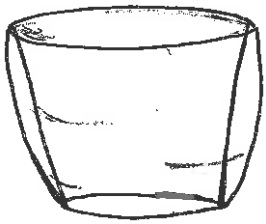
SOLUTIONS

Math 2, Winter 2016

QUIZ #6 — WEDNESDAY, MARCH 2



Consider the curves $y = \sqrt{x} + 1$ and $y = x + 1$.

(a) Find the point(s) of intersection of the two curves.

pts of intersection:

$\sqrt{x} + 1 = x + 1$

$\Rightarrow \sqrt{x} = x$

$x = 0, 1$

$(0,1)$ and $(1,2)$

the object is like a cylindrical tube that is a bit thicker in the middle, curved on the outside.

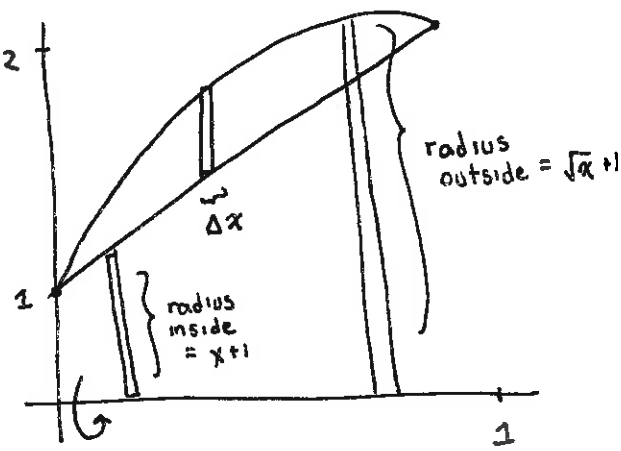
(b) Find the volume of the solid obtained by taking the region enclosed by the two curves and revolving it around the x-axis.

WASHER: The slices are disks. Volume of disk: $\pi r^2 \Delta x$

Volume solid = Volume entire (outside) − Volume hollowed out inside.

radius = $\sqrt{x}+1$ radius = $x+1$

$$\int_0^1 \pi (\sqrt{x}+1)^2 \, dx - \int_0^1 \pi (x+1)^2 \, dx$$



$$\pi \int_0^1 x + 2\sqrt{x} + 1 - \pi \int_0^1 x^2 + 2x + 1 \, dx$$

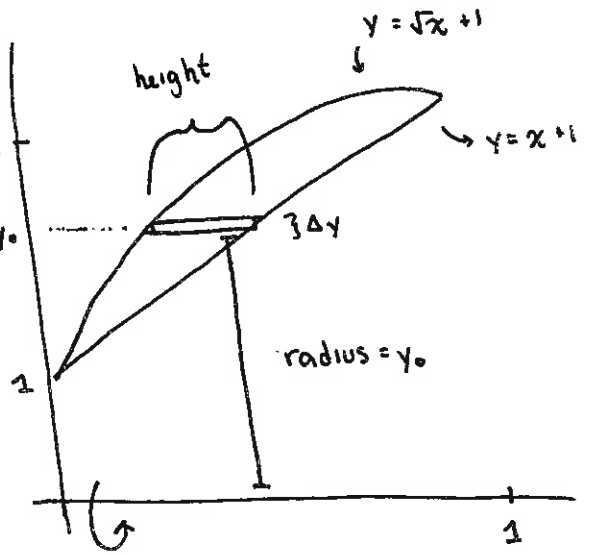
$$\pi \left(\tfrac{1}{2}x^2 + \tfrac{4}{3}x^{3/2} + x \Big|_0^1 \right) - \pi \left(\tfrac{1}{3}x^3 + x^2 + x \Big|_0^1 \right) = \pi \left(\tfrac{1}{2} + \tfrac{4}{3} + 1 - \tfrac{1}{3} - 1 - 1 \right) = \boxed{\pi/2}$$

SHELLS: The slices are cylindrical shells. Volume of shell: $2\pi r h \Delta y$

The thickness of these shells in this case is Δy. which means r and h must be in terms of y.

height = distance btwn curves (horizontal) → $\begin{cases} x = y - 1 & \leftarrow \text{TOP} \\ x = (y-1)^2 & \leftarrow \text{BOTTOM.} \end{cases}$

height: $y_0 - 1 - (y_0 - 1)^2$ radius: y_0.



$$\int_1^2 2\pi y \left(y - 1 - (y-1)^2 \right) dy = 2\pi \int_1^2 y(-y^2 + 3y - 2) \, dy$$

$$= 2\pi \int_1^2 -y^3 + 3y^2 - 2y \, dy = 2\pi \left(-\tfrac{1}{4}y^4 + y^3 - y^2 \Big|_1^2 \right) = 2\pi \left(-4 + 8 - 4 + \tfrac{1}{4} - 1 + 1 \right) = \boxed{\pi/2}$$