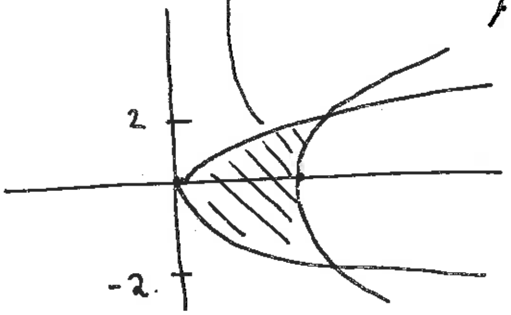
# Daily Homework #21.

5.1 #17, 5.2 # 2, 3, 5, 8, 49 (r = 4, h = 3).

17. $x = 2y^2 \quad x = 4 + y^2$




where do the curves intersect?

$2y^2 = 4 + y^2$

$y^2 = 4 \Rightarrow y = \pm 2.$

$$\boxed{\int_{-2}^{2} (4 + y^2) - 2y^2\, dy} = 2\int_0^2 4 - y^2\, dy$$

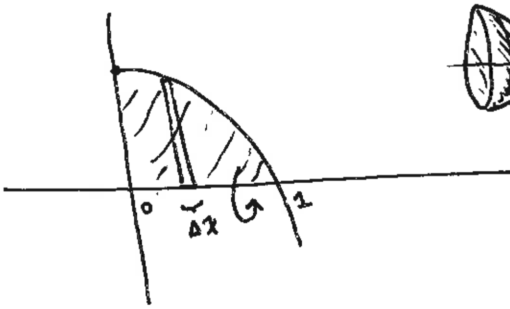
(or observing symmetry)

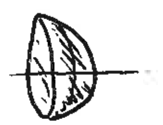
$$= 2\left(4y - \frac{1}{3}y^3\Big|_0^2\right)$$

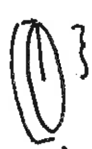
$$= 2\left(8 - \frac{8}{3}\right) = \boxed{\frac{32}{3}}$$

2. $y = 1 - x^2,\ y = 0$ about $x$-axis.

Volume disk: $\pi r^2 \Delta x$





$r = f(x) = (1 - x^2)$

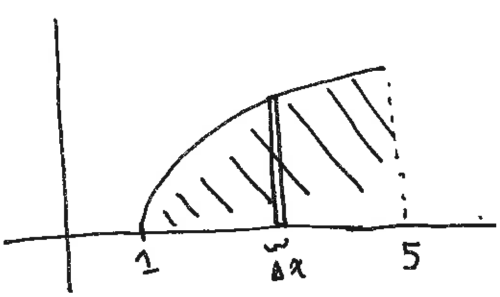
$\underbrace{}_{\Delta x}$

$$\boxed{\int_0^1 \pi (1 - x^2)^2\, dx}$$

expand out polynomial and integrate

(No u-sub)

3. $y = \sqrt{x - 1},\ y = 0,\ x = 5$ about $x$-axis.



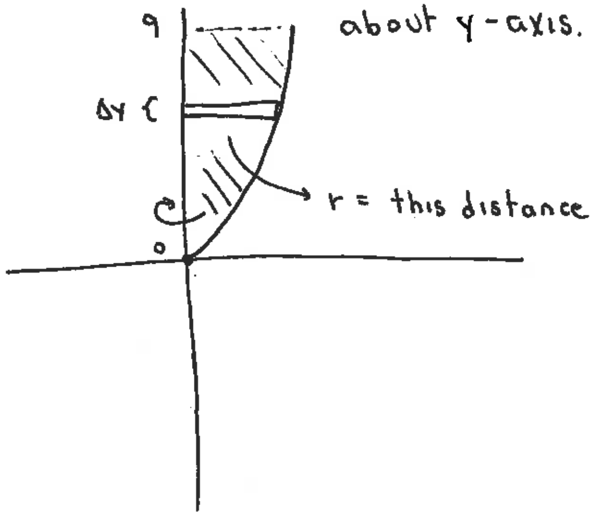

$r = f(x) = \sqrt{x-1}$

$\underbrace{}_{\Delta x}$

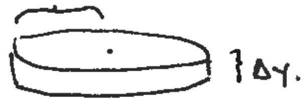
Volume disk: $\pi r^2 \Delta x$

$$\boxed{\int_1^5 \pi \left(\sqrt{x-1}\right)^2 dx}$$

5. $x = 2\sqrt{y}$, $x = 0$, $y = 9$ about $y$-axis.




