5.1 # 46, 5.2 # 40, 47. 5.3 # 17, 39.

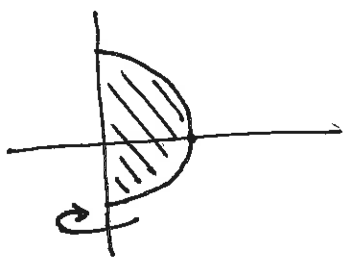
Hexagon pyramid.

46. The birth rate and death rate are given.

The area between the two curves between times $0 \le t \le 10$ represents the difference in population. In this case, the population has increased by 8800ish people in those 10 years.

40. $$\int_{-1}^{1} \pi (1-y^2)^2 \, dy.$$

we are using the washer method, $\pi r^2 \Delta y$, so spinning around y-axis.



since r is the value of our function in this case, perhaps we are spinning the region bounded by $x = 1-y^2$ around y-axis.

It looks a bit like a flattened ball.

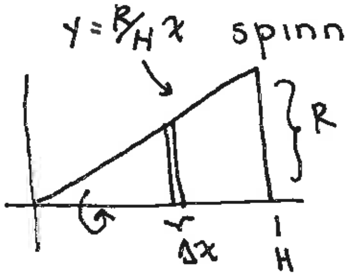
*alternatively $r = 1-y^2$ is area btwn $x=1$ and $x = y^2$, * and spin around $x = 1$.

47. A cone with height h and radius r.

We can find this region as a solid of revolution by spinning the region bounded by

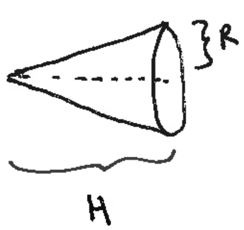
$$y = \frac{R}{H} x \quad \text{and} \quad x = H, \; y = 0$$

around x-axis.



(This does take some staring to figure out)

Volume slice = $\pi r^2 \Delta x$



$$\int_0^H \pi \left(\frac{R}{H}\right)^2 (x)^2 dx = \frac{\pi R^2}{H^2} \int_0^H x^2 \, dx$$

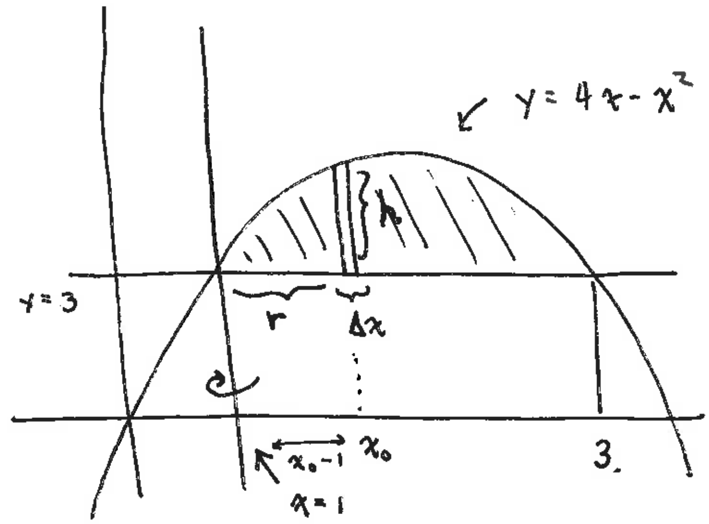
$$= \frac{\pi R^2}{H^2} \left(\frac{1}{3} x^3 \Big|_0^H \right)$$

$$= \frac{\pi R^2}{H^2} \left(\frac{1}{3} H^3 \right) = \boxed{\frac{1}{3} \pi R^2 H}$$

17. $y = 4x - x^2$

$y = 3$ use shells

and spin around $x = 1$



where do the curves intersect?

$$3 = 4x - x^2$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$\boxed{\Rightarrow x = 1, 3}$$

$$\boxed{\int_1^3 2\pi (x-1)(4x - x^2 - 3)\, dx} = \frac{8\pi}{3}.$$

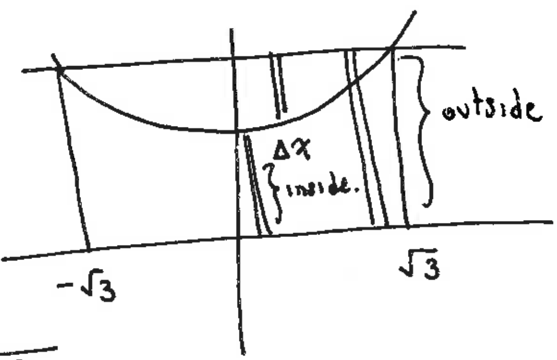
volume shell: $2\pi r h \Delta x$.

* height $= 4x_0 - x_0^2 \underline{\underline{-3}}$

$= f(x_0) - 3.$

$r = x_0 - 1$

39. $y^2 - x^2 = 1$, $y = 2$ about x-axis.



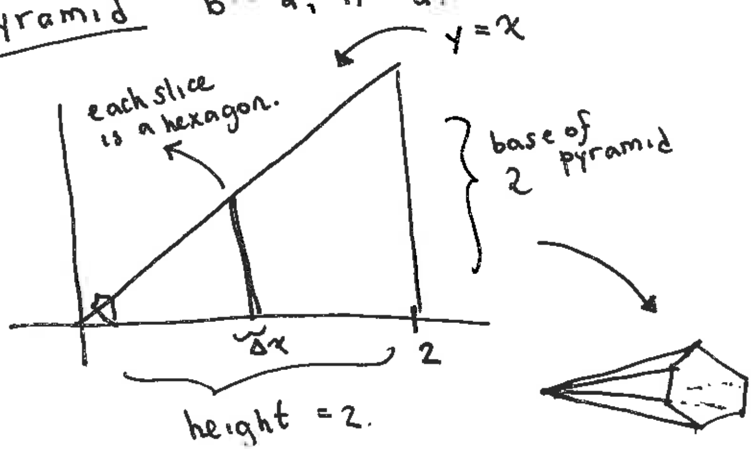
Volume slice $= \pi r^2 \Delta x$.

* $r \neq 2 - \sqrt{x^2+1}$

must do inside separately - don't try it all at once!

$$\boxed{\int_{-\sqrt{3}}^{\sqrt{3}} \pi (2)^2\, dx - \int_{-\sqrt{3}}^{\sqrt{3}} \pi \left(\sqrt{x^2+1}\right)^2 dx} = 4\pi\sqrt{3}$$

Pyramid $b = 2$, $h = 2$.



Each cross-sectional slice is a hexagon

Volume slice $= \frac{3\sqrt{3}}{2} x_0^2 \Delta x$.

$$\int_0^2 \frac{3\sqrt{3}}{2} x^2\, dx = \frac{3\sqrt{3}}{2}\left(\frac{1}{3}x^3\Big|_0^2\right)$$

$$= \frac{\sqrt{3}}{2}(8) = \boxed{4\sqrt{3}}$$