Weekly HW # 6

16. Find the derivative of $y = \int_0^{x^4} \cos^2\theta \, d\theta$ } $A(x^4)$.

$$\frac{d}{dx}\left(\int_0^{x^4} \cos^2\theta \, d\theta\right) = \cos^2(x^4)\cdot(4x^3). = \boxed{4x^3\cos^2(x^4)}$$

$$\frac{d}{dx}(A(x^4)) = A'(x^4)\cdot(4x^3).$$ chain rule.

54. If $f(x) = \int_0^{x}(1-t^2)\cos^2 t \, dt$, on what interval is f increasing?

f is increasing when $f' \geq 0$.

$$f'(x) = \frac{d}{dx}\int_0^{x}(1-t^2)\cos^2 t\, dt = (1-x^2)\cos^2 x.$$

when is $(1-x^2)\underbrace{\cos^2 x}_{\oplus} \geq 0$?

when is $1-x^2 \geq 0$?

when x is in $[-1,1]$.

its a square $(\cos(x))^2$, so $\cos^2(x) \geq 0$ always

f is increasing on the interval $[-1,1]$.

10. $f(x) = \sqrt{x}$ on $[0,4]$.

$$4^{3/2} = (4^{1/2})^3 = 2^3 = 8$$

a) Find the average value.

$$\text{Avg} = \frac{1}{4-0}\int_0^4 \sqrt{x}\, dx = \frac{1}{4}\left(\frac{2}{3}x^{3/2}\Big|_0^4\right) = \frac{1}{4}\left(\frac{2}{3}(4)^{3/2} - \frac{2}{3}0\right)$$

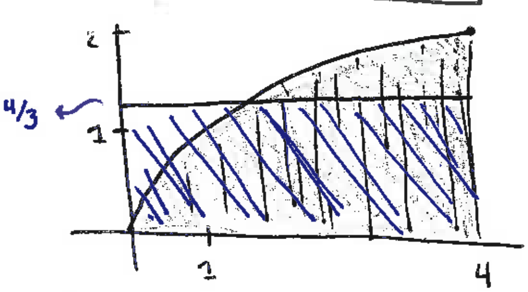
$$= \frac{1}{4}\left(\frac{2}{3}\cdot 8\right) = \frac{4}{3}$$

$$\boxed{\text{Avg value} = \frac{4}{3}}$$

b) Find c st $f(c) = f_{avg} = \frac{4}{3}$

$c = \frac{16}{9}$

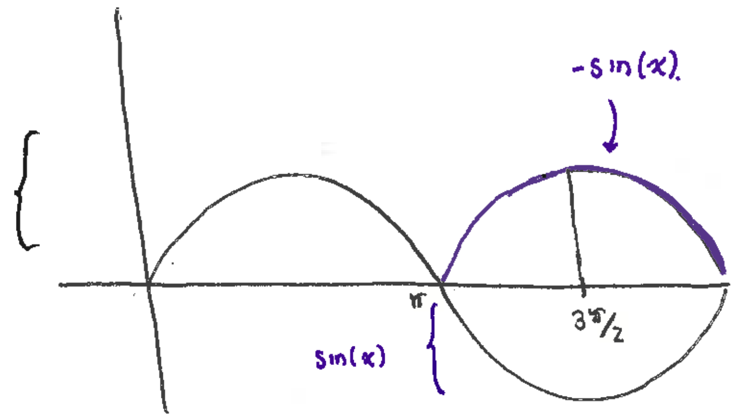
c)



$A = \frac{16}{3}$

$\int_0^4 \sqrt{x}\, dx = \frac{16}{3}$.

42. $\int_0^{3\pi/2} |\sin(x)|\,dx$



$= \int_0^{\pi} \sin(x)\,dx + \int_{\pi}^{3\pi/2} -\sin(x)\,dx$

$= -\cos(x)\Big|_0^{\pi} + \cos(x)\Big|_{\pi}^{3\pi/2}$

$= -(\overset{-1}{\cos(\pi)}) + \cos(0) + \cos(3\pi/2) - \overset{-1}{\cos(\pi)}$

$= 1 + 1 + 0 + 1 = \boxed{3}$

49. Oil leaks from a tank at a rate of $r(t)$ gal/min at time t, what does $\int_0^{120} r(t)\,dt$ represent?

The total amount of oil (in gallons) leaked from the tank between time 0 and time 120 minutes.

{ The amount of oil leaked in gallons in the first two hours. }

52. If $f(x)$ is the slope of the trail at a distance of x miles from the start of the trail, what does $\int_3^5 f(x)\,dx$ represent?

It represents the amount of elevation gain between mile 3 and mile 5.

56. $v(t) = t^2 - 2t - 8 \quad 1 \le t \le 6$

a) displacement:

$\int_1^6 t^2 - 2t - 8\,dt$

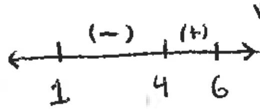
$= \frac{1}{3}t^3 - t^2 - 8t\Big|_1^6$

$= -\frac{10}{3}$ meters

b) distance (when is it going backwards?).

$0 = t^2 - 2t - 8 = (t-4)(t+2)$

$\Rightarrow t = 4, -2.$ means it's going backwards on interval $[1, 4]$.



distance $= -\int_1^4 t^2 - 2t - 8\,dt + \int_4^6 t^2 - 2t - 8\,dt$

$= -(-18) + (\frac{44}{3}) = \boxed{\frac{98}{3} \text{ meters.}}$