radius $= f(y) = 2\sqrt{y}$

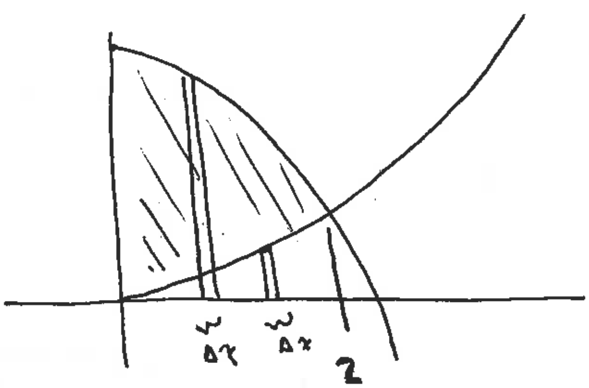


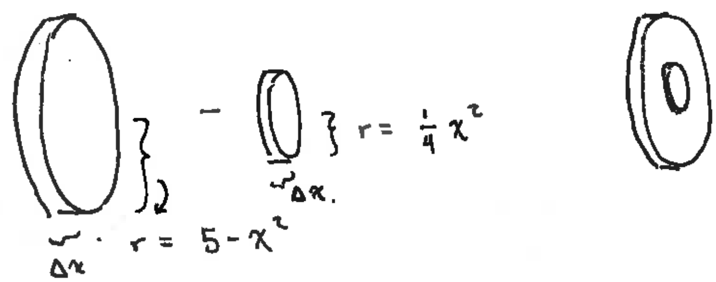

* must give radius in terms of $y$ b/c ⇒ integrate in terms of $y$.

Volume disk : $\pi r^2 \Delta y$

$$\boxed{\int_0^9 \pi \left(2\sqrt{y}\right)^2 dy}$$

8. $y = \frac{1}{4}x^2$, $y = 5 - x^2$ about $x$-axis.




Volume of a disk: $\pi r^2 \Delta x$.

where do the curves intersect?

$\frac{1}{4}x^2 = 5 - x^2$

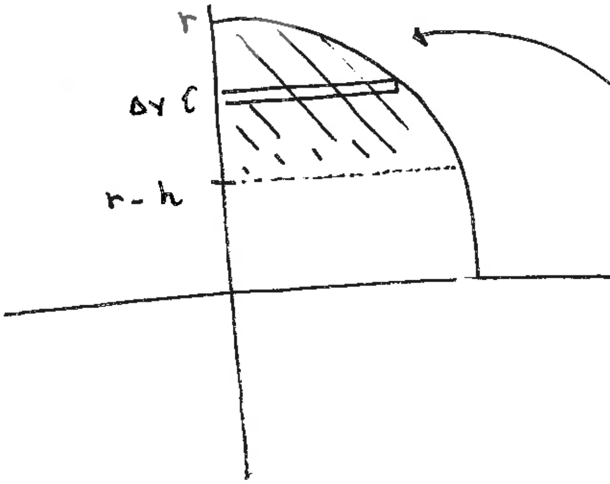
$\frac{5}{4}x^2 = 5$

$x^2 = 4$

$\Rightarrow x = \pm 2.$

$$\boxed{\begin{aligned} &\int_0^2 \pi(5-x^2)^2\,dx - \int_0^2 \pi\left(\tfrac{1}{4}x^2\right)^2 dx \\ &= \int_0^2 \pi\left((5-x^2)^2 - \left(\tfrac{1}{4}x^2\right)^2\right) dx \end{aligned}}$$

49. A cap of a sphere with radius r and height h.




The sphere cap can be obtained by spinning a portion of a circle around the y-axis.

$x^2 + y^2 = r^2$ ← sphere radius r.

$\boxed{x = \sqrt{r^2 - y^2}}$

radius = f(y)

$\Delta y$ Volume disk: $\pi r^2 \Delta y$.

$$\int_{r-h}^{r} \pi \left(\sqrt{r^2 - y^2}\right)^2 dy$$

$$= \int_{r-h}^{r} \pi (r^2 - y^2)\, dy = \pi \left( r^2 y - \frac{1}{3} y^3 \Big|_{r-h}^{r} \right)$$

(lots of simplifying).

$$= \pi h^2 \left(r - \frac{1}{3} h\right)$$

In the case: $r = 4, \ h = 3.$

$$\int_{1}^{4} \pi (16 - y^2)\, dy = \pi \left( 16y - \frac{1}{3} y^3 \Big|_{1}^{4} \right)$$

$$= \pi \left(16(3) - \frac{1}{3}(4^3 - 1)\right) = 27\pi$$