Solutions Daily HW #2 $\boxed{3.1}$ # 29, 34, $\boxed{3.3}$ #10, 20, 27

29. Find all critical numbers of $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$.

$f'(x) = \frac{1}{3} - x$.

$f'(x)$ is defined everywhere.
$f'(x) = 0$ when $x = 1/3$

$\boxed{\text{critical numbers: } 1/3}$

34. $g(t) = |3t - 4|$.

stop. compare to $f(t) = |t|$

$f'(t)$ is undefined when $t = 0$.

and $f'(t) = \begin{cases} 1 & \text{if } t > 0 \\ -1 & \text{if } t < 0 \end{cases}$ $\Rightarrow$ critical number of f: 0

now. $g(t) = |3t - 4|$ notice $3t - 4 = 0 \Rightarrow t = 4/3$

$$= \begin{cases} 3t - 4 & t \geq 4/3 \\ -(3t-4) & t \leq 4/3 \end{cases}$$

implies

$$g'(t) = \begin{cases} 3 & t > 4/3 \\ \text{undef} & t = 4/3 \\ -3 & t < 4/3. \end{cases}$$

when is $\underbrace{g'(t) = 0}_{\text{never.}}$? when is $\underbrace{g'(t) \text{ undefined}}_{\text{when } t = 4/3}$?

$\boxed{\text{critical numbers: } t = 4/3}$

Daily HW #2

10. $f(x) = 4x^3 + 3x^2 - 6x + 1$. answer local max/min
concavity questions.

① What are the critical numbers of f?

$f'(x) = 12x^2 + 6x - 6$.

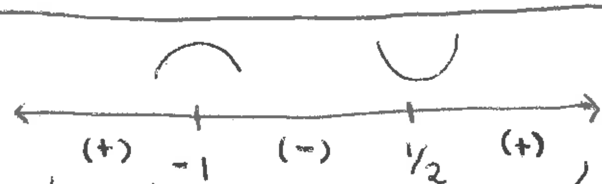
it's defined everywhere, so when is $f'(x) = 0$?

$f'(x) = 12x^2 + 6x - 6 = 0$

$2x^2 + x - 1 = 0$ $\qquad$ $2x - 1 = 0$ $\qquad$ $x + 1 = 0$

$(2x - 1)(x + 1) = 0 \Rightarrow$ $\qquad$ $x = 1/2$ $\qquad$ $x = -1$

$\boxed{\text{critical numbers of } f: -1, 1/2}$



the sign (+ or −) of f' will be the same throughout each interval b/c f' is continuous, so to pass from (+) to (−), it would have to go through 0, and we marked all places where $f'(x) = 0$.

$f'(x) > 0$ $\qquad$ $f'(x) < 0$ $\qquad$ $f'(x) > 0$

take any pt less than −1, say, −2, and see if $f'(-2)$ is positive or negative.

$f'(-2) = 12(-2)^2 + 6(-2) - 6$
$= 48 - 12 - 6 > 0$

take pt 0, it's easy.

$f'(0) = -6 < 0$

$f'(1) = 12 > 0$

a) intervals on which f is increasing: $(-\infty, -1], [1/2, \infty)$
f is decreasing: $[-1, 1/2]$.

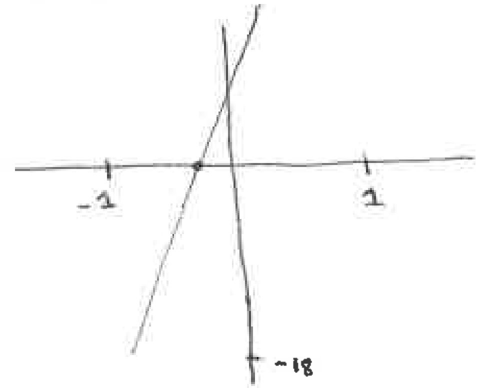
b) local max: $x = -1$ pt $(-1, 6)$
local min: $x = 1/2$ pt $(1/2, -3/4)$

For concavity, we need f''.

$$f''(x) = 24x + 6$$

when is $f''(x) = 0$? when $24x + 6 = 0 \Rightarrow 4x + 1 = 0$

when $x = -1/4$



if $x > -1/4$, $f''(x) > 0$

$x < -1/4$, $f''(x) < 0$

c) f is concave up on the interval $(-1/4, \infty)$

f is concave down on the interval $(-\infty, -1/4)$.

The point $x = -1/4$, pt $(-1/4, 21/8)$, is the inflection point of f.

20. Find local max and local min using First and Second derivative tests. $f(x) = \dfrac{x^2}{x-1}$

what are the critical points? $f'(x) = \dfrac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \dfrac{x^2 - 2x}{(x-1)^2}$

- $f'(x)$ is undefined when $x = 1$ (dividing by 0 is bad).
- $f'(x) = 0$ when $x = 0, 2$

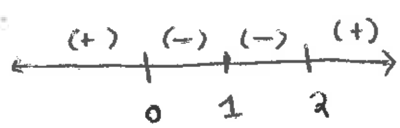
$\boxed{\text{critical pts: } 0, 1, 2}$

First Derivative Test

when is $f'(x) = \dfrac{x^2 - 2x}{(x-1)^2} > 0$. Denominator is always positive. So $x(x-2) > 0$ when $x > 2$ or $x < 0$.

$f'(x) > 0$ on intervals $(-\infty, 0)$ and $(2, \infty)$.

$f'(x) < 0$ on interval $(0, 2)$.



local maximum at $x = 0$

local minimum at $x = 2$.

Second derivative test

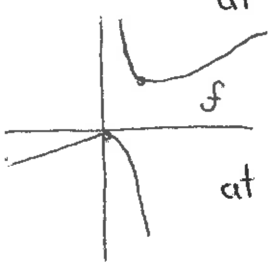
$$f'(x) = \frac{x^2 - 2x}{(x-1)^2}$$

$$f''(x) = \frac{(x-1)^2(2)(x-1) - (x^2 - 2x)(2)(x-1)}{(x-1)^4}$$

$$= \frac{2x^2 - 4x + 2 - (2x^2 - 4x)}{(x-1)^3} = \frac{2}{(x-1)^3}.$$

So. at the point $x = 0$, we have $f'(0) = 0$ and $f''(0) = \frac{2}{(-1)^3} < 0$

therefore $x = 0$, pt $(0, 0)$, is a local max.



at the point $x = 2$, we have $f'(2)$ and $f''(2) = \frac{2}{(1)^3} > 0$.

therefore, $x = 2$, pt $(2, 4)$ is a local min.

27. $f'(x) > 0$ if $|x| < 2$, $f'(x) < 0$ if $|x| > 2$.

$f'(-2) = 0$ $\quad \lim_{x \to 2} |f'(x)| = \infty$, $\quad f''(x) > 0$ if $x \neq 2$.