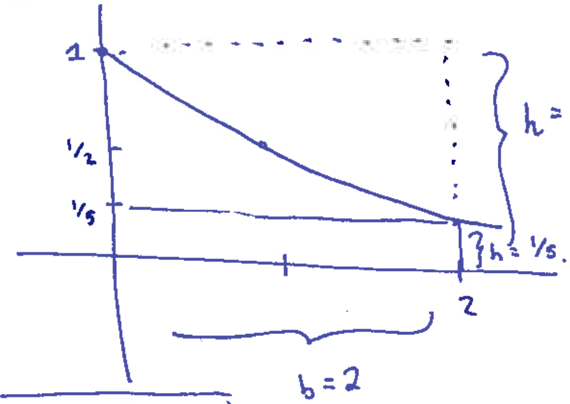
Weekly # 6

4.2 # 60, 71 4.3 # 40, 61, 4.5 # 55.

Review (p337) # 5 T/F 1, 3, 4, 14

60. $\displaystyle\int_0^2 \frac{1}{1+x^2}\,dx$



$f(x) = \dfrac{1}{1+x^2}$ has a minimum on $[0, 2]$

at $x = 2$, $f(2) = 1/5$.

Therefore, $\boxed{\displaystyle\int_0^2 \frac{1}{1+x^2}\,dx \geq \underbrace{2}_{\text{base}} \cdot \underbrace{(1/5)}_{\text{height.}} = 2/5}$

$f(x) = \dfrac{1}{1+x^2}$ has the maximum on $[0, 2]$ at $x = 0$, $f(0) = 1$.

Therefore $\boxed{\displaystyle\int_0^2 \frac{1}{1+x^2}\,dx < 2 \cdot 1 = 2}$

71. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{i^4}{n^5} = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\left(\frac{1}{n}\right)}_{\text{base.}} \overbrace{\underbrace{\left(\frac{i}{n}\right)}_{x_i^*,\text{ our sampling pts.}}^4}^{f(x_i^*)} = \lim_{n\to\infty} \sum_{i=1}^{\infty} \underbrace{\left(\frac{1}{n}\right)}_{\text{base}\,\cdot} \underbrace{f\left(\frac{i}{n}\right)}_{\text{height}}$

$\boxed{f(x) = x^4}$

$*$ sampling points

$x_1^* = \frac{1}{n}$

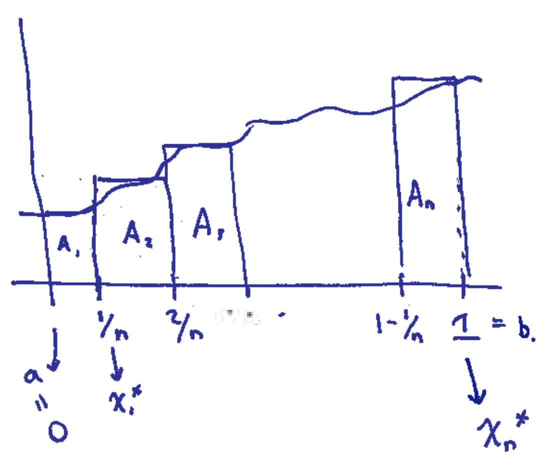
$x_2^* = \frac{2}{n}$

$x_3^* = \frac{3}{n}$

$\vdots$

$x_n^* = \frac{n}{n} = 1$

$\boxed{\displaystyle\int_0^1 x^4\,dx}$



40. $\int_{-1}^{2} \frac{4}{x^3}\,dx \neq -\frac{2}{x^2}\Big|_{-1}^{2} = \frac{3}{2}.$

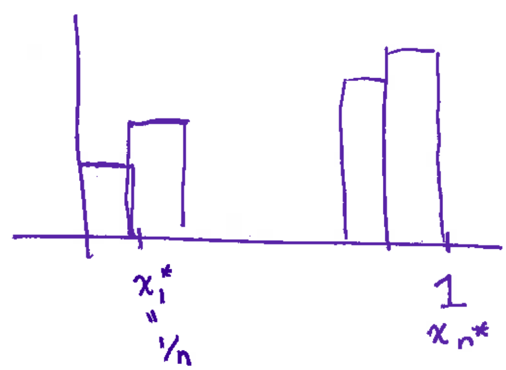
The function $f(x) = \frac{4}{x^3}$ has a vertical asymptote at $x = 0$. This means that f is not continuous there. Then we cannot apply the FTC b/c f is not continuous on the interval we want to integrate over.

61. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i^3}{n^3}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^3$

base $= \frac{1}{n}$. $f(x) = x^3$ sampling pts $x_i^* = \frac{i}{n}$.

$x_1^* = \frac{1}{n}$, $x_2^* = \frac{2}{n}$ ... $x_n^* = \frac{n}{n} = 1.$

$$\boxed{\int_0^1 x^3\,dx}$$



55. $\int_{-2}^{2} (x+3)\sqrt{4-x^2}\,dx$ $= 0 + 3(2\pi) = \boxed{6\pi}$

$= \underbrace{\int_{-2}^{2} x\sqrt{4-x^2}\,dx}_{A} + 3\underbrace{\int_{-2}^{2}\sqrt{4-x^2}\,dx}_{B \text{ a half circle.}}$

$u = 4-x^2$
$du = -2x\,dx$

A $= -\frac{1}{2}\int_{-2}^{2} -2x\sqrt{4-x^2}\,dx$

$= -\frac{1}{2}\int_0^0 \sqrt{u}\,du = \frac{-1}{2}\left(\frac{2}{3}(4-x^2)^{3/2}\right)\Big|_{-2}^{2}$

$= -\frac{1}{3}\left((4-2^2)^{3/2} - (4-(-2)^2)^{3/2}\right) = 0$

$y = \sqrt{4-x^2}$
$\Rightarrow x^2+y^2 = 4$
radius 2.
$A_{\frown} = \frac{1}{2}\pi r^2 = 2\pi$

5. A particle moves back and forth along a straight line with velocity $v(t)$ ft/sec.

a) $\int_{60}^{120} v(t)\,dt = p(120) - p(60)$. ($v(t)$ negative $\Rightarrow$ going backward).

is the displacement of the particle, it is the difference in the location of the particle between the time minute 1 and minute 2.

b) $\int_{60}^{120} |v(t)|\,dt$

is the total distance traveled by the partical in the time frame (60 seconds) from minute 1 to minute 2.

c). $\int_{60}^{120} a(t)\,dt = v(120) - v(60)$.

is the difference in velocity of the particle at $t = 120$ sec and $t = 60$ sec.

1. If f, g continuous, then $\int f + g\,dx = \int f\,dx + \int g\,dx$.

TRUE. Add areas.

3. If f is continuous on $[a,b]$, then

$$\int_a^b 5f(x)\,dx = 5\int_a^b f(x)\,dx$$

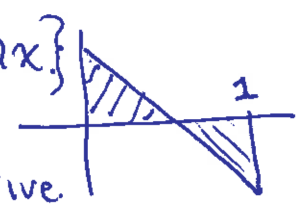
TRUE. It's just like you're scaling the y axis.

4. If f is continuous on $[a,b]$, then

$$\int_a^b x f(x)\,dx = x\int_a^b f(x)$$

FALSE. Take $f(x) = x$.

14. If $\int_0^1 f(x)\,dx = 0$, then $f(x) = 0$. $\{ f(x) = 1 - 2x \}$ $\Downarrow$



False Remember area below x-axis is negative.