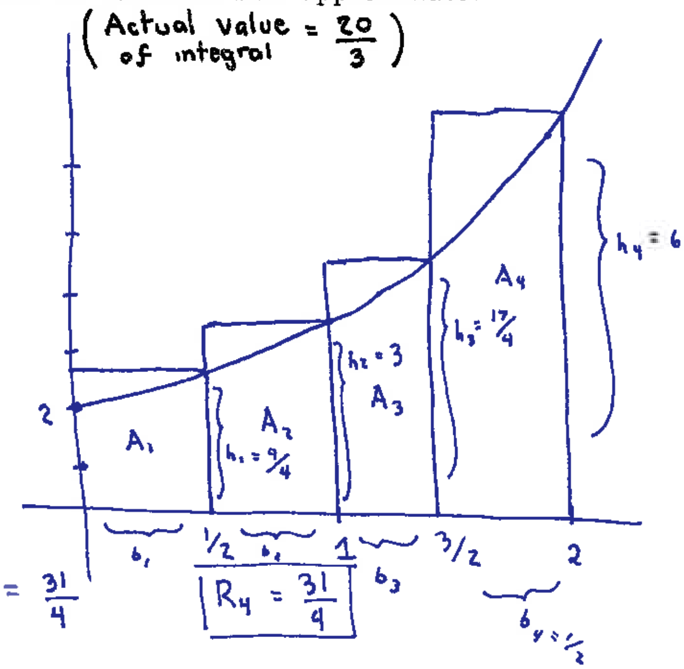
Solutions

Math 2, Winter 2016

Quiz #4 — Wednesday, February 10

1. Evaluate the Riemann sum for $f(x) = x^2 + 2$ between 0 and 2 using four subintervals, taking the sample points to be the right endpoints of the subintervals. Draw a diagram with the curve $f(x)$ and the four rectangles. What value does the Riemann sum approximate?



The Riemann sum is $A_1 + A_2 + A_3 + A_4$.

We have $b_1 = 1/2$, $b_2 = 1/2$, $b_3 = 1/2$, $b_4 = 1/2$.

We have $h_1 = f(1/2) = 9/4$ $\quad h_2 = f(1) = 3$

$h_3 = f(3/2) = 17/4$. $\quad h_4 = f(2) = 6$.

Therefore

$$R_4 = \sum_{i=1}^{4} A_i = \sum_{i=1}^{4} b_i h_i$$

$$= \frac{1}{2}(9/4) + 1/2\,(3) + 1/2\,(17/4) + 1/2\,(6)$$

$$= \frac{1}{2}\left(\frac{9}{4} + \frac{17}{4} + 3 + 6\right) = \frac{1}{2}\left(\frac{13}{2} + 9\right) = \frac{31}{4}$$

$$\boxed{R_4 = \frac{31}{4}}$$

The Riemann sum approximates $\int_0^2 x^2 + 2\,dx$, the area under the curve $f(x) = x^2 + 2$ between $x = 0$ and $x = 2$.

2. Find the average value of $f(x) = \sin(x) + 2x$ over the interval $[0, 2\pi]$.

Recall $\quad \text{Avg} = \frac{1}{b-a}\int_a^b f(x)\,dx.$

$$\text{Avg} = \frac{1}{2\pi}\int_0^{2\pi} \sin(x) + 2x\,dx = \frac{1}{2\pi}\left(-\cos(x) + x^2 \Big|_0^{2\pi}\right.$$

$$= \frac{1}{2\pi}\left(-\underbrace{\cos(2\pi)}_{1} + (2\pi)^2 + \underbrace{\cos(0)}_{1} - 0^2\right)$$

$$= \frac{1}{2\pi}\left(4\pi^2\right) = 2\pi.$$

The average value over interval $[0, 2\pi]$ is 2π.

$\boxed{\text{Why?}}$ $\sin(x)$ oscillates over this interval, on average contributing 0 to the ftn. $g(x) = 2x$ is a line, so it's avg value will happen at the midpt, which is π. so that $g(\pi) = 2\pi$.

3. Find $\frac{d}{dx}\int_4^x \tan(t) + 3t^4 + 15dt$.

By FTC, $\quad \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$

Therefore, $\quad \frac{d}{dx}\int_4^x \tan(t) + 3t^4 + 15\,dt = \tan(x) + 3x^4 + 15.$