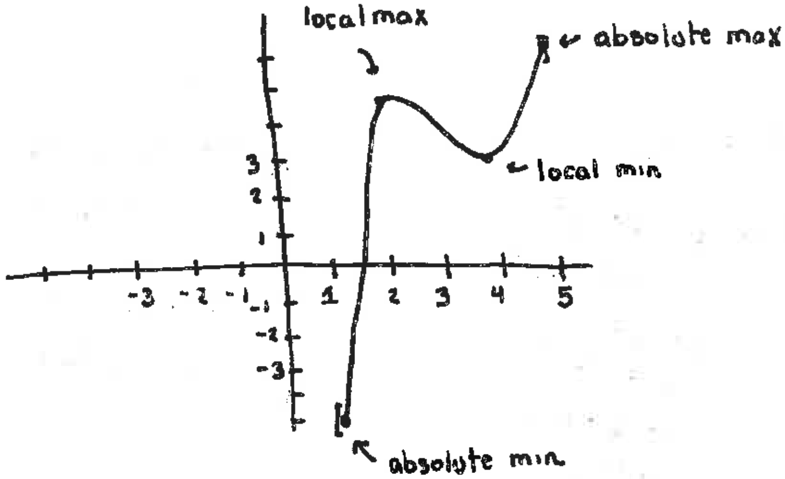
3.1 # 8, 45   3.3 # 11, 12.

8.



45. Find absolute max/min of f on interval. $f(x) = 12 + 4x - x^2$ on $[0,5]$.

critical numbers: $\boxed{x = 2}$

when is $f'(x) = 0$? $f'(x) = 4 - 2x$

(f' is defined everywhere) $4 - 2x = 0$

$x = 2$

The critical numbers and the endpoints are the potential locations of an absolute max/min. Potential abs max/min: 0, 2, 5.

$f(0) = 12$

$f(2) = 12 + 4(2) - (2)^2 = 16$

$f(5) = 12 + 4(5) - (5)^2 = 7$

Therefore, (2,16) is the absolute max and (5,7) the absolute min.

11. $f(x) = x^4 - 2x^2 + 3$

a) intervals incr./decr.
b) local max/min.
c) intervals concav. + inflection pts.

$f'(x) = 4x^3 - 4x$

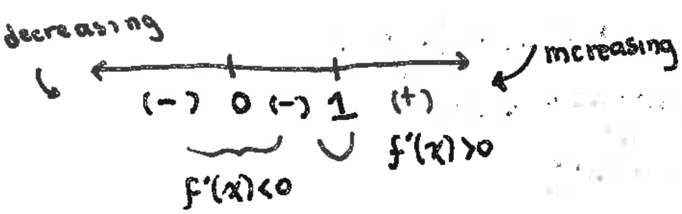
when is $f'(x) = 0$? (it's defined everywhere)

$4x^3 - 4x = 0$

$x^3 - x = 0$

$x^2(x - 1) = 0$

Critical numbers: $x = 0, 1$



$f''(x) = 12x^2 - 4$

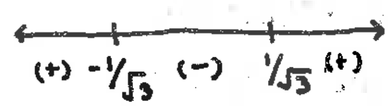
when is $f''(x) = 0$? $12x^2 - 4 = 0$

$3x^2 - 1 = 0$

$x^2 = 1/3$

$x = \pm 1/\sqrt{3}$

CONCAVITY



a) f is increasing on $[1, \infty)$
decreasing on $(-\infty, 1]$.

b) (1, 2) is a local min
(0, 3) is neither a local max nor l. min

c) concave up on $(-\infty, -1/\sqrt{3})$ and $(1/\sqrt{3}, \infty)$

concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$.

12. $f(x) = \dfrac{x}{x^2+1}$

critical numbers: $\boxed{x = 1, -1}$

$$f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

when is $f'(x) = 0$?

$$\frac{1-x^2}{(x^2+1)^2} = 0$$
$$1 - x^2 = 0$$
$$x^2 = 1$$
$$\boxed{x = \pm 1}$$

when is f undefined?
do we ever divide by 0?

$$(x^2+1)^2 = 0$$
$$x^2 + 1 = 0 \quad \underline{\text{NO SOLUTIONS}}$$

$(-)$	-1	$(+)$	1	$(-)$
$f'(x)<0$ (plug in $x=-2$ to f')		$f'(x)>0$ (plug in $x=0$ to f')		$f'(x)<0$ (plug in $x=2$ to f')

$\boxed{(-1, -1/2) \text{ is a local min. } (1, 1/2) \text{ is a local max}}$

f is increasing on $[-1, 1]$
f is decreasing on $(-\infty, -1]$ and $[1, \infty)$.

concavity:
$$f''(x) = \frac{(x^2+1)^2(-2x) - (1-x^2)(2(x^2+1)2x)}{(x^2+1)^4}$$
$$= \frac{(x^2+1)[(x^2+1)(-2x) - 4x(1-x^2)]}{(x^2+1)^4}$$
$$= \frac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3} = \frac{2x^3-6x}{(x^2+1)^3}.$$

when is $f''(x) = 0$?

$$\frac{2x^3-6x}{(x^2+1)^3} = 0$$
$$2x^3 - 6x = 0$$
$$x^3 - 3x = 0$$
$$x(x^2-3) = 0$$
$$x = 0, \pm\sqrt{3}$$

CONCAVITY

(*)

$(-)$	$-\sqrt{3}$	$(+)$	0	$(-)$	$\sqrt{3}$	$(+)$

$f''(-2) = \dfrac{-16+12}{5^3} < 0$

$f''(-1) = \dfrac{2(-1)^3 - 6(-1)}{2^3} > 0$
$= \dfrac{-2+6}{2^3}$

$f''(1) = \dfrac{2-6}{2^3} < 0$

$f''(2) = \dfrac{2(2)^3 - 6(2)}{5^3} = \dfrac{16-12}{5^3} > 0$

(*) Concave up on $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$
concave down on $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$