Final Exam
Math 3
December 7, 2010

Name:____

On this final examination for Math 3 in Fall 2010, I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:____

Instructor (circle):

Lahr (Sec. 1, 8:45) Franklin (Sec. 2, 11:15) Diesel (Sec. 3, 12:30)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided. Take a moment now to print your name and instructor's name clearly on your Scantron form and booklet and sign the affirmation. You may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 30 multiple-choice problems worth 5 points each. Check to see that you have 15 pages of questions plus the cover page for a total of 16 pages.

1. Which of the following lists exactly the vertical asymptotes of the function $f(x) = \frac{e^{-x}}{x-3}$?

(a) $x = 3$

(b) $x = 0$

(c) $y = 3$

(d) $x = 0$ and $x = 3$

(e) none of the above

2. Calculate $\cos(\arcsin(-\frac{2}{5}))$.

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $-\frac{\sqrt{21}}{5}$

(d) $\frac{\sqrt{21}}{5}$

(e) none of the above

3. Consider the function $f(x) = x^3 - \frac{9}{2}x^2 + 6x + 4$. Which of the statements below does not show that the graph of $f(x)$ has a local minimum at $x = 2$?

(a) $f(x)$ is greater than $f(2)$ when x is in $[1, 2) \cup (2, 3]$.

(b) $f'(2) = 0$.

(c) $f'(x)$ is positive immediately to the right of 2 and negative immediately to the left of 2.

(d) $f'(2) = 0$ and $f''(2)$ is positive.

4. Calculate $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{3}+h\right) - \tan\left(\frac{\pi}{3}\right)}{h}$.

(a) $\frac{1}{4}$

(b) 4

(c) $\sqrt{3}$

(d) $4h$

(e) none of the above

5. If the derivative of a function $g(x)$ is $g'(x) = 2(\ln(x))^x$ and $g(e) = 1$, what is the equation of the tangent line to the graph of $y = g(x)$ at the point $(e, 1)$?

(a) $y = \frac{1}{e}x + 2$

(b) $y = 1$

(c) $y = 2x + 2e$

(d) $y = 2e^e x + (1 - 2e^{e+1})$

(e) none of the above

6. A juggler throws a ball into the air with an initial velocity of 19.6 meters per second from a height of 1 meter. Assuming that air resistance is negligible and that the acceleration due to gravity is 9.8 meters per second squared, when will the ball start to fall back to earth, and how high will it be then?

(a) 2 seconds, 20.6 meters

(b) 2 seconds, 19.6 meters

(c) 1 second, 14.7 meters

(d) 1 second, 15.7 meters

(e) none of the above

7. Calculate $\int \frac{x^2}{\sqrt{1-x^6}}\, dx$.

(a) $\frac{1}{5}x^3\sqrt{1-x^6} + C$

(b) $\frac{1}{3}\arccos(x^3) + C$

(c) $-\frac{1}{3}\arctan(x^3) + C$

(d) $\frac{1}{3}\arcsin(x^3) + C$

(e) none of the above

8. Let $f(x) = \sin(x)$ and $g(x) = x^3$ be considered on the restricted domain $[-1, 1]$. Which of the following statements is true?

(a) $f(x)$ has an inverse, but $g(x)$ does not.

(b) $g(x)$ has an inverse, but $f(x)$ does not.

(c) $f(x)$ and $g(x)$ both have inverses.

(d) Neither $f(x)$ nor $g(x)$ has an inverse.

9. Consider the function

$$h(x) = \begin{cases} x - 1 & x < 0 \\ \ln(x + 1) & x \geq 0 \end{cases}.$$

Which of the following statements is true?

(a) $h(x)$ is continuous at every point.

(b) $h(x)$ is discontinuous at $x = 0$, but the discontinuity is removable.

(c) $h(x)$ is discontinuous at $x = 0$, and the discontinuity is not removable.

(d) $h(x)$ is differentiable at every point.

(e) none of the above

10. The absolute maximum of the function $f(x) = 3x^2 - 2x$ on the interval $[-1, 0]$ occurs at

(a) $x = -1$.

(b) $x = -\frac{1}{3}$.

(c) $x = 0$.

(d) $x = \frac{1}{3}$.

(e) nowhere; this function has no maximum on the interval $[-1, 0]$.

