1. $f(x) = \dfrac{12x^2 + 36}{(x+3)^2}$

Vertical asymptote at $x = -3$.
Horizontal asymptote at $y = 12$.

y-intercept. $f(0) = \dfrac{36}{3^2} = 4$ pt. $(0,4)$

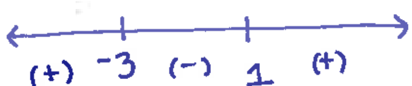
no x intercepts
$f(x) = 0$ has no solutions.

$$f'(x) = \frac{(x+3)^2(24x) - (12x^2+36)(2)(x+3)}{(x+3)^4}$$

$$= \frac{72x + 24x^2 - 24x^2 - 72}{(x+3)^3} = \frac{72(x-1)}{(x+3)^3}$$

critical numbers: $x = 1$ and $x = -3$

$x = 1$: $f'(x) = 0$; $x = -3$: f' undefined

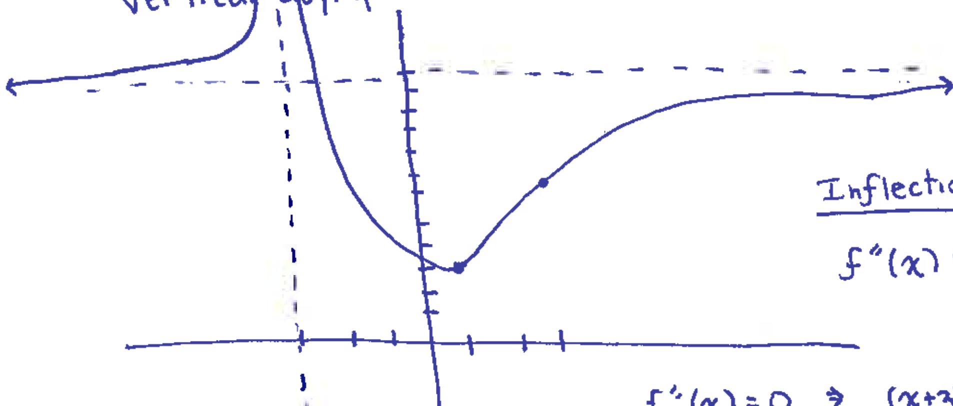


f is increasing $(-\infty, -3]$ and $[1, \infty)$
f is decreasing on $(-3, 1]$.

vertical asymptote

local min at pt $(1,3)$.

$f(1) = \dfrac{48}{16} = \dfrac{12}{4} = 3$



Inflection pts

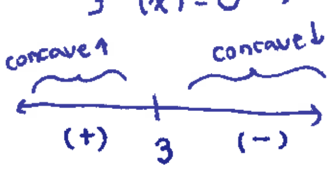
$$f''(x) = \frac{(x+3)^3(72) - 72(x-1)(3)(x+3)^2}{(x+3)^6}$$

$f''(x) = 0 \Rightarrow (x+3) - (x-1)(3) = 0$

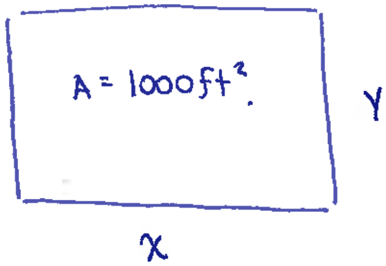
$x + 3 - 3x + 3 = 0$

$-2x + 6 = 0$

$\boxed{x = 3}$ inflection pt



2. Find the height and width of a rectangle with area $1000\ ft^2$ whose perimeter is minimized.



Area: $A = xy = 1000$
↑ restriction

$\Rightarrow y = \frac{1000}{x}$

Perimeter: $P = 2x + 2y$

write perimeter (which you want to minimize) in one variable:

$$p(x) = 2x + 2\left(\frac{1000}{x}\right)$$

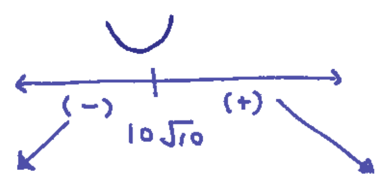
Find critical pts:

$$p'(x) = 2 - \frac{2000}{x^2} \quad \Rightarrow \quad 2 = \frac{2000}{x^2}$$

$$x^2 = 1000 \Rightarrow \boxed{x = 10\sqrt{10}}$$

why is this a mimimum?

① First derivative test:



implies a minimum.

$p'(2) = 2 - 500 < 0$

$p'(100) = 2 - \frac{2000}{10000} > 0$

And it's the only critical number $\Rightarrow$ ABSOLUTE MIN.

OR

② A reasonable domain is $[0, \infty)$, which is harder to check endpoints. But you can make this argument usually.

