Key

Math 2, Winter 2016

QUIZ #5 — WEDNESDAY, FEBRUARY 24

In both of these problems, you must solve for y in your solution.

1. What is the family of solutions to:

$$\frac{dy}{dx} = 3y$$

we can either solve this using the separable diff eq technique or by recognition.

Day 1 observation:
$y = f(x)$ $y = e^x \Rightarrow \frac{dy}{dx} = e^x = y$.

Now $y = e^{3x} \Rightarrow \frac{dy}{dx} = e^{3x} \cdot 3 = 3y$.

$y = e^{3x}$ is one solution.

$\boxed{y = Ae^{3x}}$ is the family of solutions.

separable eqn:

$$\int \frac{1}{y}\,dy = \int 3\,dx$$

$$\ln|y| = 3x + C$$

$$|y| = e^{3x+C} = e^{3x}e^C$$

$\Rightarrow \boxed{y = Ae^{3x}}$

e^C a constant, always positive

check! $\frac{dy}{dx} = A \cdot e^{3x} \cdot 3 = 3y$

a constant, can be positive or negative. $A = e^C$ or $A = -$

2. Find the solution to $\frac{dy}{dx} = y(x^3 + 2)$ through the point $(x, y) = (0, 6)$.

separable differential equation.

$$\int \frac{1}{y}\,dy = \int x^3 + 2\,dx$$

$$\ln|y| = \frac{1}{4}x^4 + 2x + C$$

$$|y| = e^{\frac{1}{4}x^4 + 2x + C} = e^{\frac{1}{4}x^4 + 2x}e^C$$

e^C a constant, always positive.

want the soln through plug in and solve for A.

$6 = Ae^{\frac{1}{4}0^4 + 2\cdot 0} = Ae^0 = A$

$\Rightarrow \boxed{A = 6}$

$\boxed{y = 6e^{\frac{1}{4}x^2 + 2x}}$ is soln through $(0,6)$

$y = e^C e^{\frac{1}{4}x^4 + 2x}$

or

$y = -e^C e^{\frac{1}{4}x^4 + 2x}$

let $A = e^C$ or $-e^C$, all of which are just constants!

$\boxed{y = Ae^{\frac{1}{4}x^4 + 2x}}$

a constant, can be negative or positive.