11. Use the Trapezoid Rule with $n = 3$ to approximate $\int_1^4 (2^{x+1})\, dx$.

(a) 28

(b) 40

(c) 42

(d) 56

(e) none of the above

12. Evaluate the integral $\int \frac{x+3}{(x^2+6x)^4}\, dx$.

(a) $-\frac{1}{3}\frac{1}{(x^2+6x)^3} + C$

(b) $-\frac{1}{6}\frac{1}{(x^2+6x)^3} + C$

(c) $\frac{1}{(x^2+6x)^3} + C$

(d) $\frac{x^2+3x}{(x^3+3x^2)^5} + C$

(e) none of the above

13. Which definite integral below will give you the arclength along the curve $y = 5\sin(x)$ from $x = 0$ to $x = \pi$?

(a) $\int_0^\pi \sqrt{1 - 25\cos^2(x)}\; dx$

(b) $\int_0^\pi \sqrt{1 + 25\sin^2(x)}\; dx$

(c) $\int_0^\pi \sqrt{1 - 25\sin^2(x)}\; dx$

(d) $\int_0^\pi 1 + 25\cos^2(x)\; dx$

(e) none of the above

14. Find the area of the region bounded by the curves $y = \cos(x)$ and $y = \sin(x)$ on $[0, \frac{\pi}{2}]$.

(a) $\sqrt{2} - 1$

(b) $2\sqrt{2} - 2$

(c) $\pi - \sqrt{2}$

(d) 1

(e) none of the above

15. What is $\int (x+1)(x^2+2)\,dx$?

(a) $\frac{x^4}{4} + \frac{x^3}{3} + x^2 + 2x + C$

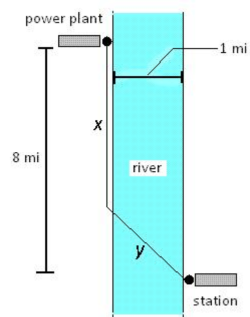
(b) $x^4 + x^3 + 2x^2 + 2x + C$

(c) $(\frac{x^2}{2} + x)(\frac{x^3}{3} + 2x) + C$

(d) $3x^2 + 2x + 2 + C$

(e) none of the above

16. A power plant sits on one bank of a river. It needs to be connected by power lines to a station on the other side of the river 8 miles downstream. The river is 1 mile wide and runs a straight course between the plant and the station. The cost to run power lines over water is \$130 per mile. The cost to run lines over land is \$120 per mile. What distance over land should the lines run to minimize the total cost?



(a) $5\frac{3}{5}$ miles

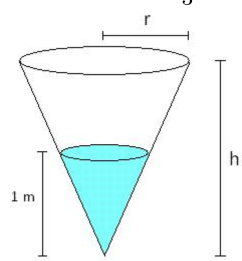
(b) $2\frac{2}{5}$ miles

(c) $2\frac{1}{2}$ miles

(d) 8 miles

(e) none of the above

17. A reservoir in the shape of a cone, vertex downward, is filled with water at a rate of 0.05 cubic meters per minute. The radius of the tank is one half its height. How fast is the height of water in the tank rising when the height of the water equals 1 m? (Recall that the volume of a cone with radius r and height h is $V = \frac{\pi}{3}r^2h$.)



(a) $\frac{3}{10\pi}$ meters per minute

(b) $\frac{1}{30\pi}$ meters per minute

(c) $\frac{1}{3\pi}$ meters per minute

(d) $\frac{1}{5\pi}$ meters per minute

(e) none of the above

18. Solve the initial value problem $\frac{dy}{dx} = \frac{2}{3}xy$, $y(0) = -1$.

(a) $y(x) = Ce^{\frac{x^2}{3}}$

(b) $y(x) = -e^{\frac{x^2}{3}}$

(c) $y(x) = e^{\frac{x^2}{3}}$

(d) $y(x) = \frac{2}{3}x - 1$

(e) none of the above

19. Let $f(x) = x^3 - 6x^2 + 12x - 21$. Which of the following three statements are true of $f(x)$?

I $x = 2$ is an inflection point of $f(x)$.

II $x = 2$ is a local maximum or local minimum of $f(x)$.

III There is a horizontal tangent line to the graph of $f(x)$ at $x = 2$.

(a) I only

(b) II only

(c) III only

(d) I and III

(e) II and III

20. The half life of Carbon-14 is 5730 years. What fraction of the original amount of Carbon-14 would be found in an artifact that is 25,000 years old?