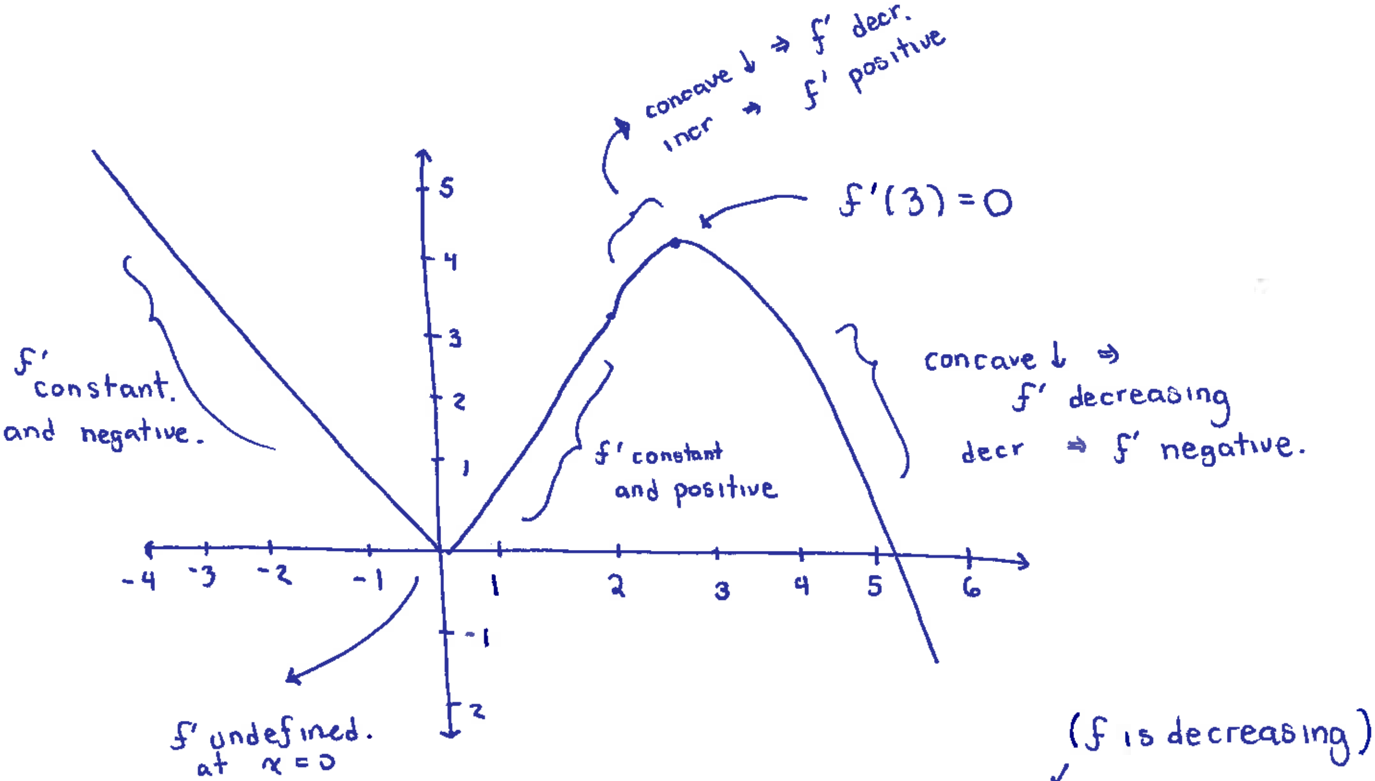
OR

③ Second derivative test.

$p''(x) = 2 \Rightarrow p$ is concave up everywhere

$\Rightarrow x = 10\sqrt{10}$ is a minimum. $\Rightarrow$ Absolute min b/c only critical pt.

3.



a) • f' is <u>negative</u> on the interval $(-4, 0)$. (f is decreasing)

• since f is a line (on this interval), f' is <u>constant</u>, (line ℓ has a constant slope)

$$f'(x) = -3/2 \text{ on this interval.}$$

• f' is <u>positive</u> on the interval $(0, 2)$. (f is increasing)

since f is a line on this interval, f' is <u>constant</u>

$$f'(x) = 3/2 \text{ on this interval.}$$

b) $f'(0)$ is undefined b/c the derivative does not exist @ corners.

c) f is increasing on $[0, 3]$, decreasing on $[-4, 0]$, and $[3, 6]$.

d) f is never concave up. f is concave down on $(2, 6]$.

e) f' is positive on $[0, 3]$, f' is negative on $[-4, 0]$ and $[3, 6]$.

f' is never increasing, f' is decreasing on $[2, 6]$.

f)