = 2\pi$

34. a) $\int_0^2 g(x)\,dx = 4$

Area$_\Delta = \frac{1}{2}\underbrace{(2)}_{b_\Delta}\underbrace{(4)}_{h_\Delta} = 4$



b) $\int_2^6 g(x)\,dx = -2\pi$

c) $\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx$ $\boxed{= 4\tfrac{1}{2} - 2\pi}$