Weekly HW #2 Solutions

50. Find the absolute max and min of $f(x) = (x^2-1)^3$ on $[-1, 2]$.

$$f'(x) = 3(x^2-1)^2(2x)$$

when $f'(x)=0$? $3(x^2-1)^2(2x) = 0$

$$x(x^2-1)^2 = 0$$

$$x(x+1)^2(x-1)^2 = 0$$

$\left\{ \text{CRITICAL PTS: } x = 0, 1, -1 \quad \text{ENDPTS: } -1, 2. \right\}$ potential abs. max/min

$f(0) = -1, \quad f(1) = 0, \quad f(-1) = 0, \quad f(2) = (4-1)^3 = 27$

Absolute minimum pt $(0,-1)$. Absolute maximum pt $(2, 27)$.

67. $V(r) = K(r_0 - r)r^2$ on the interval $[\frac{1}{2}r_0, r_0]$.

r_0 and K are constants, positive.

a) Find the value of r in $[\frac{1}{2}r_0, r_0]$ when V has abs. max.

$$V'(r) = K(-1)r^2 + K(r_0 - r)(2r)$$

$$= K(-r^2 + 2r_0 r - 2r^2) = K(2r_0 r - 3r^2).$$

when is $V'(r) = 0$? $\quad 0 = K(2r_0 r - 3r^2) = Kr(\underbrace{2r_0 - 3r}_{})$.

$2r_0 - 3r = 0$

$r = \frac{2r_0}{3}$

CRITICAL PTS: $\underbrace{r = 0}_{}$ and $r = \frac{2r_0}{3}$

is not in the interval $[\frac{1}{2}r_0, r_0]$ b/c r_0 is the radius of the throat, so $r_0 > 0$.

potential abs max/min: $\frac{1}{2}r_0, \frac{2}{3}r_0, r_0$. ← endpts + cpts.

$V(\frac{1}{2}r_0) = K(r_0 - \frac{1}{2}r_0)(\frac{1}{2}r_0)^2$
$= K(\frac{1}{2}r_0)^3$
$= \frac{1}{8}Kr_0^3$

$\underbrace{V(\frac{2}{3}r_0) = K(r_0 - \frac{2}{3}r_0)(\frac{2}{3}r_0)^2 = \frac{4}{27}Kr_0^3}_{\text{absolute maximum}}$

$V(r_0) = K(r_0 - r_0)(r_0)^2 = 0$

$\boxed{V \text{ has abs max at } r = \frac{2}{3}r_0}$

This matches experiments!

b). $V(\frac{2}{3}r_0) = \frac{4}{27}Kr_0^3$

c) (far less important) (ungraded)

10. Verify the ftn satisfies the hypotheses of MVT. Then find all numbers c that satisfy the conclusion.

$f(x) = x^3 - 3x + 2$ on $[-2, 2]$

$(-\infty, \infty)$

1) f is continuous on $[-2, 2]$ b/c continuous everywhere.

2) f is differentiable on $(-2, 2)$ b/c differentiable $(-\infty, \infty)$.

Then there is a number c in $(-2, 2)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(-2)}{2 + 2}$$

$$= \frac{8 - 6 + 2 - (-8 + 6 + 2)}{4} = \frac{4 - 0}{4} = 1.$$

Find solutions to $f'(x) = 1$

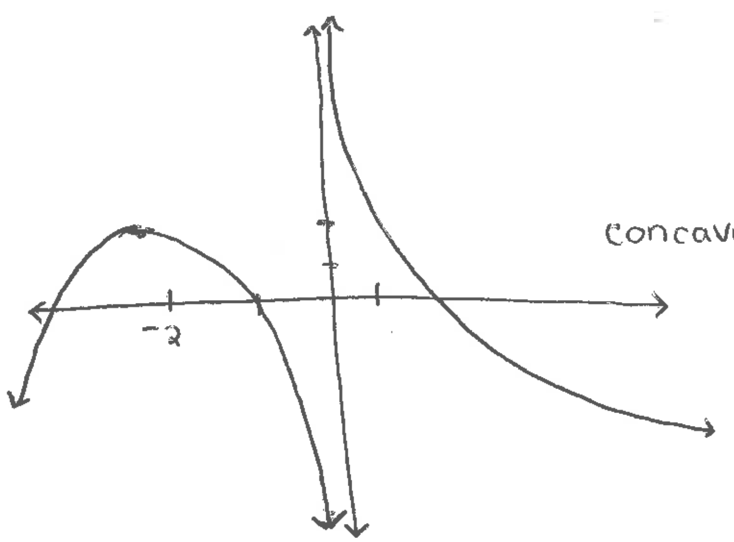
$3x^2 - 3 = 1$

$3x^2 = 4 \Rightarrow x^2 = 4/3$

The #'s c satisfying concl.

$\Rightarrow \boxed{c = 2/\sqrt{3},\ -2/\sqrt{3}}$

20.



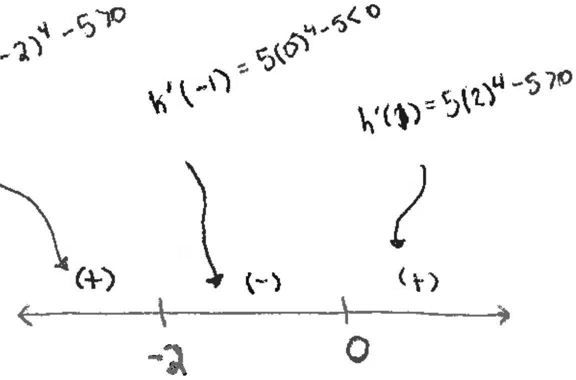
concave up and decreasing.

