

QUIZ #1 — WEDNESDAY, JANUARY 13

Let $h(x) = x^4 - 4x^3$. Answer the following questions and show your work.

- Find the interval(s) where h is increasing, and find the interval(s) where h is decreasing.
- Find the critical numbers of h , and classify each one as a local minimum, local maximum, or neither.
- Find the interval(s) where h is concave upward, and find the interval(s) where h is concave downward.
- Find the number(s) where h has an inflection point.

critical points:

$$h'(x) = 4x^3 - 12x^2$$

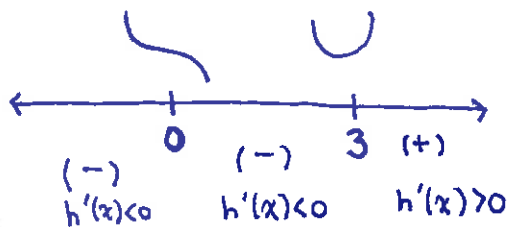
defined everywhere. when is $h'(x) = 0$?

$$4x^3 - 12x^2 = 0$$

$$x^3 - 3x^2 = 0$$

$$x^2(x - 3) = 0$$

critical points $x = 0, 3$



$$h'(-1) = 4(-1)^3 - 12(-1)^2 = -4 - 12 < 0$$

$$h'(1) = 4(1)^3 - 12(1)^2 = -8 < 0$$

$$h'(4) = 4(4)^3 - 12(4)^2 = (16 - 12)(4)^2 > 0$$

$(x, h(x)) = (3, -27)$ is a local min.

$(0, 0)$ is neither a local max nor a local min.

h is increasing on the interval $[3, \infty)$
and decreasing on the interval $(-\infty, 3]$

concavity: h is concave up when $h''(x) > 0$. concave down when $h''(x) < 0$.

$$h''(x) = 12x^2 - 24x$$

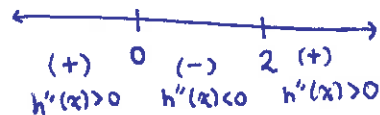
when is $h''(x) = 0$?

$$12x^2 - 24x = 0$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

CONCAVITY



$$h''(-1) = 12(-1)^2 - 24(-1) = 12 + 24 > 0$$

$$h''(1) = 12 - 24 < 0$$

$$h''(3) = 12(3)^2 - 24(3) = 12(9 - 6) > 0$$

concave up on intervals $(-\infty, 0)$ and $(2, \infty)$
concave down on $(0, 2)$.

inflection points
 $(x, h(x)) = (0, 0)$
 $(x, h(x)) = (2, -16)$