(a) $(\ln(2))^{\frac{2500}{573}}$

(b) $\frac{1}{2^{\frac{2500}{573}}}$

(c) $2^{\frac{2500}{573}}$

(d) $e^{-\frac{2500}{573}}$

(e) none of the above

21. Estimate $\cos^2(\frac{11\pi}{40})$ using linearization starting with the point $x = \frac{\pi}{4}$.

(a) $\frac{1}{2} - x + \frac{\pi}{4}$

(b) $\frac{1}{2}x + \frac{\pi}{40}$

(c) $\frac{1}{2} - \frac{\pi}{40}$

(d) $-2\cos\left(\frac{11\pi}{40}\right) \cdot \sin\left(\frac{11\pi}{40}\right) + \frac{1}{2}$

(e) none of the above

22. Find the derivative of $y = x^{\sin(x)}$.

(a) $y' = \sin(x) \cdot x^{\sin(x)-1} \cdot \cos(x)$

(b) $y' = x^{\cos(x)}\left(\frac{\sin(x)}{x} + \ln(x)\right)$

(c) $y' = x^{\sin(x)}\left(x\cos(x) + \sin(x)\right)$

(d) $y' = x^{\sin(x)}\left(\frac{\sin(x)}{x} + \ln(x)\cos(x)\right)$

(e) none of the above

23. Which describes $g(x) = 4\sin(x + \frac{\pi}{4}) + 1$ when it is restricted to the domain $[0, \pi]$? (Be careful!)

(a) $g(x)$ has an absolute maximum at $x = \frac{\pi}{4}$ and an absolute minimum at $x = \pi$.

(b) $g(x)$ has an absolute maximum at $x = 5$ and an absolute minimum at $x = -3$.

(c) $g(x)$ has an absolute maximum at $x = \frac{\pi}{2}$ and absolute minimums at $x = 0$ and $x = \pi$.

(d) $g(x)$ has an absolute maximum at $x = 5$ and no absolute minimum.

(e) none of the above

24. Simplify $\int_{-a}^{a}(\sec(x) + x^3)\,dx$.

(a) $2\int_0^a x^3\,dx$

(b) $2\int_0^a \sec(x)\,dx$

(c) $\frac{x^4}{2} + C$

(d) $2\tan(a)\sec(a) + C$

(e) none of the above

25. Evaluate $\frac{d}{dx}\int_1^{x^2} \ln(3t - t^2)\, dt$.

(a) $\frac{6x-4x^3}{3x^2-x^4}$

(b) $\ln(3 - 2x^2)$

(c) $\ln(3x^2 - x^4) - \ln(2)$

(d) $2x\ln(3x^2 - x^4)$

(e) none of the above

26. Consider $\Phi = \int_a^b f(x)\ dx$. What does the Fundamental Theorem of Calculus state about this integral if $f(x)$ is continuous on $[a, b]$?

(a) There is a c in $[a, b]$ such that $\Phi = f(c) \cdot (b - a)$.

(b) There is a c in $[a, b]$ such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.

(c) If $F(x)$ is any antiderivative of $f(x)$, then $\Phi = F(b) - F(a)$.

(d) Φ is equal to the area under the curve $f(x)$ and above the x-axis.

(e) none of the above

27. Choose the definite integral below that is equal to

$$\lim_{n\to\infty}\sum_{j=1}^{n}\frac{3}{n}\sqrt{4+\frac{3(j-1)}{n}}.$$

(a) $3\int_4^5 \sqrt{x}\,dx$

(b) $\int_4^7 \sqrt{x}\,dx$

(c) $\int_1^4 \sqrt{x}\,dx$

(d) $\int_1^n \sqrt{4+\frac{3}{n}}\,dx$

(e) none of the above

28. Evaluate $\lim_{x\to-\infty}\frac{2x^2}{\sqrt{x^2-x}}$.

(a) $+\infty$

(b) $-\infty$

(c) 2

(d) $\sqrt{2}$

(e) none of the above

29. Evaluate $\int \left(\sec^2(x) - \sec(x)\tan(x)\right)\, dx$.

(a) $\frac{1}{\cos(x)} - \frac{1}{\sin(x)} + C$

(b) $\frac{\sin(2x)}{2} + C$

(c) $\tan(x) - \cot(x) + C$

(d) $\cot(x) - \tan(x) + C$

(e) none of the above

30. Evaluate $\frac{d}{dx}\frac{\arctan(x)}{x}$.

(a) $\frac{1}{1+x^2}$

(b) $\frac{1}{x+x^3} - \frac{\arctan(x)}{x^2}$

(c) $\frac{1}{x\sqrt{1-x^2}} - \frac{\arctan(x)}{x^2}$

(d) $-\frac{1}{(x\arccos(x))^2} - \frac{\arctan(x)}{x^2}$

(e) none of the above

END OF EXAM