33. $h(x) = (x+1)^5 - 5x - 2$

CRITICAL PTS:

When is $h'(x) = 0$?

$$h'(x) = 5(x+1)^4 - 5 = 0$$
$$(x+1)^4 = 1$$
$$\Rightarrow x+1 = -1 \quad \text{or} \quad x+1 = 1$$
$$\boxed{x = -2} \qquad \boxed{x = 0}$$



a) h is increasing on $(-\infty, -2]$ and $[0, \infty)$

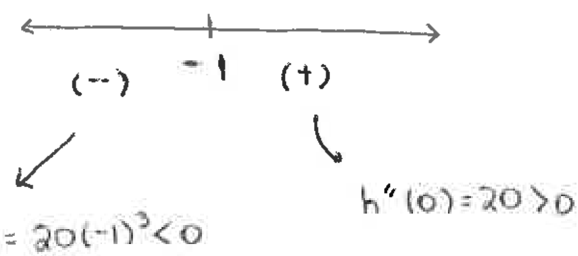
decreasing on $[-2, 0]$.

b) h has a local max at $x = -2$ pt $(-2, 7)$

a local min at $x = 0$ pt $(0, -1)$

concavity:

$$h''(x) = 20(x+1)^3 \; *$$
$$20(x+1)^3 = 0$$
$$\boxed{x = -1}$$

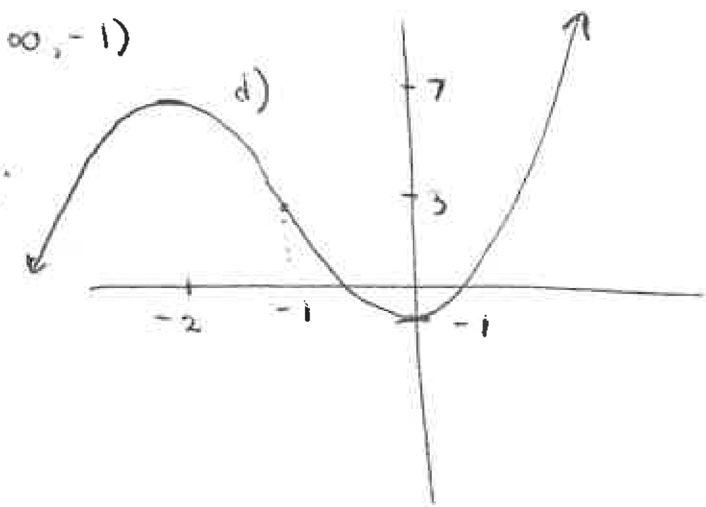


c) h is concave up on $(-1, \infty)$

and concave down on $(-\infty, -1)$

$x = -1$ is inflection point.

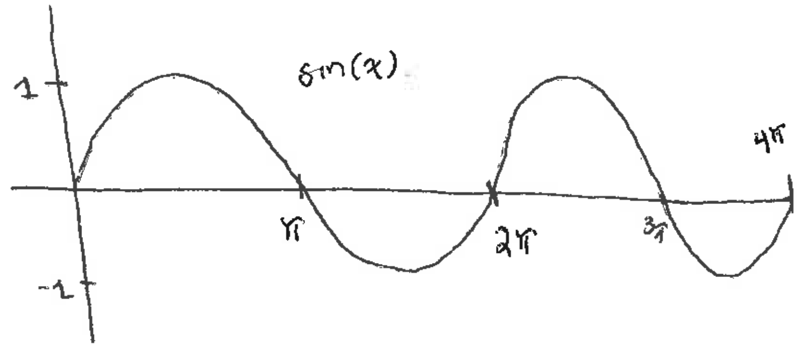
d)



For the function:

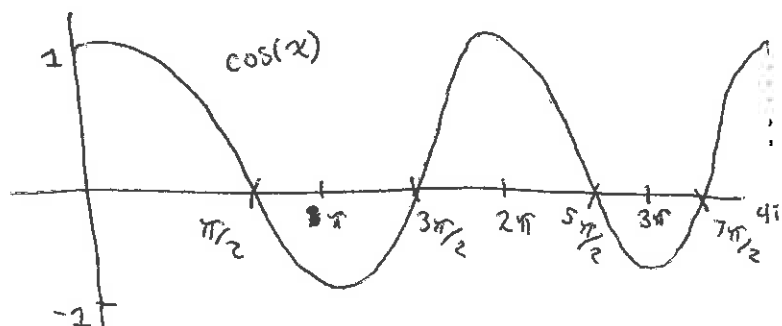
$f(x) = \sin(x)$.

a) graph f over $[0, 4\pi]$.



b) $f'(x) = \cos(x)$.

c) graph $f'(x)$ on $[0, 4\pi]$.



d) Local min/max of f? on $[0, 4\pi]$

$f'(x) = \cos(x) = 0$ (f' defined everywhere)

critical numbers: $x = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

2nd derivative test $f''(x) = -\sin x$.

$f''(3\pi/2) = f''(7\pi/2) = 1 > 0$

$\Rightarrow$ f is concave up at these pts

$\Rightarrow$ $x = 3\pi/2$ and $7\pi/2$ are local min.

$f''(\pi/2) = f''(5\pi/2) = -1 < 0$

$\Rightarrow$ f is concave down at these pts

$\Rightarrow$ $x = \pi/2$ and $5\pi/2$ are local max.

e) inflection pts.

$f''(x) = -\sin x$

$-\sin x = 0$

@ $x = , \pi, 2\pi, 3\pi$ (usually wouldn't say an endpt is inf pt b/c concavity doesn't change + to -, - to +).

0 (−) π (+) 2π (−) 3π (+)

inflection pts $x = \pi, 2\pi, 3\pi$