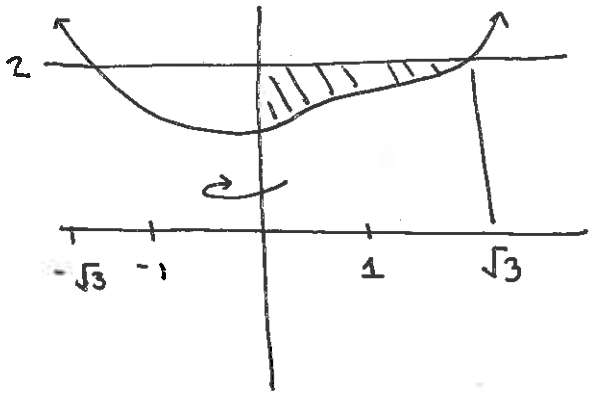
5.3 # 40 5.2 # 57 - 59.

40. $y^2 - x^2 = 1$

$y = 2$

around y-axis



where do the curves intersect?

$y = \sqrt{1+x^2}$

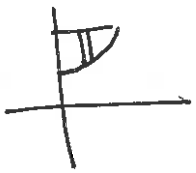
$y = 2.$

$4 - 1 = x^2 \Rightarrow \boxed{x = \pm\sqrt{3}}$

Shells

Volume shell $= 2\pi r h \Delta x$

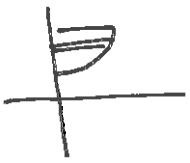
$r = x \quad h = 2 - \sqrt{1+x^2}$



$$\int_0^{\sqrt{3}} 2\pi x\left(2 - \sqrt{1+x^2}\right) dx$$

washers

Volume washer $= \pi r^2 \Delta y$



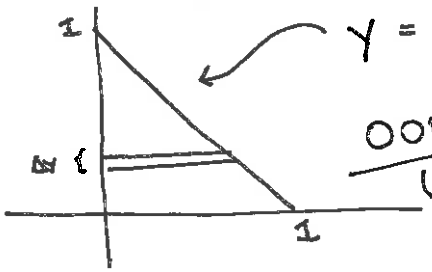
$y^2 - x^2 = 1 \Rightarrow x^2 = y^2 - 1$

$x = \sqrt{y^2 - 1}.$

$\Rightarrow r = \sqrt{y^2 - 1}$

$$\int_1^2 \pi \left(\sqrt{y^2-1}\right)^2 dy$$

57.



$y = 1 - x.$

OOPS (slices $\perp$ to y-axis)

Volume slice $= b^2 \Delta y$

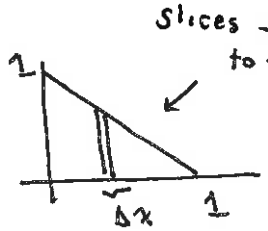
$\boxed{b = 1 - y}$

$y = 1 - x$

$x = 1 - y$

$$\int_0^1 (1-y)^2 dy$$

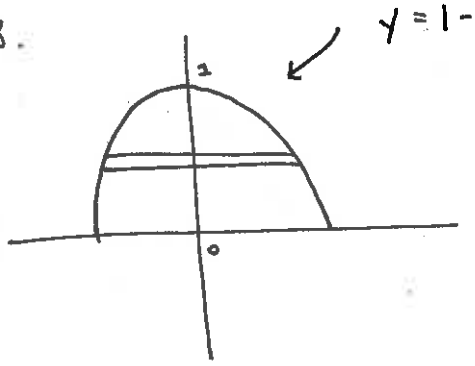
does it make sense why answer will be same?



Volume slice $= b^2 \Delta x \quad b = 1 - x$

$$\int_0^1 (1-x)^2 dx$$

58.



cross-sections are squares.

Volume slice $= b^2 \Delta y$

$y = 1 - x^2$

$\Rightarrow x^2 = 1 - y$

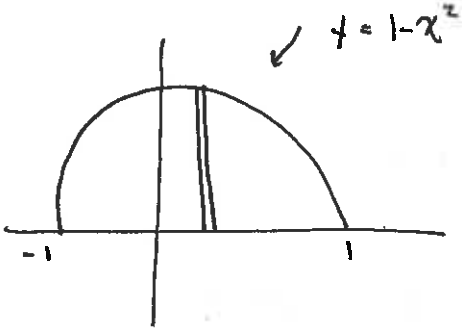
$\Rightarrow x = \pm\sqrt{1-y}$

$\Rightarrow b = 2\sqrt{1-y}$

distance y-axis to curve.

$$\int_0^1 \left(2\sqrt{1-y}\right)^2 dy$$

59.



cross-sections are isoceles triangles with $b = h$

Volume slice $= \frac{1}{2} bh \Delta x = \frac{1}{2} b^2 \Delta x$

$b = 1 - x^2$

$$\int_{-1}^{1} \frac{1}{2}\left(1 - x^2\right)^2 dx$$