

Daily HW #14

(4.2 #48. 4.3 #32. 4.4 #14, 24, 26, 35)

48. If $\int_1^5 f(x) dx = 12$ and $\int_4^5 f(x) dx = 3.6$, find $\int_1^4 f(x) dx$.

$$\int_1^5 f(x) dx = \int_1^4 f(x) dx + \int_4^5 f(x) dx$$

$$12 = \int_1^4 f(x) dx + 3.6$$

$$\Rightarrow \boxed{\int_1^4 f(x) dx = 8.4}$$

32. $\int_0^{\pi/4} \sec \theta \tan \theta d\theta$ $\frac{d}{d\theta} (\sec \theta) = \sec \theta \tan \theta$

$$\begin{aligned} \Rightarrow \sec \theta \Big|_0^{\pi/4} &= \sec(\pi/4) - \sec(0) \\ &= \frac{1}{\cos(\pi/4)} - \frac{1}{\cos(0)} = \frac{1}{\sqrt{2}/2} - \frac{1}{1} \\ &= \sqrt{2} - 1 \end{aligned}$$

$$\boxed{\int_0^{\pi/4} \sec \theta \tan \theta d\theta = \sqrt{2} - 1}$$

14. $\int \sec t (\sec t + \tan t) dt$

$$= \int \sec^2 t + \sec t \tan t dt$$

$$= \boxed{\tan(t) + \sec(t) + C}$$

$$\frac{d}{dt} \tan t = \sec^2 t$$

$$\frac{d}{dt} \sec t = \sec t \tan t$$

$$\begin{aligned}
24. \quad & \int_{-1}^1 t(1-t)^2 dt \\
&= \int_{-1}^1 t(1-2t+t^2) dt \\
&= \int_{-1}^1 t - 2t^2 + t^3 dt \\
&= \left. \frac{1}{2} t^2 - \frac{2}{3} t^3 + \frac{1}{4} t^4 \right|_{-1}^1 \\
&= \frac{1}{2} (1)^2 - \frac{2}{3} (1) + \frac{1}{4} (1) - \left[\frac{1}{2} (-1)^2 - \frac{2}{3} (-1)^3 + \frac{1}{4} (-1)^4 \right] \\
&= \frac{1}{2} - \frac{2}{3} + \frac{1}{4} - \left[\frac{1}{2} + \frac{2}{3} + \frac{1}{4} \right] \\
&= -\frac{2}{3} - \frac{2}{3} = -\frac{4}{3} \quad \Rightarrow \boxed{\int_{-1}^1 t(1-t)^2 dt = -\frac{4}{3}}
\end{aligned}$$

$$\begin{aligned}
26. \quad & \int_1^2 x^{-2} - 4x^{-3} dx \\
&= -x^{-1} + 2x^{-2} \Big|_1^2 \\
&= -\frac{1}{x} + \frac{2}{x^2} \Big|_1^2 \\
&= \underbrace{-\frac{1}{2} + \frac{2}{4}}_0 - \underbrace{\left(-\frac{1}{1} + \frac{2}{1}\right)}_1 \\
&= -1
\end{aligned}$$

$$\boxed{\int_1^2 \frac{1}{x^2} - \frac{4}{x^3} dx = -1}$$

$$\begin{aligned}
35. \quad & \int_1^{64} \frac{1 + \sqrt[3]{x}}{\sqrt{x}} dx \\
&= \int_1^{64} x^{-1/2} + \frac{x^{1/3-1/2}}{x^{-1/6}} dx \\
&= \left. 2x^{1/2} + \frac{6}{5} x^{5/6} \right|_1^{64} \\
&\quad \rightarrow 2(\sqrt{64}) + \frac{6}{5} (64^{5/6}) - 2 - \frac{6}{5} \\
&= 16 + \frac{192}{5} - 2 - \frac{6}{5} \\
&= 14 + \frac{186}{5} = \boxed{\frac{256}{5}}
\end{aligned}$$

